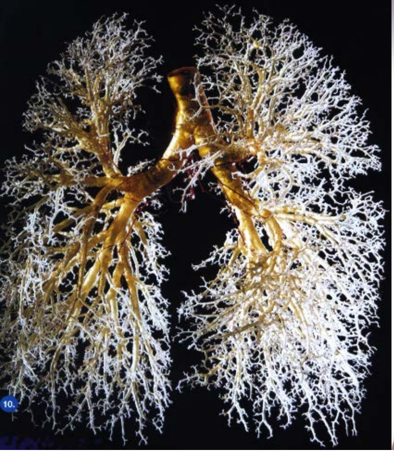



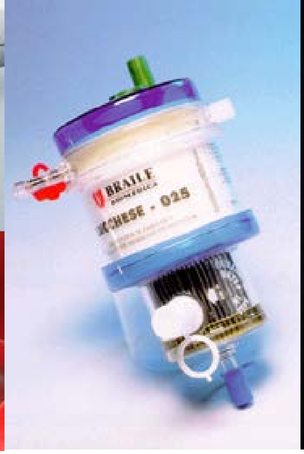


Biological and Medical Fluid Mechanics I

3. Rheology of blood

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Lecture contents

3. Rheology of blood	Exercise
3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
3.2 Viscosity of blood 3.2.1 Viscosity measurement methods 3.2.2 Viscosity models for blood plasma and whole blood 3.2.3 Influences on the viscosity of blood	
3.3 Non-Newtonian blood analog fluid 3.3.1 Requirements and composition 3.3.2 Xanthan gum 3.3.3 Influence of temperature	

3.1.1 Definitions

- dynamic viscosity = viscosity coefficient of the fluid
- derivation for a **Newtonian fluid**: flow between two infinitely extended flat plates, lower plate is fixed, upper plate can move

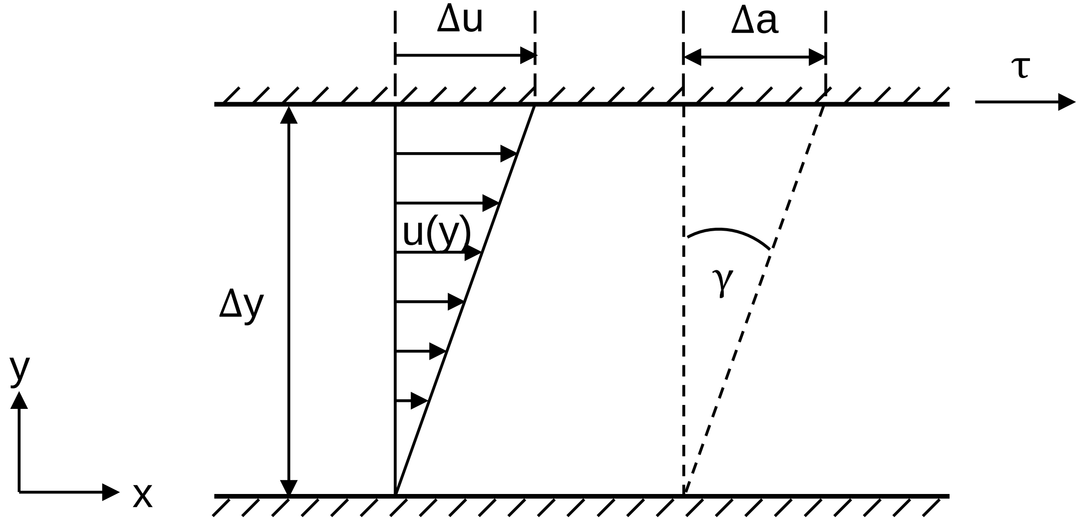

Fig. 3.1: Derivation of the dynamic viscosity

$\rightarrow$ for small deformations: $\tan(\gamma) \approx \gamma = \dfrac{\Delta a}{\Delta y}$

3.1.1 Definitions

$\rightarrow$ with $\Delta a = \Delta u \cdot dt$;

$$\Rightarrow \dot{\gamma} = \frac{d\gamma}{dt} = \frac{\Delta u \cdot dt}{\Delta y \cdot dt} = \frac{\Delta u}{\Delta y} = \frac{du}{dy} \quad \text{with } \dot{\gamma}\text{: shear velocity}$$

$\rightarrow$ with $\tau \backsim \Delta u$ and $\tau \backsim \frac{1}{\Delta y}$: $\Rightarrow \tau \backsim \frac{\Delta u}{\Delta y} \quad \left[= \frac{du}{dy} = \dot{\gamma} \right]$

- the shear stress is proportional to the shear rate: $\rightarrow \tau \backsim \dot{\gamma} \quad \Rightarrow \boxed{\tau = \eta \cdot \dot{\gamma}}$

  with η: dynamic viscosity

- unit of the dynamic viscosity: $[\eta] = \frac{[\tau]}{\left[\frac{\partial w}{\partial y}\right]} = \frac{g / (cm \cdot s^2)}{cm / (cm / s)} = \frac{g}{cm \cdot s} = 1$ Poise

- the normal stresses are identical in all directions and are equal to the static pressure: $\sigma_x = \sigma_y = \sigma_z = p$

3.1.1 Definitions

- for **non-Newtonian fluids**: $\tau \nsim \dot{\gamma}$

  → there is no proportionality factor such as the dynamic viscosity

  → to characterize the flow properties: flow curve $\tau = f(\dot{\gamma})$

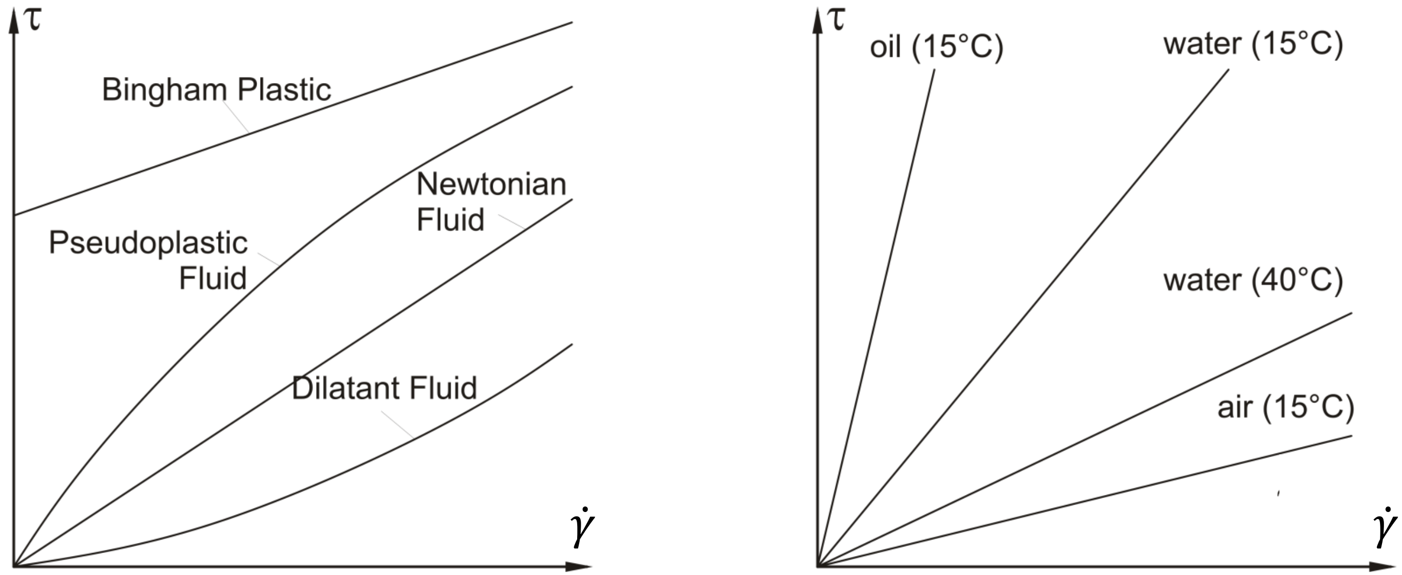

Fig. 3.2: Flow curves for different fluid types

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3.1.1 Definitions

- **visco-elastic fluids (non-Newtonian)**
  - the parallel layer flow can be described by a flow curve
  - the normal stresses depend on the direction:

$$\sigma_x = -p^* + f_1(\dot{\gamma}^2) \qquad \sigma_y = -p^* + f_2(\dot{\gamma}^2) \qquad \sigma_z = -p^*$$

    $\rightarrow$ with $p^* = f(\dot{\gamma})$ describing the isotropic part of the normal stress

    $\rightarrow$ for $\dot{\gamma} = 0$, this part is equal to the hydrostatic pressure p

  - for $\dot{\gamma} = 0$: particles or molecules (e.g. RBCs) are suspended in the fluid with an arbitrary orientation

3.1 Classification of blood

3.1.1 Definitions

- for $\dot{\gamma} \neq 0$: orientation phenomena can be observed
- when the shear stress returns to zero, the particles again are distributed with an arbitrary orientation $\rightarrow$ elastic effect
- in a shear field a reversible deformation of the particles can occur

$\rightarrow$ elastic effects can be neglected!

$\rightarrow$ In many cases: <u>sufficient to determine the flow curve of the non-Newtonian fluid</u>!

- pronounced, characteristic decrease of the pipe friction coefficient in a turbulent pipe flow

Lecture contents

3. Rheology of blood	Exercise
3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
3.2 Viscosity of blood 3.2.1 Viscosity measurement methods 3.2.2 Viscosity models for blood plasma and whole blood 3.2.3 Influences on the viscosity of blood	
3.3 Non-Newtonian blood analog fluid 3.3.1 Requirements and composition 3.3.2 Xanthan gum 3.3.3 Influence of temperature	

3.1.2 Flow behavior of suspensions

- blood is not a pure liquid → bearer fluid (blood plasma) with suspended blood corpuscles (flexible)

⇒ ***blood = suspension of flexible particles of high concentration***

- different models:

1. **Suspension of stiff spheres in a Newtonian fluid**
2. **Suspension with non spherical stiff particles (e.g. ellipsoids) in a shear field**
3. **Drops in a shear field**

3.1.2 Flow behavior of suspensions

1. Suspension of stiff spheres in a Newtonian fluid

- question: do stiff spheres influence the viscosity of the suspension?

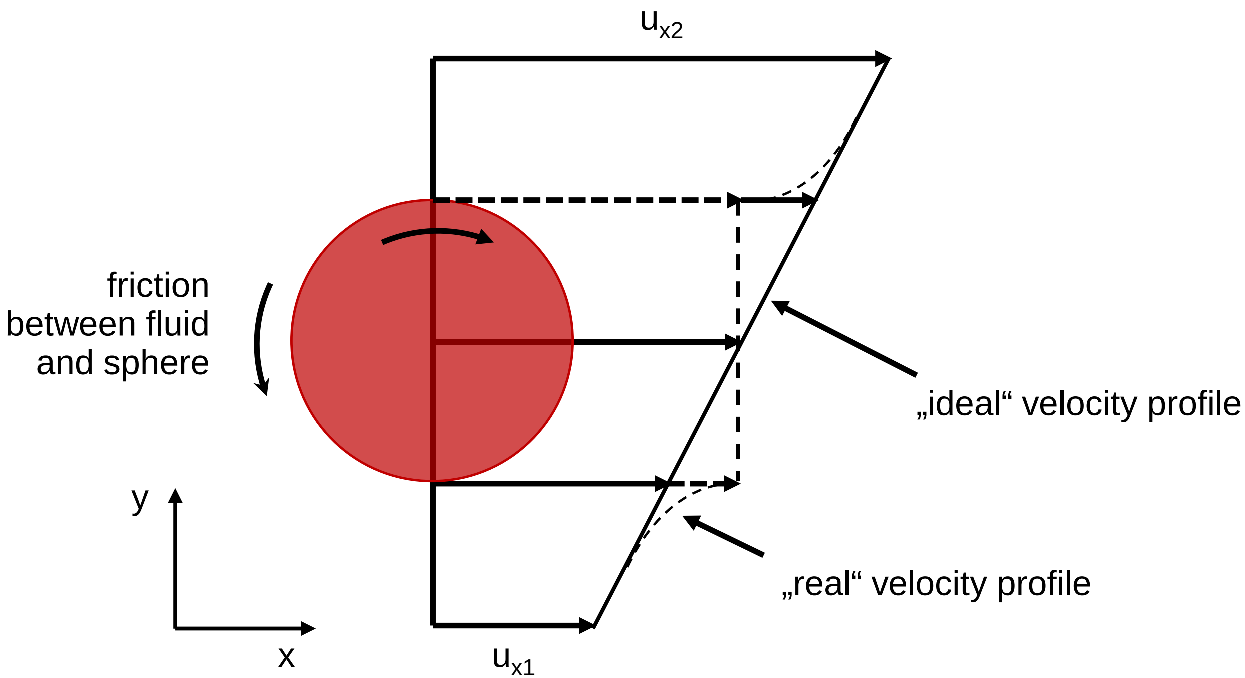

Fig. 3.3: Effect of a stiff sphere to the velocity gradient in a shear field

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3.1.2 Flow behavior of suspensions

- due to friction exerted by the fluid on the sphere, the flow at points A and B can't move around the sphere with the predetermined distribution profile
  - → dashed velocity distribution profile (steeper velocity gradient between A and B)
- result: to produce a velocity difference $u_{x2} - u_{x1}$ higher shear stresses are necessary compared to a homogenous fluid (without a sphere)
  - → resistance to deformation of a suspension ("s") with embedded stiff spheres is higher than that of a homogenous fluid ("fl"): $\eta_s > \eta_{fl}$

3.1.2 Flow behavior of suspensions

- **low particle concentration C:**
  - relative volume fraction of the spheres ≤ 1% (i.e., no interaction between the spheres)

    → $\eta_s = \eta_0 \cdot (1 + 2.5C)$
  - flow behavior remains Newtonian

- **medium particle concentration C:**
  - $1\% < C \leq 30\%$

    → $\eta_{rel} = \eta_s / \eta_0 = 1/(1 + 2.5C) = 1 + 2.5C + 6.25C^2 + \ldots$ (Oliver, 1953)
  - η_{rel} increases quickly with concentration C
  - flow behavior still remains Newtonian up to C = 40%

3.1.2 Flow behavior of suspensions

2. Suspension with non-spherical particles (e.g. Ellipsoids) in a shear field

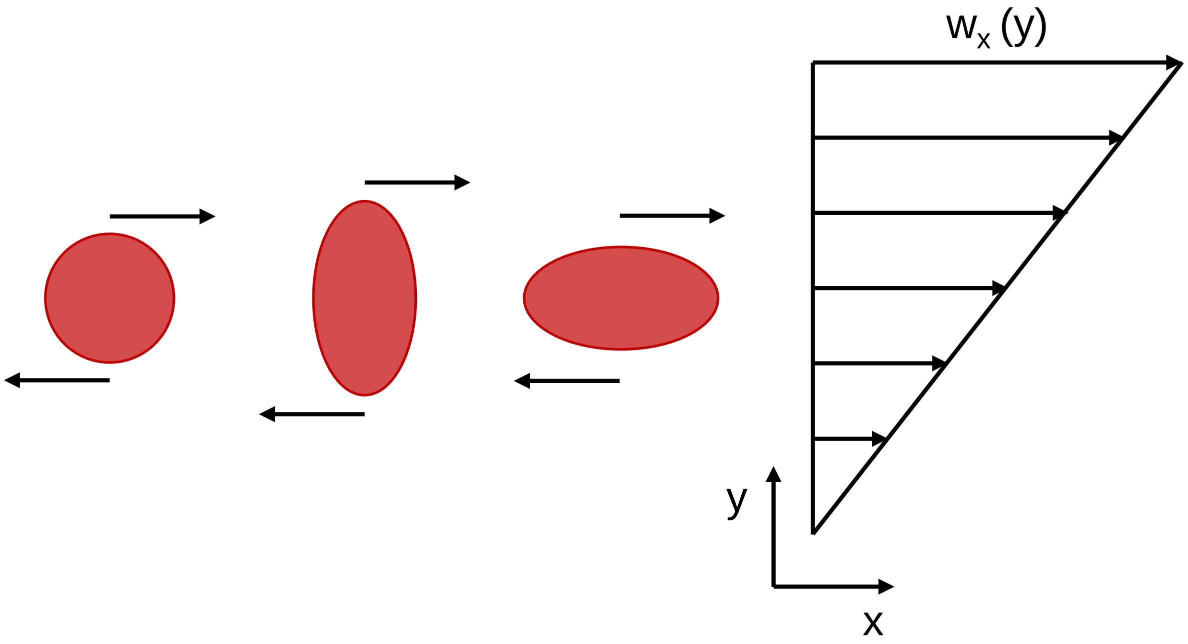

Fig. 3.4: Possible orientations of ellipsoids in a shear field

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3.1.2 Flow behavior of suspensions

- particles are mostly oriented s.t. the long axis is parallel to the shear plane
- rotation of the Ellipsoids in the shear field is hindered strongly
- occasionally a turn down process takes place
  - → increases impulse transport in y-direction
  - → steeper velocity gradients in the fluid
  - → higher viscosities

- **for low concentrations:**
  - → $\eta_{rel} = 1 + K_1 \cdot C$ with $K_1 > 2{,}5$ (grows with asymmetry)
- for small particles (<1 µm) the Brownian movement plays a role

3.1.2 Flow behavior of suspensions

3. Behavior of drops in a shear field

- equilibrium between:
  - → tension on the surface borders
  - → pressure distribution ΔP_G due to dynamic flow relations in and around the drop
  - → deformation of the drop
- increase or decrease of the viscosity η_{rel} depends on the viscosity relation between the carrier fluid and the drop

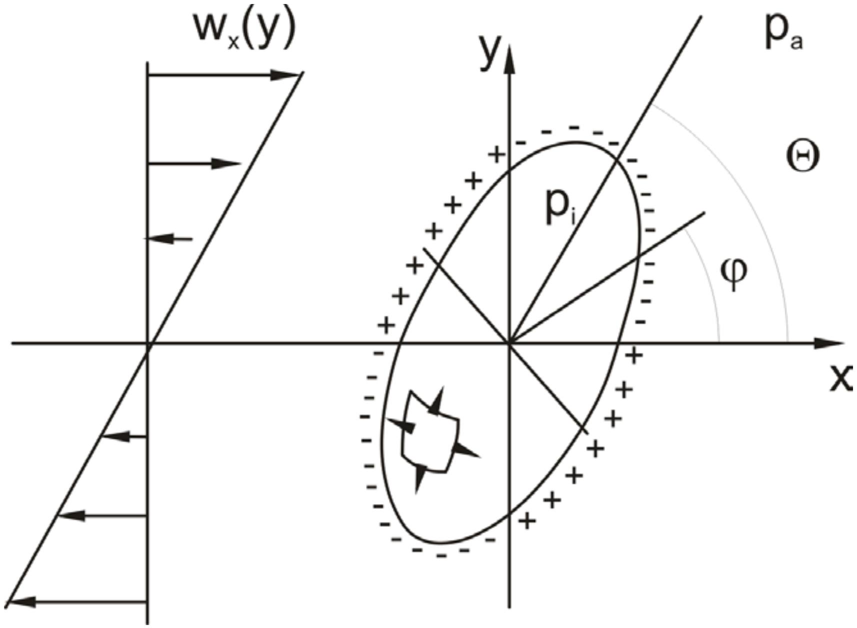

Fig. 3.5: Drop in a shear field

Lecture contents

3. Rheology of blood	Exercise
3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
3.2 Viscosity of blood 3.2.1 Viscosity measurement methods 3.2.2 Viscosity models for blood plasma and whole blood 3.2.3 Influences on the viscosity of blood	
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3.1.3 Blood as a suspension of flexible particles

1. **Carrier fluid:**
   - plasma = aqueous solution of macromolecules (proteins)

     → macromolecules orient and deform under shear forces

     → configuration changes depending on shear velocity

     → viscosity change of the solution depends on shear velocity

     => **non-Newtonian** flow behavior!

3.1.3 Blood as a suspension of flexible particles

2. Suspension of erythrocytes in blood plasma:

- remaining blood corpuscles can be neglected from a rheological point of view
- main characteristics:
  - particle concentration C > 40% (Hct = 45%)
  - particles can not be considered as spheres

    $\rightarrow$ orientation phenomena, turn-over processes
  - particle shape depends on the shear stress
  - membrane can rotate around the viscous cell content (see Bull, Fischer)
  - Behavior of a single RBC in the shear field is similar to that of a drop in a liquid of low viscosity (see Taylor)

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3.1.3 Blood as a suspension of flexible particles

Particle behavior as a function of the shear stress

1. **At rest (no shear stress)**
   - aggregation:
     - → money-roll formation
     - → network formation
   - aggregation depends on proteins in the plasma (e.g. by infections)

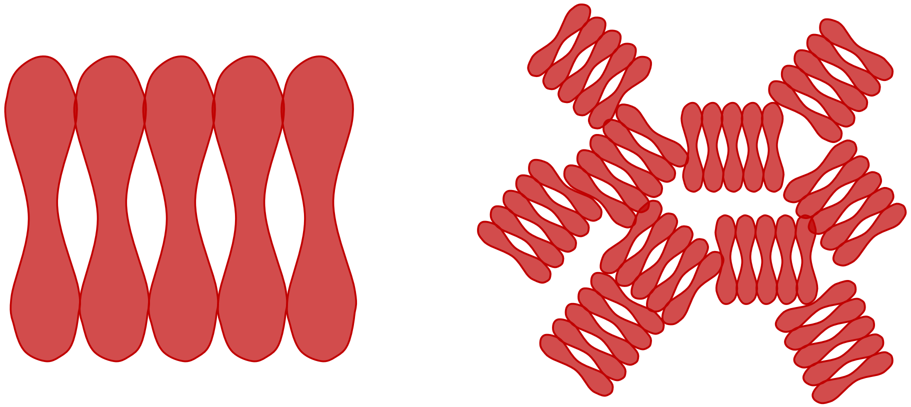

Fig. 3.6: Money roll formation (left) and network formation (right) of RBCs

3.1.3 Blood as a suspension of flexible particles

Particle behavior as a function of the shear stress

2. **Low shear stresses**
   - aggregates are torn apart
     → high viscosity
   - blood cells still behave as stiff discs
     and perform corresponding librations
     → relatively high viscosity

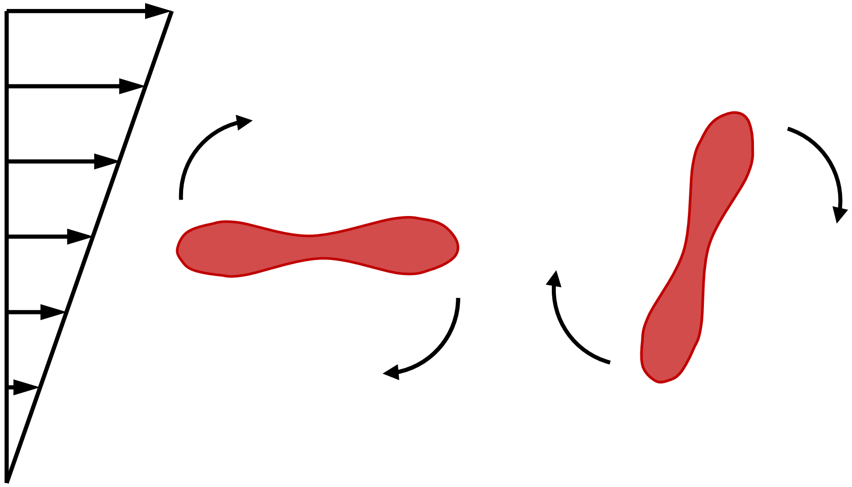

Fig. 3.7: Tumbling of the RBC when a shear is applied to the flow

3.1.3 Blood as a suspension of flexible particles

Particle behavior as a function of the shear stress

3. **Higher shear stresses**
   - starting deformation and alignment in the shear field
   - membrane rotates around the cell content

   → viscosity decreases

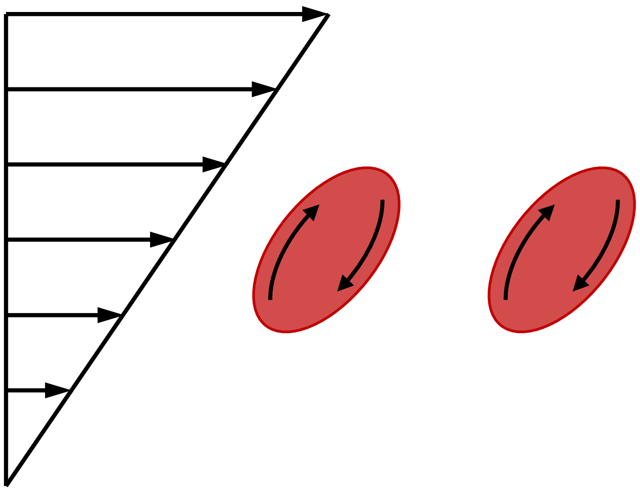

Fig. 3.8: Rotation of the membrane of the RBC around its content

3.1.3 Blood as a suspension of flexible particles

Particle behavior as a function of the shear stress

4. **Further increase of shear stress**
   - stronger deformation towards slim bodies until maximum deformation degree is reached (depending on surface/volume-relation)

     → then: no further decrease of viscosity

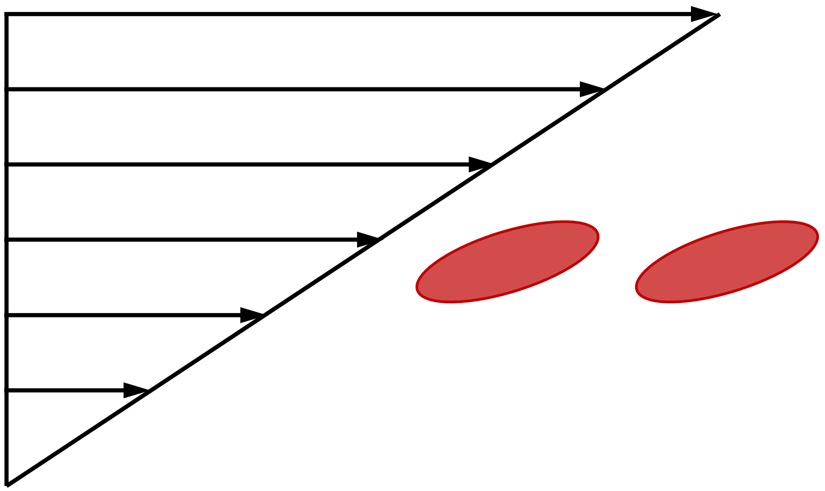

Fig. 3.9: Stronger deformation towards slim bodies

→ whole blood possesses a viscosity depending on the shear rate: $\eta = f(\dot{\gamma})$

→ **non-Newtonian** flow behavior!

Lecture contents

3. Rheology of blood	Exercise
3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
3.2 Viscosity of blood 3.2.1 Viscosity measurement methods 3.2.2 Viscosity models for blood plasma and whole blood 3.2.3 Influences on the viscosity of blood	
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3.2.1 Viscosity measurement methods

1. Sphere drop viscosimeter

- **balance of forces** for a constant drop velocity of the sphere: $\sum F = G - L - D = 0$
  - **G**: weight $\quad G = \rho_s \cdot g \cdot \frac{4}{3} \cdot \pi \cdot \frac{d^2}{8}$
  - **L**: lift (Archimedes) $\quad L = \rho_F \cdot g \cdot \frac{4}{3} \cdot \pi \cdot \frac{d^2}{8}$
  - **D**: drag $\quad D = c_D \cdot \pi \cdot \frac{d^2}{4} \cdot \frac{\rho_F}{2} \cdot v^2$

$$\rightarrow \boxed{c_D = \frac{3 \cdot g \cdot d}{2 \cdot \rho_F \cdot v^2} \cdot (\rho_s - \rho_F)}$$

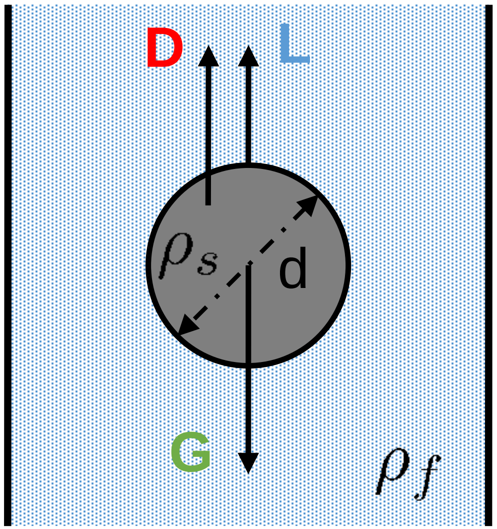

Fig. 3.10: Sphere drop viscosimeter

3.2.1 Viscosity measurement methods

- **Stokes:** relationship between Reynolds-Number and Drag coefficient of a sphere in a **Newtonian fluid**:

$$\rightarrow \boxed{c_D = \frac{24}{Re} = \frac{24 \cdot \eta}{\rho_F \cdot d \cdot v}}$$

with the previous result:

$$\rightarrow \boxed{\eta = \frac{g \cdot d^2}{16 \cdot v} \cdot (\rho_s - \rho_F)}$$

→ limitation: sphere drop viscosimeter can **only** be used for a **Newtonian fluid**!

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3.2.1 Viscosity measurement methods

2. Capillary viscosimeter

- assumptions:
  - laminar layer flow
  - steady flow: $v \neq f(t)$
  - constant pressure gradient

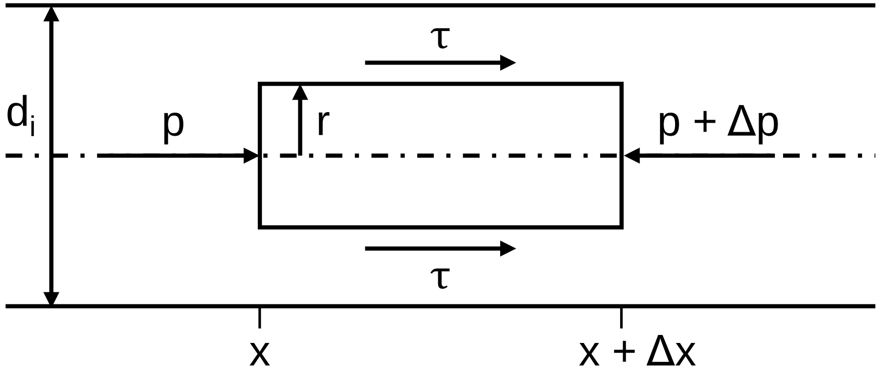

Fig. 3.11: Capillary viscosimeter

- **balance of forces for an infinitesimal volume element:** $\sum F = 0$

$$\rightarrow -(p + \Delta p) \cdot \pi \cdot r^2 + p \cdot \pi \cdot r^2 + \tau \cdot 2 \cdot \pi \cdot r \cdot \Delta x = 0$$

$$\rightarrow \boxed{\tau = \frac{\Delta p}{\Delta x} \cdot \frac{r}{2}}$$: this equation does not contain any information about the fluid!

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3.2.1 Viscosity measurement methods

- **<u>Newtonian fluid</u>:** $\boxed{\tau = \eta \cdot \frac{du}{dr}}$ $\rightarrow$ $\boxed{du = \frac{1}{2\eta} \cdot \frac{\Delta p}{\Delta x} \cdot r \cdot dr}$
- integration: $u(r) = \frac{1}{2\eta} \cdot \frac{\Delta p}{\Delta x} \cdot \frac{r^2}{2} + C$
- boundary condition: $u(r = R) = 0$

$$\rightarrow C = -\frac{1}{2\eta} \cdot \frac{\Delta p}{\Delta x} \cdot \frac{R^2}{2}$$

- finally: $\boxed{u(r) = \frac{1}{4\eta} \cdot \frac{\Delta p}{\Delta x} \cdot (r^2 - R^2)}$
- **flow rate:** $\dot{Q} = \int_0^R 2\pi \cdot r \cdot u(r) \cdot dr$

3.2.1 Viscosity measurement methods

$$\rightarrow \dot{Q} = \int_0^R 2\pi \cdot r \cdot \frac{1}{4\eta} \cdot \frac{\Delta p}{\Delta x} \cdot (r^2 - R^2) \cdot dr = \frac{\pi}{2\eta} \cdot \frac{\Delta p}{\Delta x} \int_0^R (r^3 - rR^2) \cdot dr$$

- hence:

$$\boxed{\dot{Q} = \frac{\pi}{2\eta} \cdot \frac{\Delta p}{\Delta x} \left[\frac{r^4}{4} - \frac{r^2}{2} \cdot R^2\right]_0^R}$$

- **Hagen-Poiseuille (1840):**

$$\dot{Q} = -\frac{\pi}{8\eta} \cdot \frac{\Delta p}{\Delta x} \cdot R^4$$

$$\rightarrow \boxed{\eta = \frac{\pi}{8\dot{Q}} \cdot \frac{\Delta p}{\Delta x} \cdot R^4}$$

- given parameter: R (geometry)
- to measure: $\dot{Q}$, $\frac{\Delta p}{\Delta x}$

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3.2.1 Viscosity measurement methods

- **non-Newtonian fluid**: $\boxed{\tau = f(\dot{\gamma})}$
- maximum shear stress is reached at the wall r = R (wall shear stress): $\boxed{\tau_w = \frac{\Delta p}{\Delta x} \cdot \frac{R}{2}}$
- **flow rate:** $\dot{Q} = \int_0^R 2\pi \cdot r \cdot u(r) \cdot dr = 2\pi \int_0^R r \cdot u(r) \cdot dr = 2\pi \left[u(r) \left[\frac{r^2}{2}\right]_0^R - \int_0^R \frac{r^2}{2} du(r) \right]$
- boundary conditions: $u(r = R) = 0$ and $u(r = 0) \neq \infty$

$\rightarrow$ $\boxed{\dot{Q} = -2\pi \int_0^R \frac{r^2}{2}\, du(r) = -\pi \int_0^R r^2 \frac{du(r)}{dr} dr}$

- substitutions: 1.) $-\frac{du(r)}{dr} = F(\tau)$ 2.) $\frac{\Delta p}{\Delta x} = p'$ 3.) $r = R\frac{\tau}{\tau_w} \rightarrow dr = \frac{R}{\tau_w} d\tau \rightarrow dr = \frac{2}{p'} d\tau$

3.2.1 Viscosity measurement methods

- finally: $$\boxed{\dot{Q} = -\pi \cdot \int_0^{\tau_w} R^2 \frac{\tau^2}{\tau_w^2} (-F(\tau)) \frac{2}{p'} \, d\tau = \frac{2\pi R^2}{\tau_w^2 p'} \cdot \int_0^{\tau_w} \tau^2 F(\tau) \, d\tau}$$
- substitution: $$\frac{2}{p'} = \frac{R}{\tau_w}$$

$\rightarrow$ $$\boxed{\dot{Q} = \frac{\pi R^3}{\tau_w^3} \cdot \int_0^{\tau_w} \tau^2 F(\tau) \, d\tau}$$

- substitution: $$\Phi = \frac{\dot{Q}}{\pi R^3}$$

$\rightarrow$ $$\boxed{\Phi \cdot \tau_w^3 = \int_0^{\tau_w} \tau^2 F(\tau) \, d\tau}$$

- derivative: $$\boxed{\frac{d\Phi}{d\tau_w} \cdot \tau_w^3 + 3 \cdot \Phi \cdot \tau_w^2 = \tau_w^2 \cdot F(\tau_w)}$$

3.2.1 Viscosity measurement methods

with: $F(\tau_w) = -\dfrac{du(R)}{dr} = \dot{\gamma}_w$

$$\rightarrow \boxed{-\dot{\gamma}_w = F(\tau_w) = \frac{d\Phi}{d\tau_w} \cdot \tau_w + 3 \cdot \Phi}$$

Rabinowitsch-Mooney

- correlation between $\dot{\gamma}_w$ and τ_w

- to measure: $\dot{Q}, \dfrac{\Delta p}{\Delta x}$
- yields the curve: $\boxed{\Phi = f(\tau_w)}$

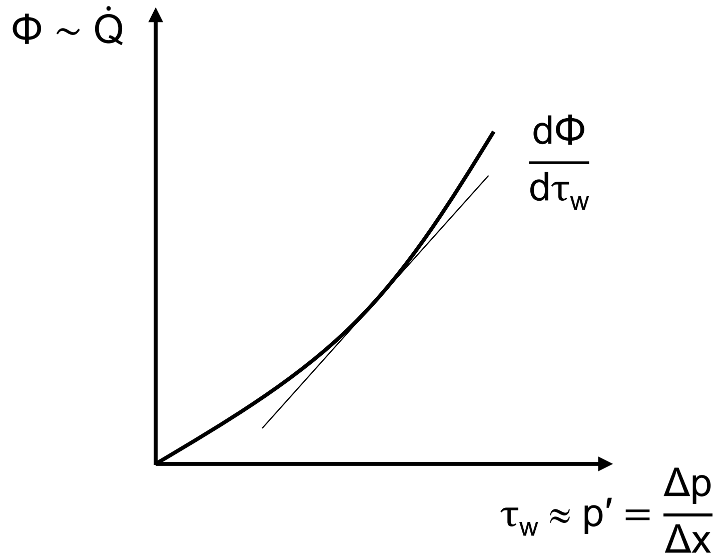

Fig. 3.12: Rabinowitsch-Mooney

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3.2.1 Viscosity measurement methods

3. Cone-Plate-Rheometer

- technically complex
+ easy handling, also for **non-Newtonian fluids**
• **Newtonian fluid:** viscosity is determined by the (constant) gradient of the flow curve
• **non-Newtonian fluid:** viscosity must be reconstructed from the flow curve using pointwise measurements
- secondary flow phenomena for higher angular velocities
  $\longrightarrow$ precise instrument is necessary
  $\longrightarrow$ high costs

3.2.1 Viscosity measurement methods

- in the gap:
  - constant angular velocity: Ω=const.
  - steady, parallel flow: $\boxed{\dot{\gamma} = \frac{v_\phi}{h} = \frac{\Omega \cdot r^*}{H}}$
  - with: $\boxed{\frac{H}{r^*} = \tan\alpha}$ $\rightarrow$ $\boxed{\dot{\gamma} = \frac{\Omega}{\tan\alpha}}$

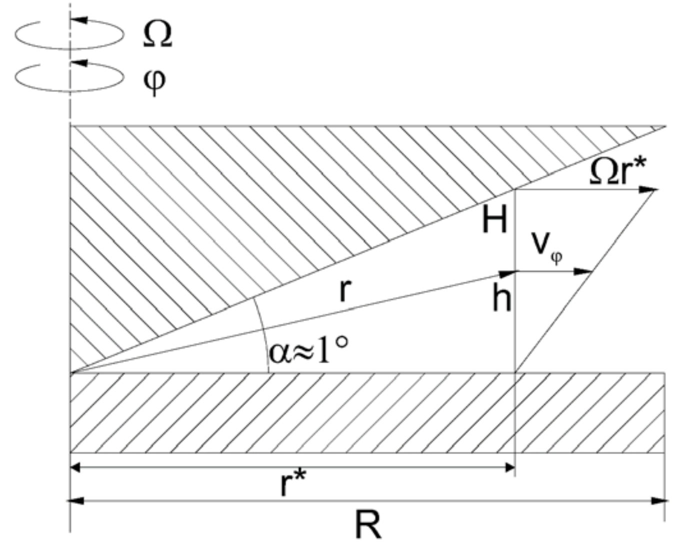

Fig. 3.13: Cone-plate rheometer

- torque: $M = \int_0^R \tau \cdot 2\pi r^* \cdot r^* dr^* = \frac{2}{3} r^{*3} \cdot \pi \cdot \tau \Big|_0^R = \frac{2}{3} R^{*3} \pi \cdot \tau$ $\rightarrow$ $\boxed{\tau = \frac{3}{2} \frac{M}{\pi R^3}}$

- given parameter: R, α
- to measure: $\Omega \rightarrow \dot{\gamma}$, $M \rightarrow \tau$

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3.2.1 Viscosity measurement methods

4. Couette-Rheometer

- similar principle as the cone-plate rheometer
- additionally to the cone-plate-system: very small cylindrical gap
- flow instabilities (Taylor vortices)

  → driven inner cylinder: instabilities appear at lower angular velocities compared to a driven outer cylinder
- double gap also possible

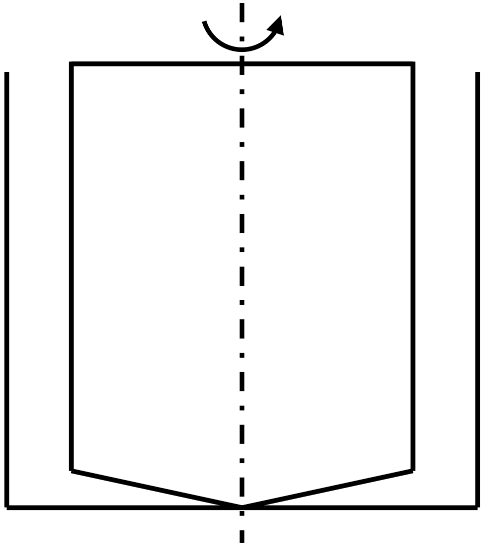

Fig. 3.14: Couette -Rheometer

Lecture contents

3. Rheology of blood	Exercise
3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
3.2 Viscosity of blood 3.2.1 Viscosity measurement methods 3.2.2 Viscosity models for blood plasma and whole blood 3.2.3 Influences on the viscosity of blood	
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3.2.2 Viscosity models for blood plasma and whole blood

- **blood plasma:**
  - $\eta_{pl} = 1{,}92 - 2{,}2$ cP
  - the viscosity is independent from the shear velocity over a wide range:
  - pronounced, characteristic decrease of the pipe friction coefficient in a turbulent pipe flow

→ **viscous-elastic fluid** with: $\tau \backsim \dot{\gamma} \quad \rightarrow \quad \tau \backsim \eta_{pl} \cdot \dot{\gamma}$

- **viscosity models of whole blood:**

1. Copley
2. Chmiel
3. Casson
4. Merville and Pelletier

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3.2.2 Viscosity models for blood plasma and whole blood

1. Viscosity model of Copley (1965)

- steeper increase of the viscosity at low shear rates ($\dot{\gamma} < 10\ \text{sec}^{-1}$)
- viscosity stays approx. constant above $\dot{\gamma} = 20\ \text{sec}^{-1}$, $\eta = 8$ cP

→ blood was assumed to have a **flow limit**:

for $\tau \leq \tau_\gamma$: $\dot{\gamma}' = 0$; $\eta = \infty$

- aggregation affinity of the RBCs at $\dot{\gamma}' = 0$
- minimal force necessary to dissociate the aggregate
- only after the dissociation the fluid will move

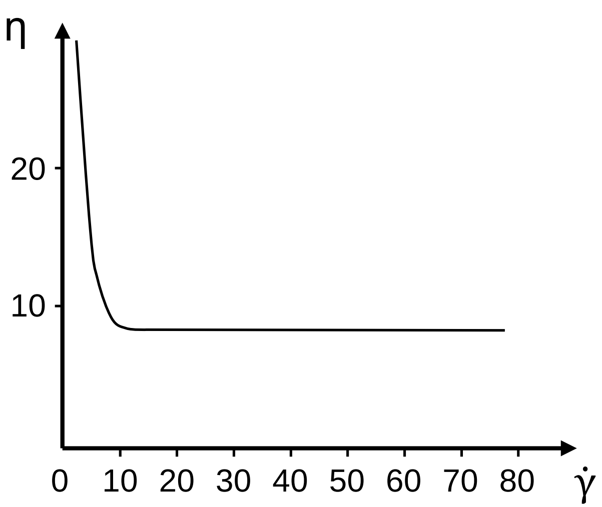

Fig. 3.15: Behavior of whole blood according to Copley

3.2.2 Viscosity models for blood plasma and whole blood

2. Viscosity model of Chmiel (1973)

- measurements with low shear rheometer
  - broader shear gap
  - very slow rotation

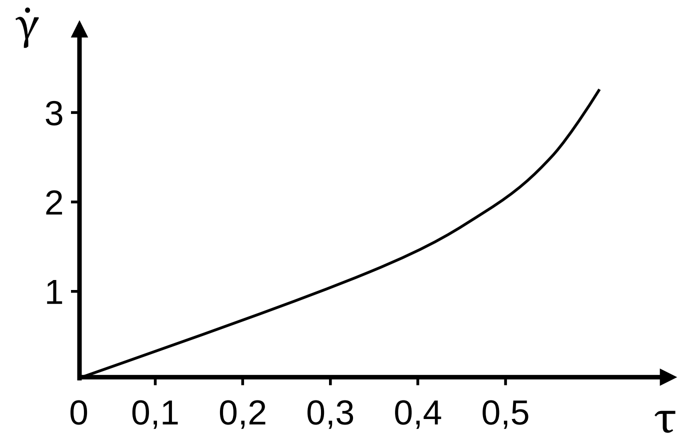

Fig. 3.16: Behavior of whole blood according to Chmiel (Hct = 44%)

- for the range [$5 \cdot 10^3 \le \dot{\gamma} \le 5\ \mathrm{sec}^{-1}$] the curve can be expresses through a polynomial

of 7. degree: $\rightarrow\ \dot{\gamma} = c \cdot \left[\frac{\tau}{a} + \left(\frac{\tau}{a}\right)^3 + d \cdot \left(\frac{\tau}{a}\right)^5 + e \cdot \left(\frac{\tau}{a}\right)^7\right]$

$\rightarrow$ constants a, c, d, e for different Hct values

3.2.2 Viscosity models for blood plasma and whole blood

- double logarithmic representation:
  - $\eta = f(\dot{\gamma})$ has an **inflection point**
  - all curves $\eta = f(\dot{\gamma})$ of fluids without a flow limit have an inflection point!

  → conclusion of Chmiel: blood has **no flow limit**

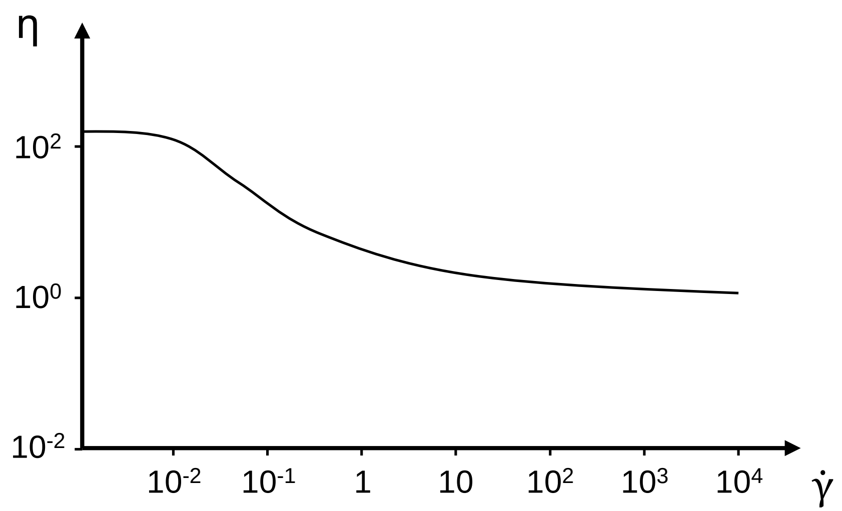

Fig. 3.17: Double logarithmic representation (Chmiel)

  - main argument against Chmiel: by logarithmic representation an inflection point can be created even if a fluid possesses a flow limit so long measurements are not successful at $\dot{\gamma} \rightarrow 0$!

3.2.2 Viscosity models for blood plasma and whole blood

3. Viscosity model of Casson

- assumption of flow limit $\dot{\gamma} = 0$ at $\tau \leq \tau_\gamma$
- for high shear rates $\dot{\gamma}$: Newtonian flow behavior
- empirical equation of the flow curve:

$$\left| \sqrt{\tau} = \sqrt{\tau_\gamma} + c \cdot \sqrt{\frac{du}{dy}} \right|$$

$\rightarrow$ square: $\left| \tau = \tau_\gamma + 2 \cdot \sqrt{\tau_\gamma} \cdot c \cdot \sqrt{\frac{du}{dy}} + c^2 \cdot \frac{du}{dy} \right|$

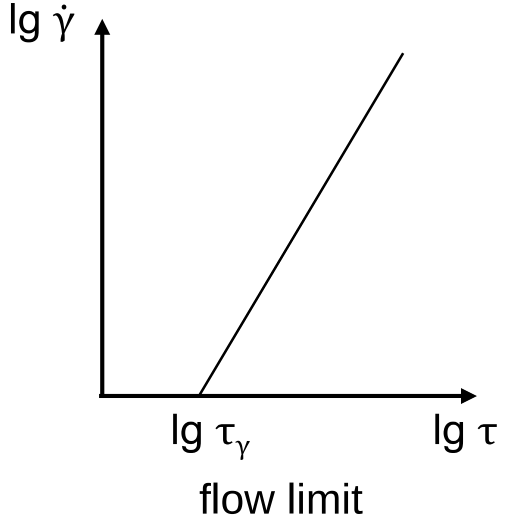

Fig. 3.18: Double logarithmic representation (Casson)

3.2.2 Viscosity models for blood plasma and whole blood

→ $\left| \tau - \tau_\gamma = \underbrace{2 \cdot \sqrt{\tau_\gamma} \cdot c \cdot \sqrt{\dot{\gamma}}}_{\text{1. term}} + \underbrace{c^2 \cdot \dot{\gamma}}_{\text{2. term}} \right|$ with $2 \cdot \sqrt{\tau_\gamma} \cdot c = \text{const.}$

→ for $\dot{\gamma} < 1$: 1. term dominates

→ for $\dot{\gamma} > 1$ (very big values): 2. term dominates

→ for $c^2 = \eta$ and for large values of $\dot{\gamma}$: **Newtonian behavior**

→ corresponds to the asymptotic approximation of η to a constant value for large shear rates by Copley and Chmiel

3.2.2 Viscosity models for blood plasma and whole blood

4. Viscosity model of Merville and Pelletier

- assumption of a flow limit τ_γ
- for $\frac{du}{dy} < 31{,}5\ \text{sec}^{-1}$: Casson model
- for $\frac{du}{dy} > 105\ \text{sec}^{-1}$: Newtonian behavior
- transition domain in between

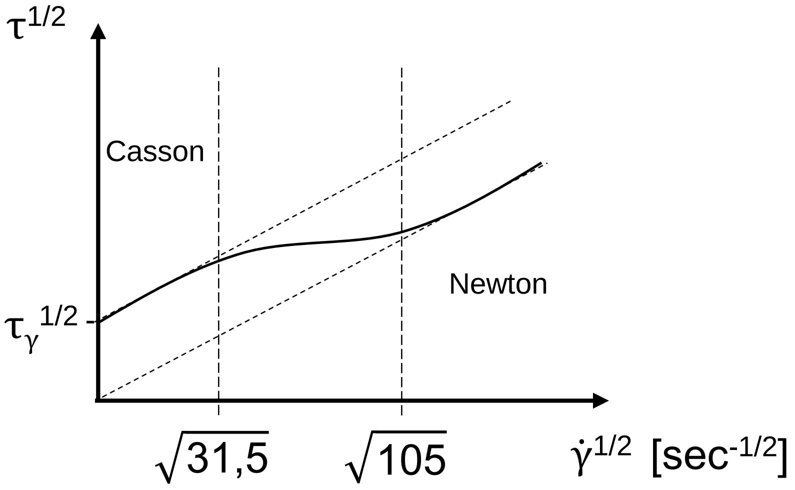

Fig. 3.19: Model of Merville and Pelletier

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3.2.2 Viscosity models for blood plasma and whole blood

Different real blood data from literature:

→ asymptotic viscosity (approx. 3,1 mPas at 10^3 sec^{-1})

→ difference of temperature minor variations of the curve

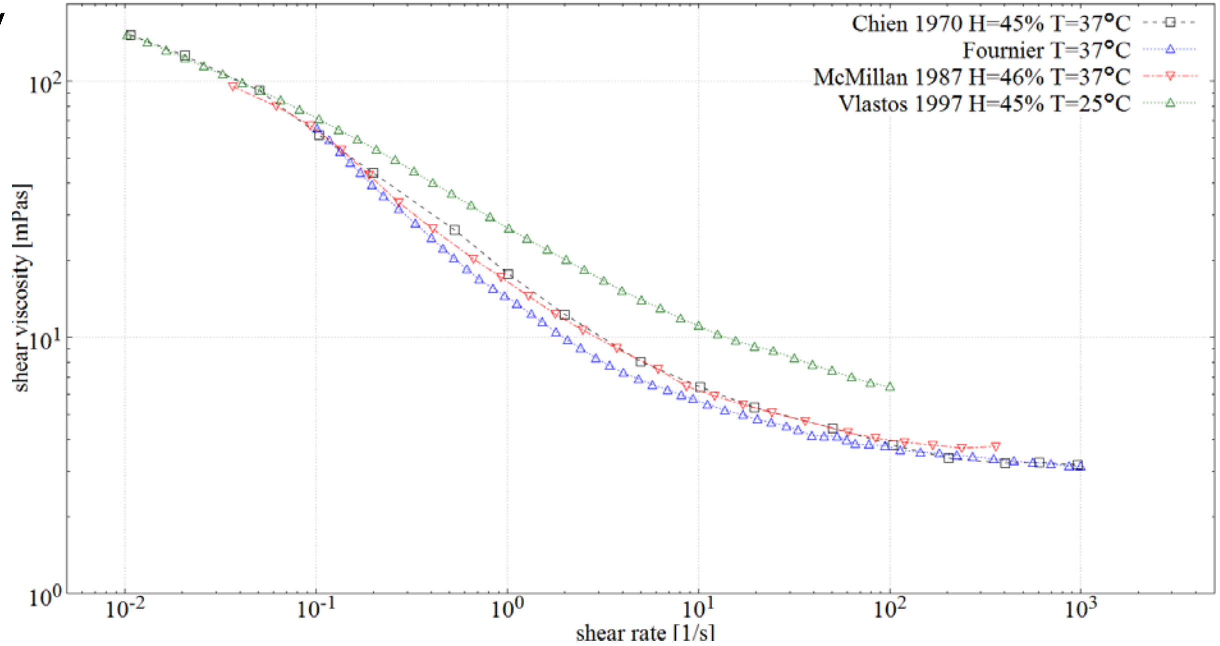

Fig. 3.20: Different real blood data from literature

3.2.2 Viscosity models for blood plasma and whole blood

Flow limit:

- a distinct clarification whether blood possesses a flow limit or not is not yet solved!
- consequences of a flow limit of the blood on the velocity profile:

$\longrightarrow$ in the vicinity of the center line $\tau < \tau_\gamma$ can occur

$\longrightarrow$ then: $\dot{\gamma} = 0$ dominates Z around the middle line

$\longrightarrow$ measurements show such a behavior!

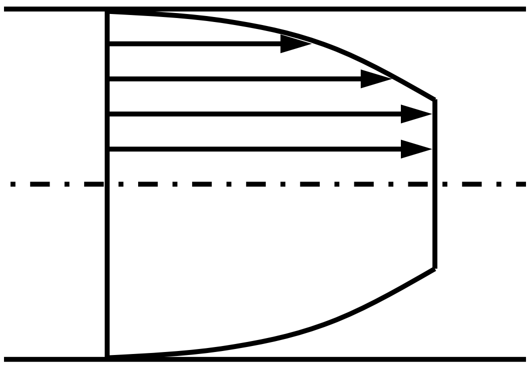

Fig. 3.21: Oblated velocity profile with core domain with $\dot{\gamma} = 0$

$\Rightarrow$ **blood must have a flow limit whereas τ_γ is a function of the hematocrit value**

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3.2.2 Viscosity models for blood plasma and whole blood

- with decreasing hematocrit the flow limit τ_γ also decreases
- for very small Hct (less than 5%): $\tau_\gamma \rightarrow 0$ (no yield stress)
- for Hct = 45%:

$\tau_\gamma \approx$ 0,015 dynes/cm²

= 0,0015 Pa

= 1,5 mPa

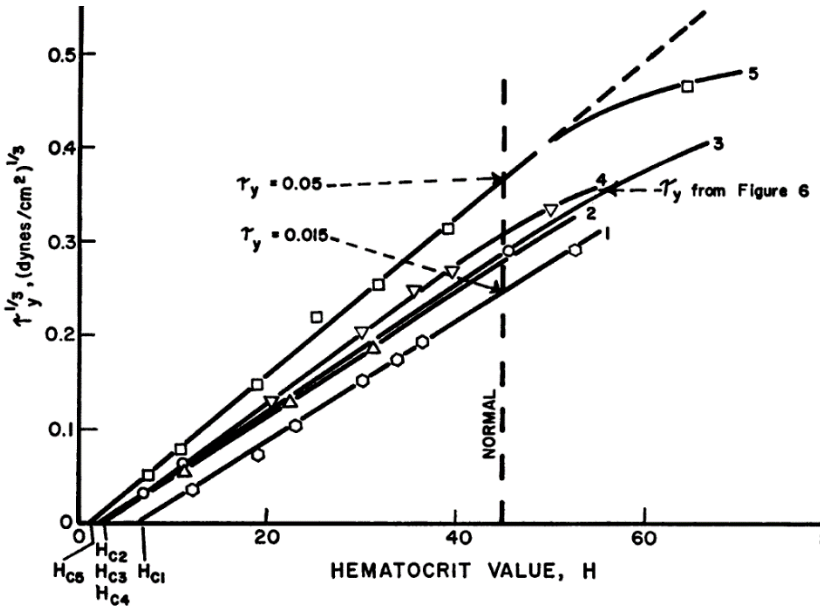

Fig. 3.22: Variation of the yield stress τ_γ with hematocrit (3 different blood samples)

Lecture contents

3. Rheology of blood	Exercise
3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
3.2 Viscosity of blood 3.2.1 Viscosity measurement methods 3.2.2 Viscosity models for blood plasma and whole blood 3.2.3 Influences on the viscosity of blood	
3.3 Non-Newtonian blood analog fluid 3.3.1 Requirements and composition 3.3.2 Xanthan gum 3.3.3 Influence of temperature	

3.2.3 Influences on the viscosity of blood

1. **Surface coating**
    - glass surface
    - fibrin coated surface → low viscosity
2. **Hematocrit**
    - Hct ↑ : viscosity ↑ (according to Chmiel)

      → corresponds to Einstein's equation for suspensions with stiff particles
3. **Temperature**
    - temperature ↑ : viscosity ↓
    - according to ratios in the water

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3.2.3 Influences on the viscosity of blood

→ Chmiel: $$2 \cdot \frac{\eta_T}{\eta_{T0}} = \exp\left(\frac{g(T_0-T)}{(T+h)\cdot(T_0+h)}\right)$$

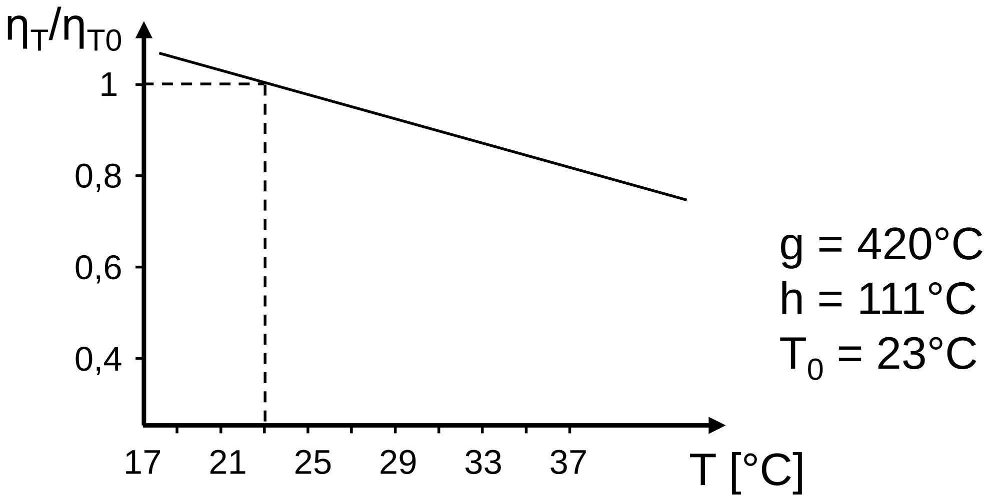

Fig. 3.23: Influence of the temperature on the viscosity of blood

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3.2.3 Influences on the viscosity of blood

4. Vessel diameter

- important vessel diameters:
  - Arterioles: $d_A = 0.02$ - 0.04 mm
  - Capillaries: $d_C = 0.008 \text{ mm} \approx d_{Erythrocytes}$ $\longrightarrow$ other conditions apply!
- **$d_{Vessel} \geq 0.5$ mm:** blood can be handled as a continuum independently of Re
- **$0.03 \text{ mm} \leq d_{Vessel} \leq 0.5$ mm:** decomposition in wall vicinity
  - in the direct vicinity of the wall: fewer blood cells than in the central flow

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3.2.3 Influences on the viscosity of blood

→ $\eta_{wall} < \eta_{central}$ due to:

→ packing density

→ transversal forces applied on the blood cells in the shear field („wall migration“)

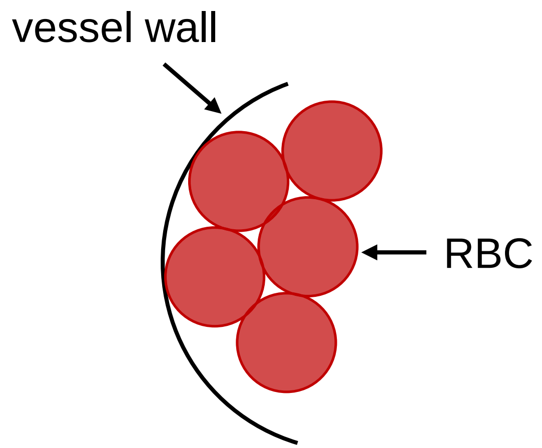

Fig. 3.24: Packing density

- for thin capillaries the cell poor layers become relatively large

- **d_{Vessel} ≤ 0.03 mm:** blood cannot be handles as a liquid any more!
- **d_{Vessel} ≤ 0.02 mm:** additional effect due to the impeded inflow of erythrocytes

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3.2.3 Influences on the viscosity of blood

- **Fahraeus-Lindqvist-Effect (1931)**
  - capillary diameter d ↓ (d < 1.5 mm) : viscosity η_{eff} ↓ (effective viscosity of the blood)
  - a formal explanation of the Fahraeus-Lindqvist-Effect is not given yet

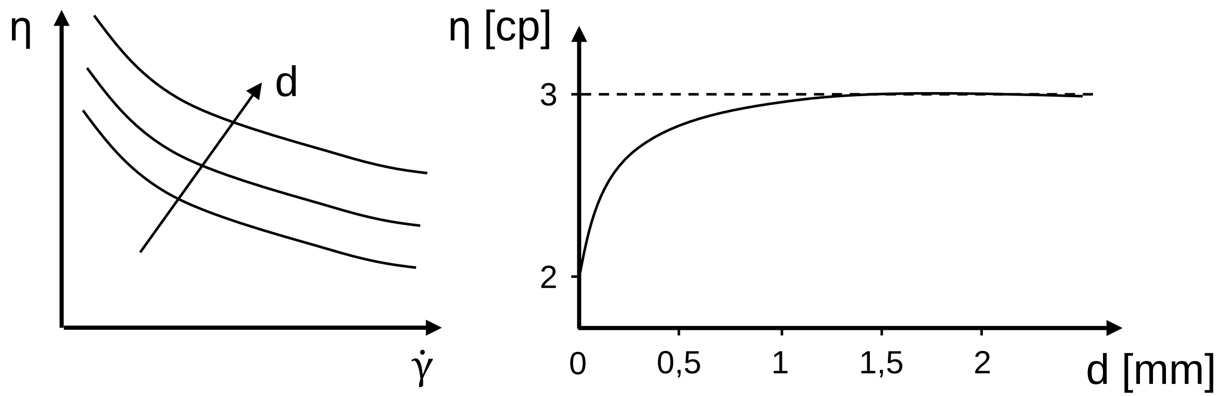

Fig. 3.25: Influence of capillary diameter on the viscosity of blood

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3.2.3 Influences on the viscosity of blood

- physiological meaning:
  - → resistance in very small vessels of the body doesn‘t increase in the proportions suggested by the decrease of the radius!
  - → according to Hagen- Poiseuille: $\Delta p = \dot{Q} \cdot \frac{8 \cdot \eta \cdot l}{\pi \cdot R^4}$

    → R^4 should have a strong influence
  - → the strong influence of the radius is partly compensated by the decrease of the viscosity

3.2.3 Influences on the viscosity of blood

- **Inflow from a large vessel in a small vessel**
  - wall vicinity: cell poor domain is stopped and expands
  - central cell rich domain is accelerated

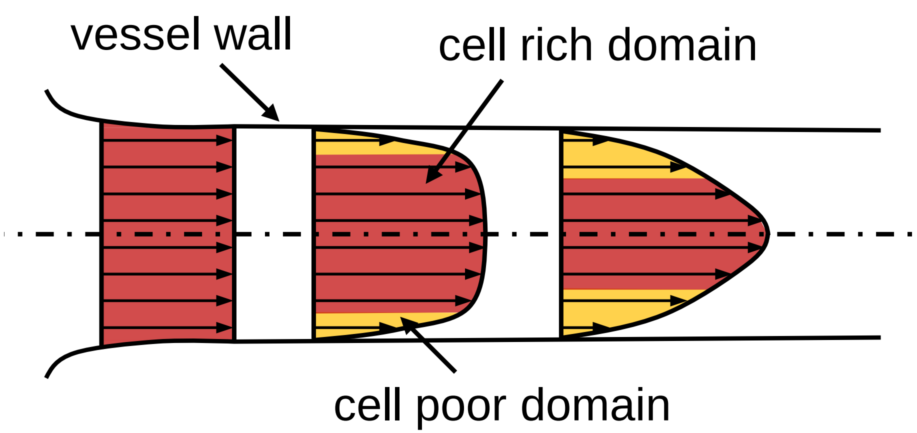

Fig. 3.26: Inflow from large to small vessel

→ RBCs flow in the capillaries with a constant velocity and equal distribution

→ after ending the intake process: RBCs are found on the flow rails which have a higher velocity than the decelerated layers in wall vicinity

3.2.3 Influences on the viscosity of blood

5. Patho-physiological influences

- **plasma:**
  - plasma proteins have an influence on
    → the RBC-aggregation (flow limit)
    → blood sedimentation rate = indication for infections

1. **Fibrinogen:** long molecules, aggregation affinity ↑
2. **Globulin:** aggregation affinity ↑
3. **Albumin:** neg. charged, aggregation affinity ↓

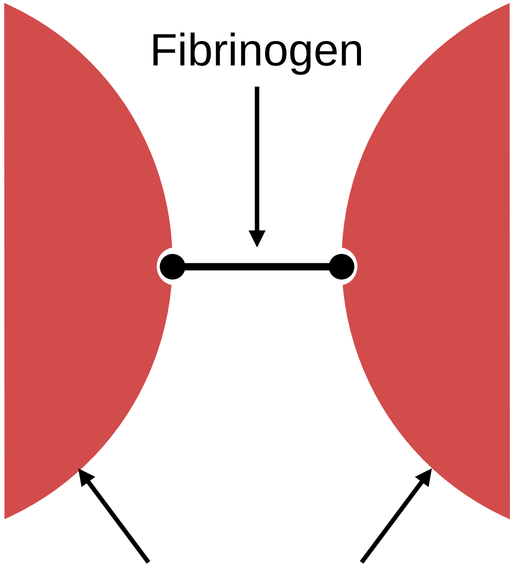

Fig. 3.27: Impact of Fibrinogen

3.2.3 Influences on the viscosity of blood

- **corpuscular components:**
  - influence of hematocrit
  - influence of cell shape:

    → e.g. **Elliptocytosis**

    (hereditary, 0.02-0.05% of the population)

    → RBCs become ellipsoid-shaped

    (bigger cell content at same surface)

    → flexibility ↓

    → **Newtonian flow behavior**

    at high viscosity

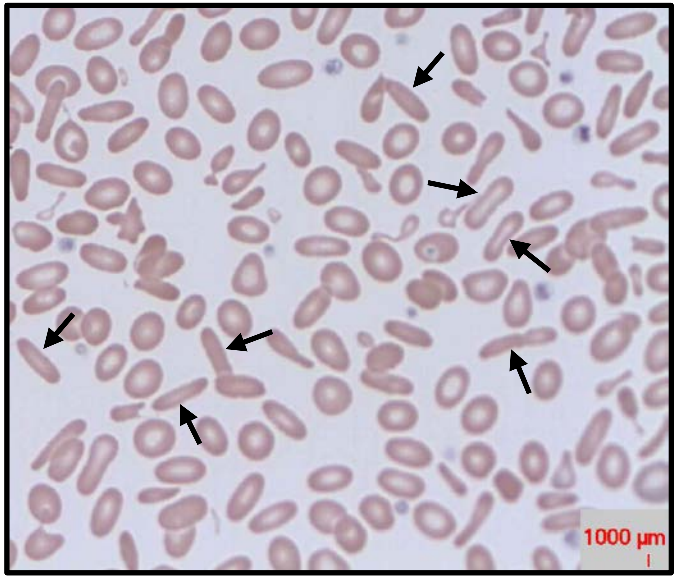

Fig. 3.28: Hereditary Elliptocytosis [2]
(some samples marked)

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3.2 Viscosity of blood

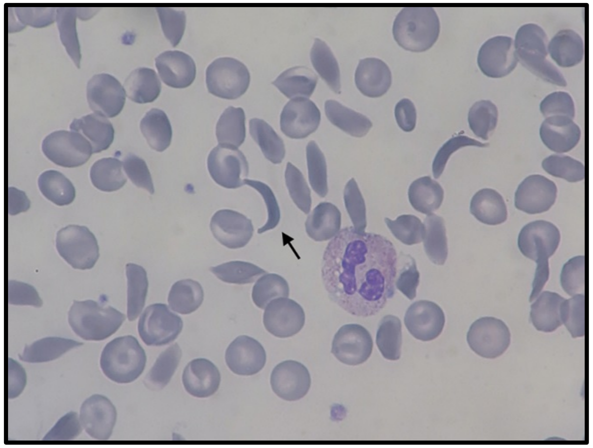
3.2.3 Influences on the viscosity of blood

→ e.g. **Sickle-Cell-Anemia** (hereditary, spread under Negroid population)

→ Hemoglobin crystallizes at low pO_2

→ weird cell forms, not flexible

→ viscosity ↑ (Dintenfass at Hct = 30%: $\eta_{oxy} = 0{,}06 \rightarrow \eta_{desoxy} = 0{,}1$)



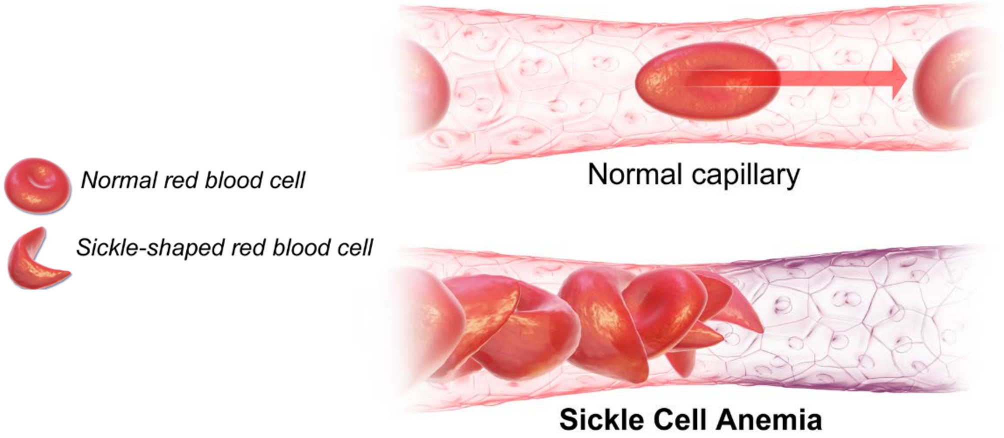


Fig. 3.29: Microscopic image (left, [3]) and impact of Sickle-Cell-Anemia (middle and right, [4])

3.2.3 Influences on the viscosity of blood

- **other influences:**
  - heavy burns
  - severe bleedings
  - excessive absorption of blood fats

    → membrane of the RBCs will be crenated („Echinocytes“)

    → flexibility ↓

    → liquid inclusion

    → effective radius ↑

    → viscosity ↑

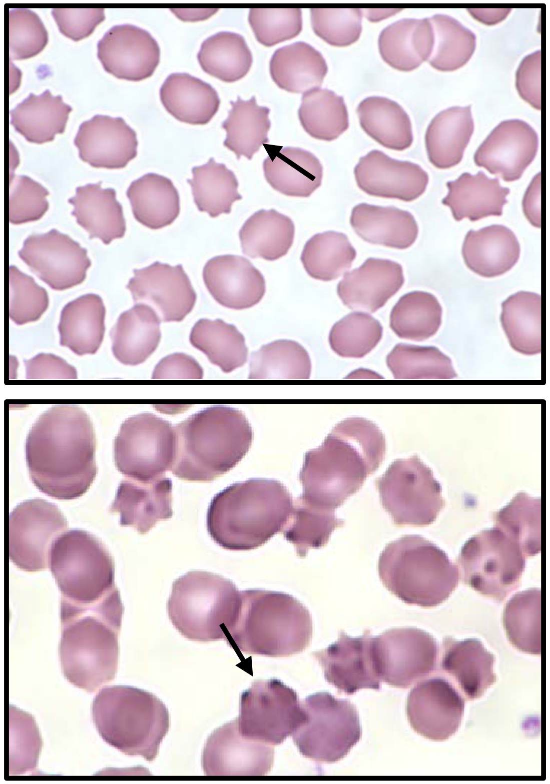


Fig. 3.30: Echinocytes [5], [6] (samples marked)

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3.1 Classification of blood 3.1.1 Definitions 3.1.2 Flow behavior of suspensions 3.1.3 Blood as a suspension of flexible particles	
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3.3 Non-Newtonian blood analog fluid 3.3.1 Requirements and composition 3.3.2 Xanthan gum 3.3.3 Influence of temperature	

3.3 Non-Newtonian blood analog fluid

3.3.1 Requirements and composition

- refractive index matched for optical access (PIV – Particle image Velocity)
- comparable non-Newtonian behavior as blood
- same asymptotic viscosity at high shear rates
- non-toxic

Substance	Specification	Effect
Distilled water	free of ions	filler, viscosity
Glycerol	sugar alcohol	viscosity, refractive index, density
Xanthan gum	polysaccharide, stabilizer/ thickener	non-Newtonian viscosity
Sodium Iodid	salt	refractive index matching, density
Sodium Thiosulfate	salt	prevents discoloration of Sodium Iodide

3.3 Non-Newtonian blood analog fluid

3.3.2 Xanthan gum

- shear-thinning behavior
- higher shear

  → breaking entanglements
- at low shear rates:

  distanglement is slower

  than entanglement

  → **Newtonian behavior**
- no destruction at high

  shear rates

  → **Newtonian behavior**

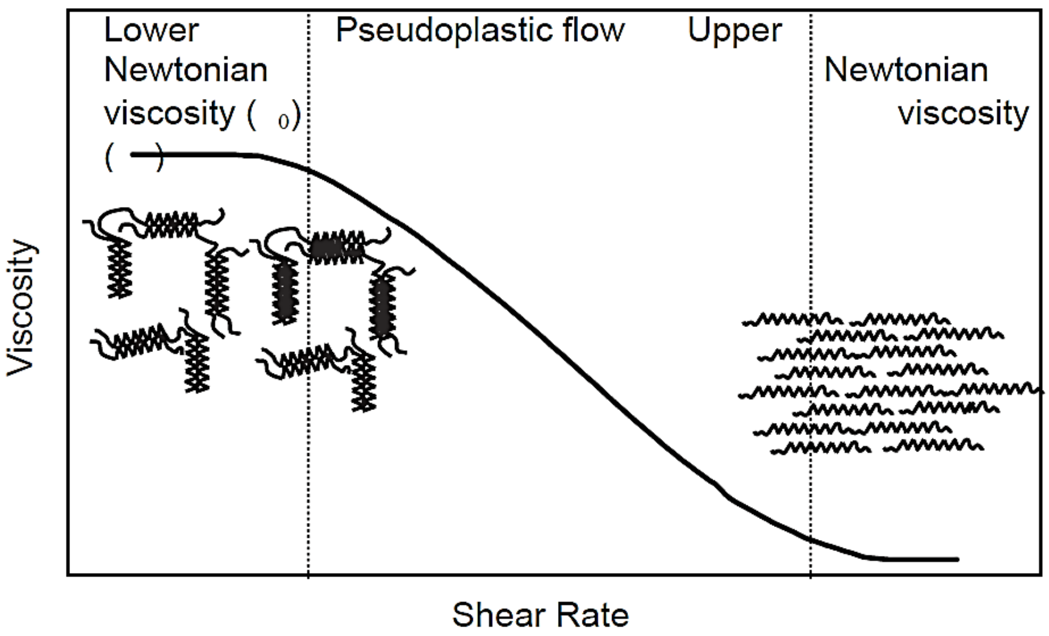

Fig. 3.31: Viscosity of Xanthan gum [6]

3.3.2 Xanthan gum

- **molecular structure:**
  - conformational isomerism:

    → temperature ↑ , Ion ↓ : random coil

    → temperature ↓ , Ion ↑ : ordered helical structure
  - model concept (salt free):

    → rigid rods (anionic side chain → rigid helical structure)

    → repulsive interaction

    → no parallelization possible

    → at high concentration: frozen points

    → high viscosity

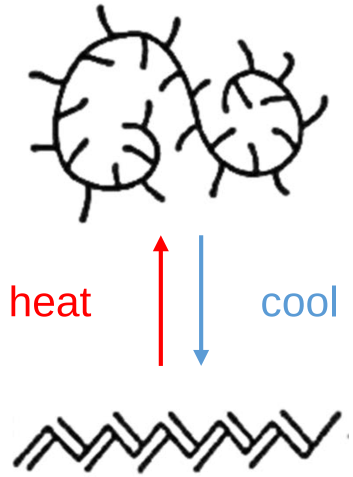

Fig. 3.32: Orientation of the Xanthan molecule

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3.3.2 Xanthan gum

- $\dot{\gamma} >> \dot{\gamma}_0$: $E_{shear} > E_{repulsion}$ $\rightarrow$ „parallelization“

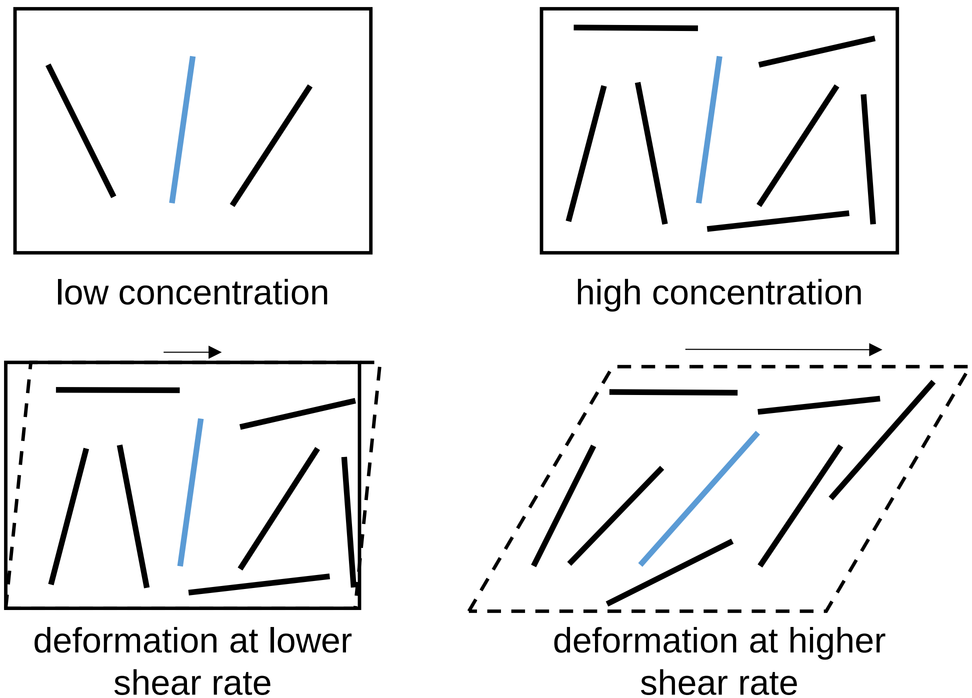

Fig. 3.33: Behavior of the Xanthan molecule at different concentrations and shear rates

3.3.3 Influence of temperature

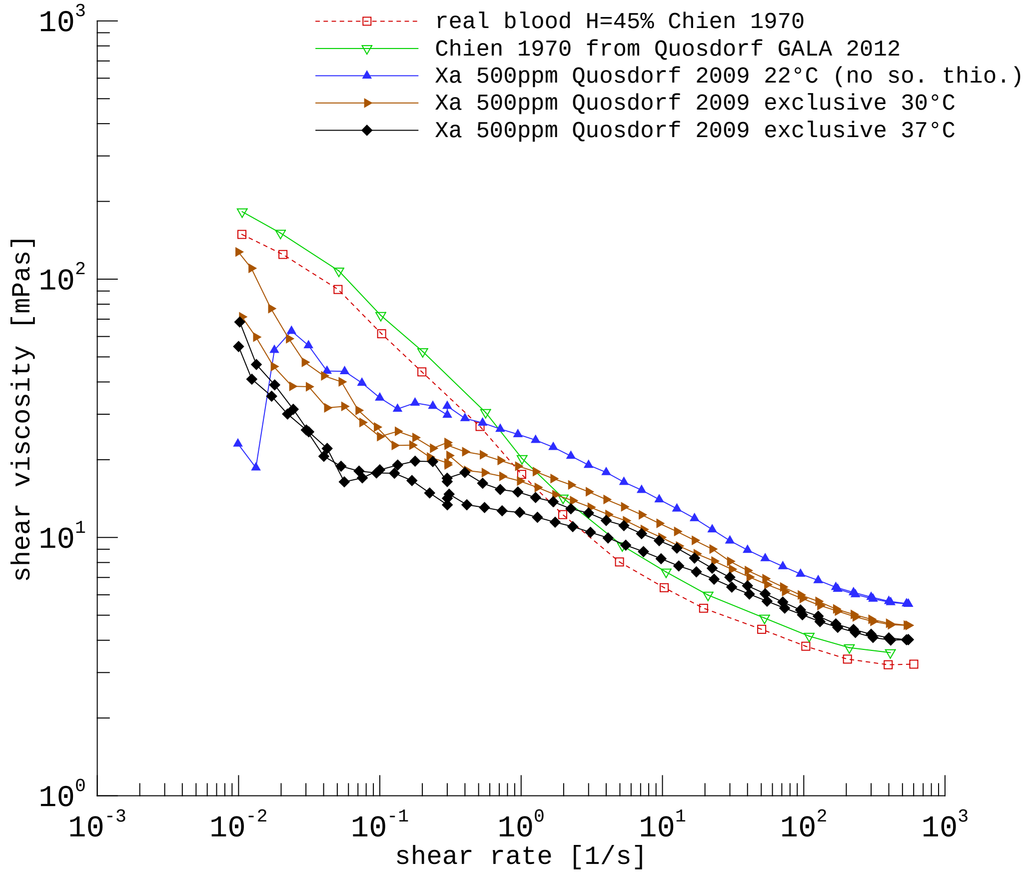

Fig. 3.34: Temperature influence on the flow curve

3.3.3 Influence of temperature

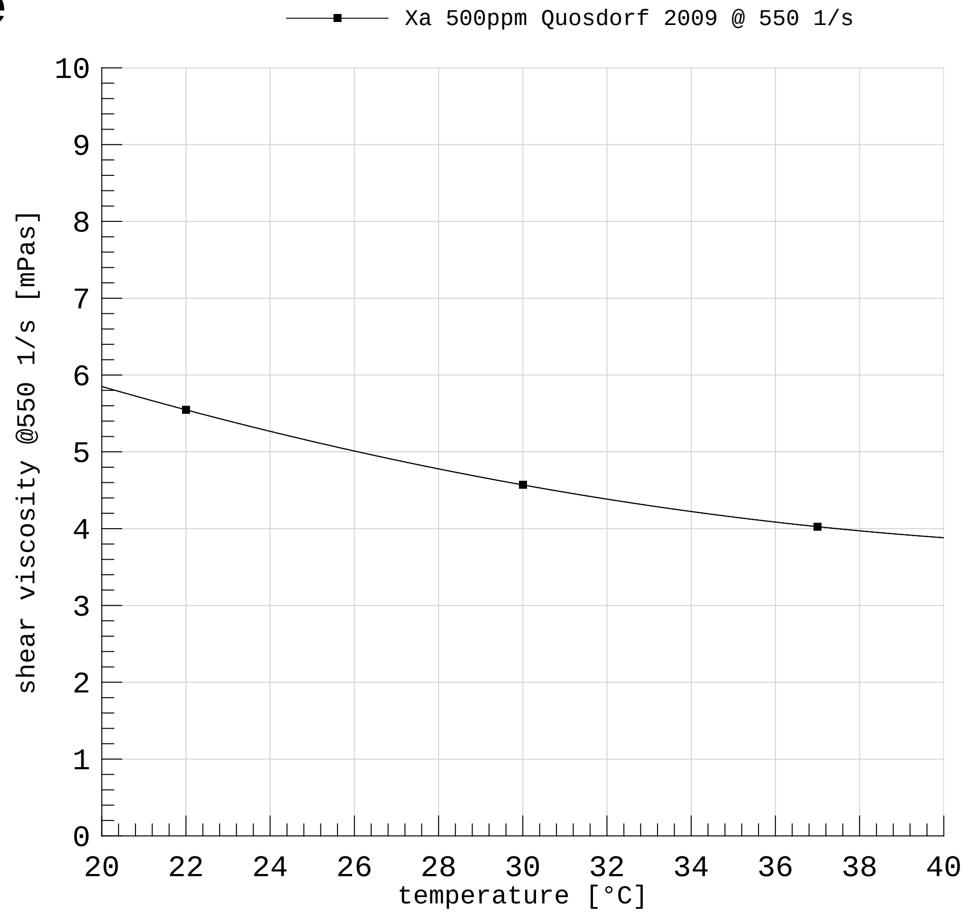

Fig. 3.35: Temperature influence on the shear viscosity

3.3 Non-Newtonian blood analog fluid

3.3.3 Influence of temperature

- viscosity decreases

  comparable to blood

  or water

- $\Delta T = 10K$

  $\rightarrow$ $\Delta\eta_{\infty} \approx -20\%$

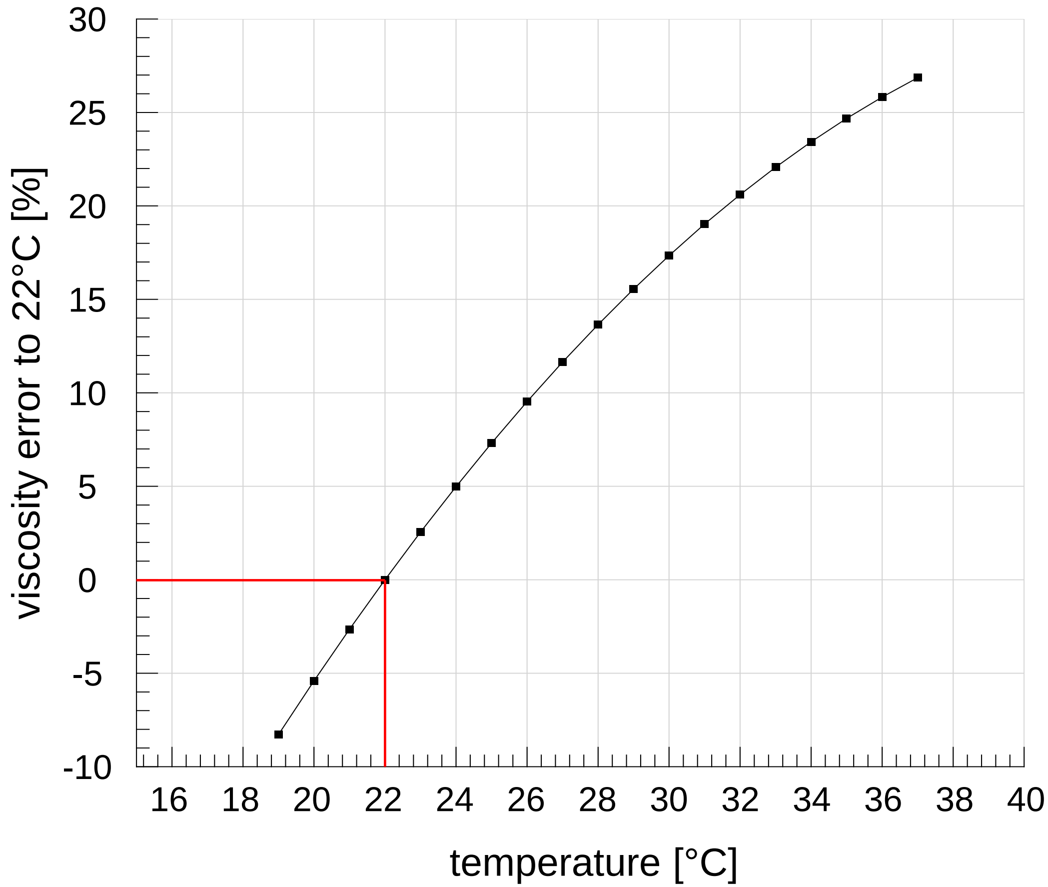

Fig. 3.36: Viscosity error to 22°C depending on temperature

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Sources

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[3] https://commons.wikimedia.org/wiki/File:Sickle_Cell_Anemia.png (modified)
[4] https://commons.wikimedia.org/wiki/File:Echinocytes-11.JPG (modified)
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[6] Sworn, G. (2009). Xanthan gum. Food Stabilisers, Thickeners and Gelling Agents, 325-342

Thank you for your attention!