1a) Aufgabe 1

Seilkraft?

Kräftegleichgewicht:

$A = G + S$

$\rho_w V_0 g = m \cdot g + S$

$\Leftrightarrow \vec{S} = \frac{1}{3}\pi R_0^2 h_0 g \rho_w - m \cdot g$

1b)

Masse der Luft bleibt konstant:

$m_L R T_L = \text{konst}$ mit $h_0 \ll H$

$\Rightarrow p_a V_0 = p_w V_H = (p_a + \rho_w g H) V_H$

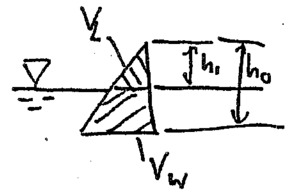
$\Rightarrow V_H = \frac{p_a}{p_a + \rho_w g H} V_0$

mit $A = \rho_w g V_H$

$S = A - G = \left[\frac{p_a}{p_a + \rho_w g H} \rho_w V_0 - m\right] g$

$= \left[\frac{p_a}{p_a + \rho_w g H} \cdot \frac{1}{3}\pi R_0^2 h_0 \rho_w - m\right] g$

1c) $A = G$

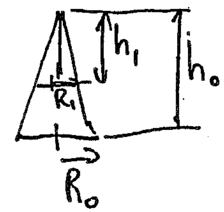


$\Rightarrow \rho_w V_W g \overset{!}{=} m \cdot g$

Es ist

$V_W = V_0 - V_L = \frac{1}{3}\pi \left[R_0^2 h_0 - R_1^2 h_1\right]$ (*)

Strahlensatz:



$\frac{h_1}{R_1} = \frac{h_0}{R_0}$

Damit folgt $V_W = \frac{1}{3}\pi R_0^2 h_0 \left[1 - \left(\frac{h_1}{h_0}\right)^3\right]$

(*) einsetzen:

$\frac{1}{3}\pi R_0^2 h_0 \left[1 - \left(\frac{h_1}{h_0}\right)^3\right] = \frac{m}{\rho_w}$

$\Leftrightarrow 1 - \left(\frac{h_1}{h_0}\right)^3 = \frac{3}{\pi R_0^2 h_0} \cdot \frac{m}{\rho_w}$

$\Leftrightarrow h_1 = \sqrt[3]{h_0^3 - \frac{3 h_0^2}{\pi R_0^2} \cdot \frac{m}{\rho_w}}$

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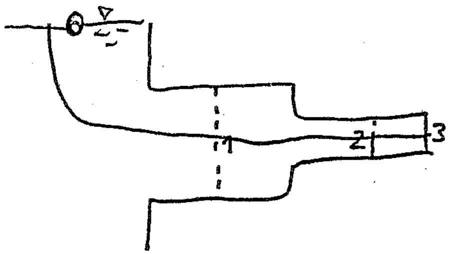
2. Aufgabe:

a) gesucht Δp^2



$$\Delta p = p_1 - p_2$$

Bernoulli von 1–2:

$$p_1 + \frac{\rho_w}{2} v_1^2 = p_2 + \frac{\rho_w}{2} v_2^2 \left[1 + \frac{\lambda}{2}\frac{L_2}{D_2}\right] + \frac{\rho_w}{2} v_1^2 \frac{\lambda}{2}\frac{L_1}{D_1}$$

Konti:

$$v_1 \cdot \pi \frac{D_1^2}{4} = v_2 \cdot \pi \frac{D_2^2}{4} \Leftrightarrow \frac{v_1}{v_2} = \left(\frac{D_2}{D_1}\right)^2$$

Es ist

$$v_2 = v_3 \quad \text{und} \quad p_3 = p_a$$

$$\Rightarrow p_1 - p_2 = \frac{\rho_w}{2} v_2^2 \left[1 + \frac{\lambda}{2}\frac{L_2}{D_2} + \left(\frac{D_2}{D_1}\right)^4 \left[\frac{\lambda}{2}\frac{L_1}{D_1} - 1\right]\right]$$

Bestimmung von v_2:

mit $v_2 = v_3$

Bernoulli von 0 – 3:

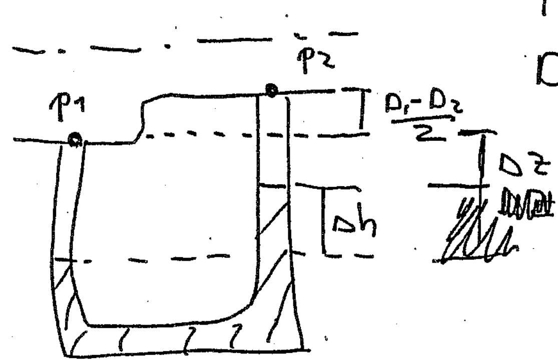
$$p_a + \rho_w g H = p_a + \frac{\rho_w}{2} v_3^2 \underbrace{\left[1 + \lambda\frac{L_2}{D_2} + \left(\frac{D_2}{D_1}\right)^4 \lambda \frac{L_1}{D_1}\right]}_{K_1}$$

$$\Rightarrow v_3 = \sqrt{\frac{2gH}{K_1}}$$

Damit folgt für $\Delta p = p_1 - p_2$

$$\Delta p = \rho_w g H \frac{1 + \frac{\lambda}{2}\frac{L_2}{D_2} + \left(\frac{D_2}{D_1}\right)^4 \left[\frac{\lambda}{2}\frac{L_1}{D_1} - 1\right]}{1 + \lambda\frac{L_2}{D_2} + \left(\frac{D_2}{D_1}\right)^4 \lambda\frac{L_1}{D_1}}$$

b) Hydrostatik:



Im Rohrquerschnitt ist Druck konstant

$$p_1 + \Delta z \rho_w g + \Delta h \rho_w g = p_2 + \Delta z \rho_w g + \frac{D_1 - D_2}{2}\rho_w g + \Delta h \rho_{HG} g$$

$$\Rightarrow p_1 - p_2 = (\rho_{HG} - \rho_w) g \Delta h + \frac{D_1 - D_2}{2} \rho_w g$$

einsetzen in a):

$$\Delta h = \left(\frac{\left[1 + \frac{\lambda}{2}\frac{L_2}{D_2} - \left(\frac{D_2}{D_1}\right)^4 \left(1 - \frac{\lambda}{2}\frac{L_1}{D_1}\right)\right]}{\left[1 + \lambda\frac{L_1}{D_1}\left(\frac{D_2}{D_1}\right)^4 + \lambda\frac{L_2}{D_2}\right]} H - \frac{D_1 - D_2}{2}\right) \frac{\rho_w}{(\rho_{HG} - \rho_w)}$$

c) Bernoulli für instationäre Strömungen:

$$p_a + \rho_w g H = p_a + \frac{\rho_w}{2} v_3^2 + \rho_w \int_0^3 \frac{\partial v}{\partial t} ds \; ; \; L \gg D_{1/2}$$

$$\Leftrightarrow gH = \frac{v_3^2}{2} + \frac{dv_3}{dt} \left(L_1 \left(\frac{D_2}{D_1}\right)^2 + L_2 \right)$$

$$\Delta t = \int_0^{0{,}99\, v_{stat}} \frac{2\left(L_1 \left(\frac{D_2}{D_1}\right)^2 + L_2\right)}{2gH - v_3^2} \, dv_3$$

Integralvorschrift aus Hinweis:

$$\Delta t = \frac{2\left(L_1 \left(\frac{D_2}{D_1}\right)^2 + L_2\right)}{2 v_{3\,stat}} \left[\ln \frac{v_{3stat} - v_3}{v_{3stat} - v_3} \right]_0^{0{,}99\, v_{3stat}}$$

mit $v_{3stat} = \sqrt{2gH}$

$\Rightarrow$

$$\Delta t = \frac{\left[L_1 \left(\frac{D_2}{D_1}\right)^2 + L_2\right]}{\sqrt{2gH}} \cdot \ln(199)$$

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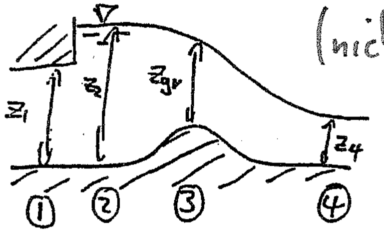
3. Aufgabe:



(nicht in TSL)

ⓐ $y_{gr} = H_1 - H_{min}$

① $\dot{V} = v_1 z_1 b = 50 \frac{m^3}{s}$

$\Rightarrow y_{gr} = z_1 + \frac{v_1^2}{2g} - \frac{3}{2}\sqrt[3]{\frac{v_1^2 z_1^2}{g}} = 1{,}801\ m$

ⓑ
$y_W > y_{gr}$: Aufstau des Wassers vor Wehr

mit $Fr = 1$ am Wehr $\Rightarrow z_W = z_{gr}$

$z_W = z_{gr} = \sqrt[3]{\frac{z_1^2 v_1^2}{g}} = 6{,}3\ m$

$v_{gr} = \frac{\dot{V}}{z_{gr} b} = \frac{v_1 z_1 b}{z_{gr}\, b} = 7{,}937 \frac{m}{s}$

ⓒ
Diagramm $\frac{H}{H_{min}} = \frac{z}{z_{gr}}$

Es gilt:

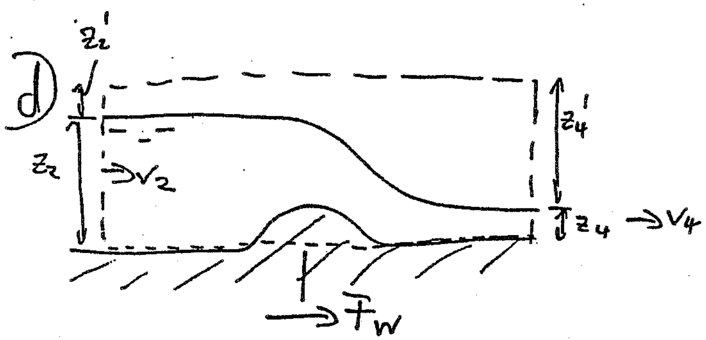
$y_W + H_{min} = H_4 \Leftrightarrow \frac{y_W}{H_{min}} + 1 = \frac{H_4}{H_{min}}$

mit $H_{min} = \frac{3}{2}\sqrt[3]{\frac{\dot{V}^2}{g b^2}} = 3{,}449\ m$

$\frac{H_4}{H_{min}} = 1{,}212 \xrightarrow{\text{Diagramm}} \frac{z_4}{z_{gr}} = 0{,}7 \Rightarrow z_4 = 4{,}41\ m$

mit $v_4 = \frac{\dot{V}}{z_4 b} = 11{,}338 \frac{m}{s}$

ⓓ



x-Impuls:

$-v_2^2 z_2 b \rho + \rho b z_4 v_4^2 = p_a b z_2' + \int_0^{z_2} (p_a + \rho g z) b\, dz - p_a b z_4' - \int_0^{z_4} (p_a + \rho g z) b\, dz + F_W$

$\Leftrightarrow -v_2^2 z_2 b \rho + \rho b z_4 v_4^2 = \rho b g \left(\frac{z_2^2}{2} - \frac{z_4^2}{2}\right) + F_W$

$\Rightarrow F_W = \rho b \left(v_4^2 z_4 - v_2^2 z_2\right) + \rho g b \left(\frac{z_4^2}{2} - \frac{z_2^2}{2}\right)$

(z_2 durch Aufstau des Wassers)

$\frac{H_2}{H_{min}} = \frac{H_4}{H_{min}} = 1{,}212$ aus c)

Diagramm ($Fr < 1$)

$\frac{z_2}{z_{gr}} = 1{,}65 \Rightarrow z_2 = 10{,}395$

mit $v_2 = \frac{\dot{V}}{z_2 b} = 4{,}81 \frac{m}{s}$

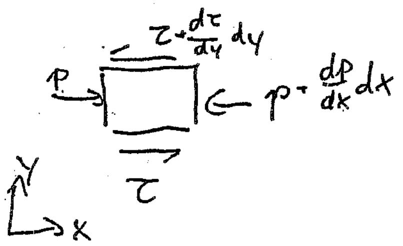
alle Werte einsetzen:

$F_W = -116.632{,}8 \frac{N}{m^2}$

(Kraft auf das Fluid)

4. Aufgabe (TSL: 3. Aufgabe)

ⓐ Kraftbilanz am Volumenelement:



x-Richtung:

$$\tau dx - \left(\tau + \frac{d\tau}{dy}dy\right)dx + p\,dy - \left(p + \frac{dp}{dx}dx\right)dy = 0$$

$$\Leftrightarrow \frac{d\tau}{dy} + \frac{dp}{dx} = 0$$

ⓑ

$\tau = -\frac{dp}{dx}y + C$; ausgebildete Strömung $\Rightarrow \frac{dp}{dx} = \frac{p_2 - p_1}{L}$ = konst.

Fluid I: $\tau_I = \frac{\Delta p}{L}y + C_1$; Fluid II: $\tau_{II} = \frac{\Delta p}{L}y + C_2$

Rb: $y = s$: $\tau_I = \tau_{II} \Rightarrow C_1 = C_2$

Newtonsches Fluid: $\tau = -\eta\frac{du}{dy}$

Fluid I: $u_I(y) = -\frac{\Delta p}{2\eta_I L}y^2 - \frac{C_1}{\eta_I}y + C_3$

Fluid II: $u_{II}(y) = -\frac{\Delta p}{2\eta_{II} L}y^2 - \frac{C_1}{\eta_{II}}y + C_4$

Rb:

$y = 0$: $u_{II} = 0 \Rightarrow C_4 = 0$

$y = h$: $u_I = u_w \Rightarrow C_3 = \frac{\Delta p h^2}{2\eta_I L} + \frac{C_1 h}{\eta_I} + u_w$

Rb:

$y = s$: $u_I = u_{II} \Rightarrow -\frac{\Delta p s^2}{2\eta_I L} - \frac{C_1 s}{\eta_I} + C_3 = -\frac{\Delta p s^2}{2\eta_{II} L} - \frac{C_1 s}{\eta_{II}}$

$$\Rightarrow C_1\left[s\left(\frac{1}{\eta_I} - \frac{1}{\eta_{II}}\right) - \frac{h}{\eta_I}\right] + \frac{\Delta p}{2L}\left[s^2\left(\frac{1}{\eta_I} - \frac{1}{\eta_{II}}\right) - \frac{h^2}{\eta_I}\right] = u_w$$

$$\Rightarrow C_1 = -\left(\frac{h-s}{\eta_I} + \frac{s}{\eta_I}\right)^{-1}\left[\left(\frac{h^2-s^2}{\eta_I} + \frac{s^2}{\eta_{II}}\right)\frac{\Delta p}{2L} + u_w\right]$$

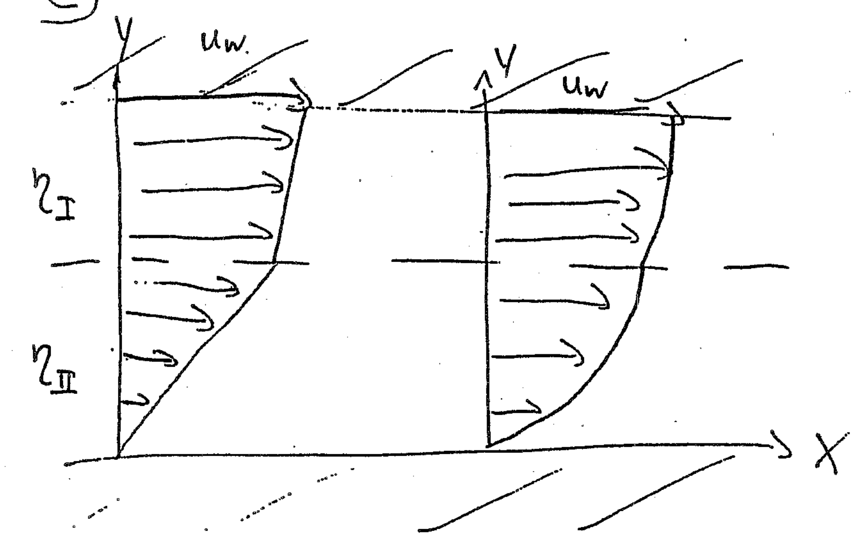
$$\Rightarrow C_3 = u_w + \frac{\Delta p}{2L}\frac{h^2}{\eta_I} - \left(\frac{h-s}{\eta_I} + \frac{s}{\eta_{II}}\right)^{-1}\left[u_w\frac{h}{\eta_I} + \left(\frac{h^2-s^2}{\eta_I} + \frac{s^2}{\eta_{II}}\right)\frac{h}{\eta_I}\frac{\Delta p}{2L}\right]$$

Geschwindigkeitsverteilung:

$0 \le y \le s$: $u(y) = -\frac{\Delta p}{2\eta_{II} L}y^2 - \frac{C_1}{\eta_{II}}y$

$s < y \le h$: $u(y) = -\frac{\Delta p}{2\eta_I L}y^2 - \frac{C_1}{\eta_I}y + C_3$

ⓒ



mit $\eta_I > \eta_{II}$

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5. Aufgabe: (TSL: 4. Aufgabe)

a) Konti, inkompressibel: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

; $v \equiv 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$

$$\Rightarrow \underbrace{\frac{d\vec{v}}{dt}}_{\text{substantielle Beschleunig.}} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{lokale B.}} + \underbrace{u \frac{\partial \vec{v}}{\partial x} + v \frac{\partial \vec{v}}{\partial y}}_{=0 \text{ konvektive B.}} = \frac{\partial \vec{v}}{\partial t}$$

Impulsgleichungen:

reibungsfrei: $\eta \nabla^2 \vec{v} = 0$

mit (1):

$$\varrho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \varrho g_x$$

$$\underbrace{\varrho \frac{\partial v}{\partial t}}_{=0} = -\frac{\partial p}{\partial y} + \varrho g_y$$

oder $\varrho \frac{\partial \vec{v}}{\partial t} = -\nabla p + \varrho \vec{g}$

b) $\varrho \frac{\partial \vec{v}}{\partial t} = -\nabla p + \varrho \vec{g}$

dim. Grossen:

$\bar{t} = t \cdot f$; $\bar{\nabla} = h \cdot \nabla$; $\bar{\vec{v}} = \frac{\vec{v}}{f \cdot h}$; $\bar{p} = \frac{p}{\Delta p}$; $\bar{\vec{g}} = \frac{\vec{g}}{g_{ref}}$

Einsetzen:

$$\varrho f^2 h \frac{\partial \bar{\vec{v}}}{\partial \bar{t}} = -\frac{\Delta p}{h} \bar{\nabla} \bar{p} + \varrho g_{ref} \bar{\vec{g}}$$

$$\Rightarrow \frac{\partial \bar{\vec{v}}}{\partial \bar{t}} = -\overbrace{\frac{\Delta p}{\varrho f^2 h^2}}^{K_1} \bar{\nabla} \bar{p} + \overbrace{\frac{g_{ref}}{f^2 h}}^{K_2} \bar{\vec{g}}$$

Kennzahlen: $K_1 = \frac{\Delta p}{\varrho f^2 h^2}$ $[= Eu]$; $K_2 = \frac{g_{ref}}{f^2 h}$ $\left[= \frac{1}{Fr^2}\right]$

6. Aufgabe: (TSL: 5. Aufgabe)

a) $F(z) = u_\infty(x+iy) + \frac{i\Gamma}{2\pi}\ln\left(r_1 e^{i\theta_1}\right) - \frac{i\Gamma}{2\pi}\ln\left(r_2 e^{i\theta_2}\right)$

$r_1 = \sqrt{x^2+(y-b)^2}$; $r_2 = \sqrt{x^2+(y+b)^2}$

$\theta_1 = \arctan\left(\frac{y-b}{x}\right)$; $\theta_2 = \arctan\left(\frac{y+b}{x}\right)$

$\Psi = \operatorname{Im}(F(z)) = u_\infty y + \frac{\Gamma}{4\pi}\left[\ln\left(x^2+(y-b)^2\right) - \ln\left(x^2+(y+b)^2\right)\right]$

b)

$$\begin{cases} u = \frac{\partial\Psi}{\partial y} = u_\infty + \frac{\Gamma}{4\pi}\left[\frac{2(y-b)}{x^2+(y-b)^2} - \frac{2(y+b)}{x^2+(y+b)^2}\right] & (1) \\ v = -\frac{\partial\Psi}{\partial x} = -\frac{\Gamma}{4\pi}\left[\frac{2x}{x^2+(y-b)^2} - \frac{2x}{x^2+(y+b)^2}\right] & (2) \end{cases}$$

Staupunkte: $u = v = 0$

$v = 0$: $2x\left[x^2+(y+b)^2 - x^2 - (y-b)^2\right] = 0$

1. Fall:

$x = 0$: in (1) $\Rightarrow -\frac{4\pi u_\infty}{\Gamma} = \frac{2(y+b)-2(y-b)}{y^2-b^2}$

$y = \pm\sqrt{b^2 - \frac{b\Gamma}{\pi u_\infty}}$, wenn $b^2 \geq \frac{b\Gamma}{\pi u_\infty}$

2. Fall: $(y+b)^2 - (y-b)^2 = 0 \Rightarrow$

$y = 0$: in (1) $\Rightarrow -\frac{4\pi u_\infty}{\Gamma} = -\frac{4b}{x^2+b^2}$

$\Rightarrow x = \pm\sqrt{\frac{b\Gamma}{\pi u_\infty} - b^2}$, wenn $b^2 < \frac{b\Gamma}{\pi u_\infty}$

c)

1. Fall: $x = 0$; $y = \pm\sqrt{b^2 - \frac{b\Gamma}{\pi u_\infty}}$

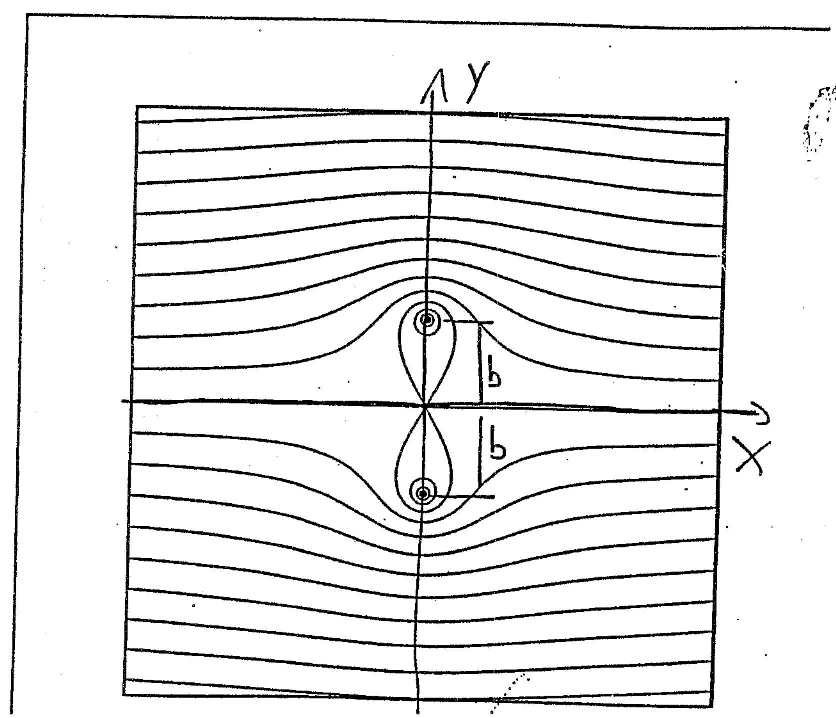
für $y = 0 \Leftrightarrow \Gamma = u_\infty b \pi$

oder:

2. Fall: $y = 0$; $x = \pm\sqrt{\frac{b\Gamma}{\pi u_\infty} - b^2}$

für $x = 0 \Leftrightarrow \Gamma = u_\infty b \pi$

d)



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7. Aufgabe: (TSL: 6. Aufgabe)

a)

Konti:

$u_0 H_0 = U(x) \cdot H(x) \Leftrightarrow U(x) = \frac{u_0 H_0}{H(x)} = \frac{u_0 H_0}{H_0 + \frac{c}{L} \cdot x}$

mit $U(x=L) = \frac{u_0}{2} \Rightarrow \frac{1}{2} = \frac{1}{1 + \frac{c}{H_0}} \Leftrightarrow c = H_0$

$U(x) = \frac{u_0}{1+\frac{x}{L}}$

Bernoulli:

$\frac{dp}{dx} = -\rho U \frac{dU}{dx}$ mit $\frac{dU}{dx} = -\frac{u_0/L}{(1+\frac{x}{L})^2}$

$\Rightarrow \frac{dp}{dx} = \rho \frac{u_0^2}{L(1+\frac{x}{L})^3}$

b)

$\frac{\partial\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{\delta}\right)} = a_1 + 2a_2\left(\frac{y}{\delta}\right) + 3a_3\left(\frac{y}{\delta}\right)^2 ; \quad \frac{\partial^2\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{\delta}\right)^2} = 2a_2 + 6a_3\left(\frac{y}{\delta}\right)$

1. R.B.: $y/\delta = 0$ $\Rightarrow u = 0 \Rightarrow a_0 = 0$

2. R.B.: $\frac{y}{\delta} = 1$: $u = U \Rightarrow 1 = a_1 + a_2 + a_3$

3. R.B.: $\frac{y}{\delta} = 0$: $\eta \frac{U}{\delta^2} \frac{\partial^2\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{\delta}\right)^2}\Bigg|_{\frac{y}{\delta}=0} = \frac{dp}{dx} \Rightarrow \eta \frac{u_0}{\delta^2}\left(\frac{1}{1+\frac{x}{L}}\right) 2a_2 = \rho \frac{u_0^2}{L(1+\frac{x}{L})^3}$

$\Rightarrow a_2 = \frac{\rho\delta^2}{2\eta} \frac{u_0}{L(1+\frac{x}{L})^2}$

4. R.B.: $\frac{y}{\delta} = 1 \Rightarrow \frac{\partial\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{\delta}\right)}\Bigg|_{\frac{y}{\delta}=1} = 0 = a_1 + 2a_2 + 3a_3$

$\Rightarrow$ mit $a_1 = 1 - a_2 - a_3 \Rightarrow 1 + a_2 + 2a_3 = 0$

$\Rightarrow a_3 = -\frac{1}{2} - \frac{\rho\delta^2}{4\eta} \frac{u_0}{L(1+\frac{x}{L})^2}$

$\Rightarrow a_1 = \frac{3}{2} - \frac{\rho\delta^2}{4\eta} \frac{u_0}{L(1+\frac{x}{L})^2}$

c) Ablösung:

$\frac{\partial u(x=L, y=0)}{\partial y} = 0 = a_1(L)$

$\Leftrightarrow \frac{3}{2} = \frac{\rho\delta^2}{4\eta} \frac{u_0}{4L} \Rightarrow \frac{L^2}{\delta^2} = \frac{L u_0 \rho}{24\eta}$

$\Rightarrow \frac{L}{\delta} = \sqrt{\frac{Re_L}{24}}$

d)

turbulente Umschlag, bewegte Wand, Absaugung, Ausblasen, usw.

8. Aufgabe:

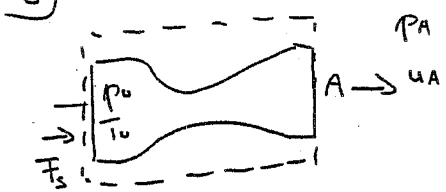
(TSL: 7. Aufgabe)

a) $c_p T_0 = c_p T + \frac{v^2}{2}$

mit $c_p = \frac{\gamma R}{\gamma - 1} \Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} \frac{v^2}{\gamma R T}$

$= 1 + \frac{\gamma-1}{2} M^2$

b)



Impulssatz:

$\rho_A u_A^2 A_A = F_s$

mit

$\frac{p_0}{p_A} = \left[1 + \frac{\gamma-1}{2} M_A^2\right]^{\frac{\gamma}{\gamma-1}} \Rightarrow M_A = 2$

mit $\frac{T_0}{T_A} = 1 + \frac{\gamma-1}{2} M_A^2 \Rightarrow T_A = 287\ K$

mit $p_A = \rho_A R T_A$

$\Rightarrow \rho_A = \frac{p_A}{R T_A} = 1{,}21\ kg/m^3$

$u_A = M_a \cdot a_A = M_a \cdot \sqrt{\gamma R T_A} = 679{,}2\ \frac{m}{s}$

$\Rightarrow A_A = \frac{F_s}{\rho_A u_A^2} = 0{,}5\ m^2$

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