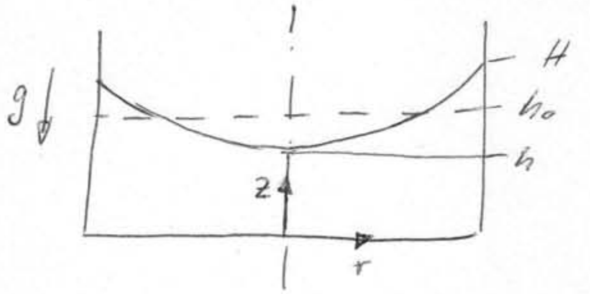
1\. Aufgabe



$$dp = \rho\omega^2 r\,dr - \rho g\,dz \quad 1^{1}$$

$$\int_{p_a}^{p} dp = \int_{r=0}^{r} \rho\omega^2 r\,dr - \int_{z=h}^{z} \rho g\,dz$$

$$p - p_a = \rho\omega^2 \frac{r^2}{2} - \rho g(z-h) = \rho g(h-z) + \frac{1}{2}\rho\omega^2 r^2 \quad 1^{2}$$

a) $p_a + \rho g h = p_a + \rho g z(r) - \frac{1}{2}\rho\omega^2 r^2 \quad 1^{3}$

$$z(r) = h + \frac{\omega^2 r^2}{2g} \qquad r = R:\ z = H \Rightarrow H = h + \frac{\omega^2 R^2}{2g} \quad 1^{4}$$

$$\rightarrow \omega^2 = \frac{2(H-h)g}{R^2}$$

$$z(r) = h + \frac{2(H-h)g r^2}{R^2 \cdot 2g} = h + (H-h)\left(\frac{r}{R}\right)^2 \quad 1^{5}$$

Volumen $V(\omega = 0) = V(\omega)$

$$\pi R^2 h_0 = \int_0^R z(r)\, 2\pi r\,dr = 2\pi \int_0^R \left[h + (H-h)\frac{r^2}{R^2}\right] r\,dr = \frac{\pi R^2}{2}(h+H) \quad 1_{6}$$

$$\Leftrightarrow h = 2h_0 - H \quad 1_{7}$$

$$\Rightarrow \underline{\underline{\omega}} = \sqrt{\frac{2(H - 2h_0 + H)g}{R^2}} = \underline{\underline{\sqrt{\frac{4(H-h_0)g}{R^2}}}} \quad 1^{8}$$

b)

$$r = R:\quad p = p_a + \rho g(H - z) \quad 1_{9}$$

$$z = 0:\quad p = p_a + \rho g h + \frac{1}{2}\rho\omega^2 r^2 \quad 1_{10}$$

2 |

$$p_a + \rho g H =$$

a) Bernoulli

$$= p_a + \rho g h + \frac{1}{2}\rho v_1^2 \left(1 + \lambda \frac{h}{D}\right) + \frac{1}{2}\rho v_0^2 \lambda \frac{L}{D} \quad — 1 \;\; 1$$

$$= p_a + \rho g h + \frac{1}{2}\rho v_2^2 \left(1 + \lambda \frac{L+h}{D}\right) + \frac{1}{2}\rho v_0^2 \lambda \frac{L}{D} \quad — 1 \;\; 2$$

Konti: $v_0 D^2 = (v_1 + v_2) D^2$ — 1 3

$$v_1^2 \left(1 + \lambda \frac{h}{D}\right) = v_2^2 \left(1 + \lambda \frac{L+h}{D}\right) \Rightarrow v_2 = \sqrt{\frac{1 + \lambda\, h/D}{1 + \lambda \frac{h+L}{D}}}\; v_1 = C_1 \cdot v_1$$

$$v_0 = (1 + C_1)\, v_1 \quad — 1 \;\; 3$$

$$p_a + \rho g H = p_a + \rho g h + \frac{1}{2}\rho v_1^2 \left(1 + \lambda \frac{h}{D} + (1+C_1)^2 \lambda \frac{L}{D}\right) \quad — 1 \;\; 4$$

$$v_1 = \sqrt{\frac{2g(H-h)}{1 + \lambda \frac{h}{D} + (1+C_1)^2 \lambda \frac{L}{D}}} \quad — 1 \;\; 7$$

$$\underbrace{v_0 = (1 + C_1)\, v_1 \;;\; v_2 = C_1 \cdot v_1}_{1 \;\; 5}$$

b) $$p_k = p_a + \rho g H - \frac{1}{2}\rho v_0^2 \lambda \frac{L}{D} - \frac{1}{2}\rho v_2^2 \left(1 + \lambda \frac{L}{2D}\right) \quad — 1 \;\; 8$$

$$= p_a + \rho g H - \frac{1}{2}\rho (1+C_1)^2 v_1^2 \lambda \frac{L}{D} - \frac{1}{2}\rho C_1^2 v_1^2 \left(1 + \lambda \frac{L}{2D}\right)$$

$$= p_a + \rho g h + \frac{1}{2}\rho v_2^2 \left(\lambda \frac{L}{2D} + \lambda \frac{h}{D}\right) \quad — 1 \;\; 9$$

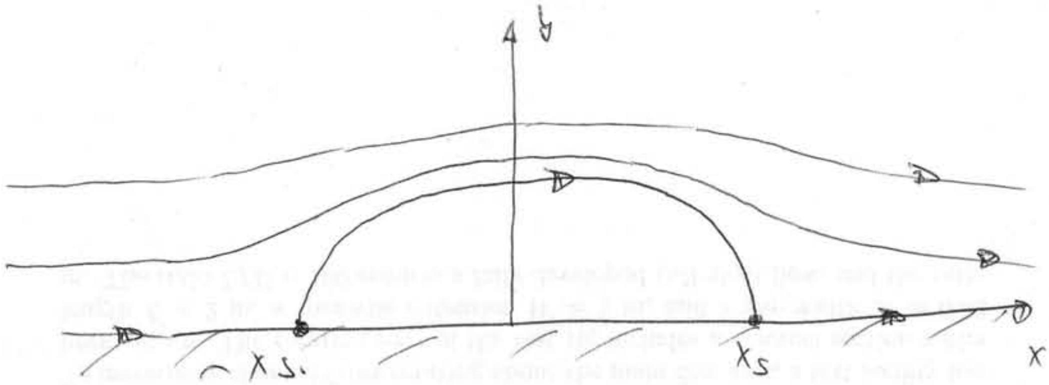
c) Die Verluste in den beiden Leitungen müssen gleich bleiben

- in der längeren Leitung eine Pumpe einbauen — 1 10
- durch bessere Oberflächenbeschaffenheit λ verringern — 1 11
- durch Vergrösserung der Querschnitte die Verluste verringern und die Strömung in einer verlustfreien Düse beschleunigen

$\Psi_u = \Psi(x_S, y_S) = 0$

$\Rightarrow$ Best. Glg.: $u_\infty y + \frac{E}{2\pi}\left[\arctan\frac{y}{x+a} - \arctan\frac{y}{x-a}\right] = 0$ (1)

d)



(1)

Alternativer Lösungsweg (für c))

$$d\Psi = \frac{\partial\Psi}{\partial x}dx + \frac{\partial\Psi}{\partial y}dy = -v\,dx + u\,dy$$

Hinweis

$$\int_0^y u\,dy \overset{\downarrow}{=} u_\infty y + \frac{E}{2\pi}\left(\arctan\frac{y}{x+a} - \arctan\frac{y}{x-a}\right) + C_1(x)$$

$$-\int_0^x v\,dx = -\frac{E}{2\pi}\left(\arctan\frac{x+a}{y} - \arctan\frac{x-a}{y}\right) + C_2(y)$$

Vergleich liefert, daß $C_1(x) = 0$

s.o.

$$\Rightarrow \Psi(x,y) = u_\infty + \frac{E}{2\pi}\left(\arctan\frac{y}{x+a} - \arctan\frac{y}{x-a}\right) = 0$$

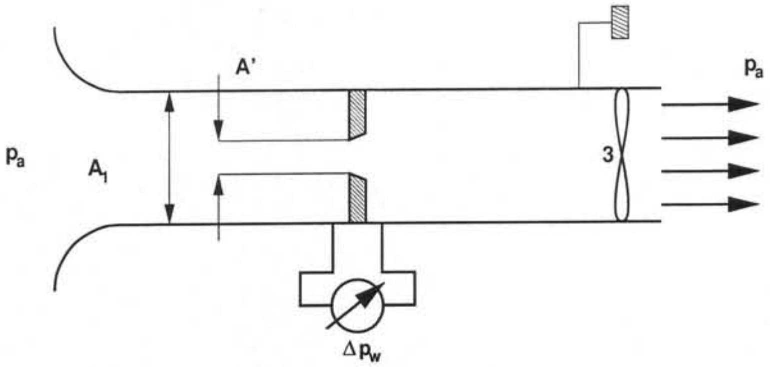
e) • Körperkanten sind Stromlinien

$\Rightarrow$ Haftbedingung ist nicht erfüllt (1)

• Wirbelfreiheit $\vec{\omega} = 0$ (1)

3. Aufgabe (17 Punkte)

Die Leistung eines Gebläses wird mit einer Blende bestimmt (geometrisches Öffnungsverhältnis $m = A'/A_1$, Kontraktionszahl Ψ, Durchflusszahl α).



a) Bestimmen Sie den theoretisch erreichbaren und den realen Volumenstrom (Herleitung).

b) Bestimmen Sie den statischen Druck in Punkt 3 vor dem Gebläse.

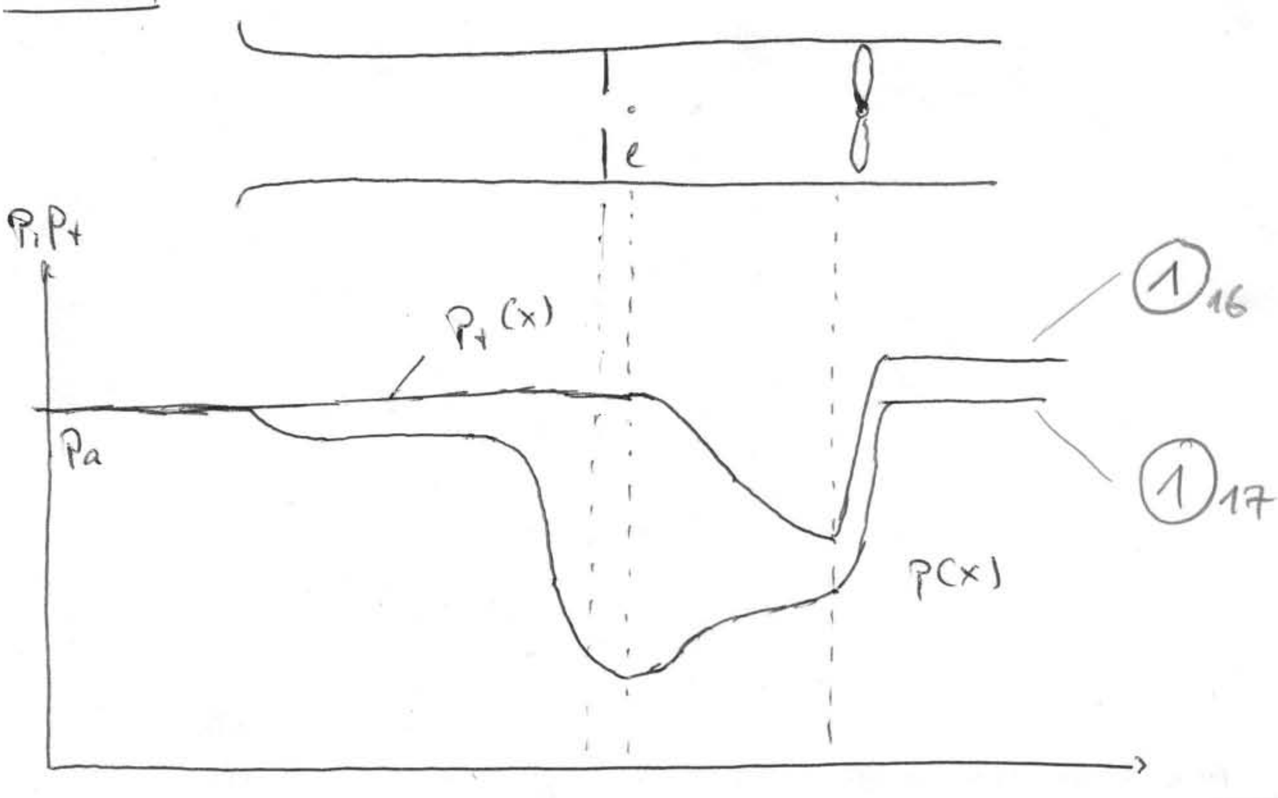
c) Bestimmen Sie die Gebläseleistung.

d) Skizzieren sie sorgfältig den Totaldruck $p_t(x)$ und den statischen Druck $p(x)$ entlang der Mittelachse.

Gegeben: $\rho = 1.2 kg/m^3$, $p_a = 10^5 N/m^2$, $\Delta p_W = 400 N/m^2$, $A_1 = 10^{-2} m^2$

$m = 0.6$, $\Psi = 0.6$, $\alpha = 0.8$

3d)



3 | Bernoulli bis in die Blende $p_1 + \frac{1}{2}\rho v_1^2 = p' + \frac{1}{2}\rho v'^2$ (1) 1

kont: $v_1 A_1 = v' A' = v' m A_1$ (1) 2

$\rightarrow p_1 - p' = \Delta p_w = \frac{1}{2}\rho v_1^2 (1 - m^2)$

$\rightarrow v' = \sqrt{\frac{2\Delta p_w}{\rho(1-m^2)}}$ (1) 3

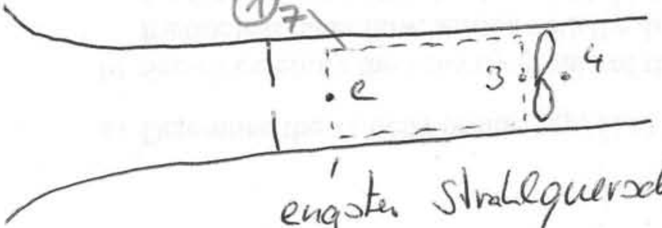
theoretische Volumenstrom (1) 4

$\dot{Q}_{th} = v' A' = A' \sqrt{\frac{2\Delta p_w}{\rho(1-m^2)}} = 0.198 \frac{m^3}{s}$

Der reale Volumenstrom wird durch Reibung, die Blendenform, m, etc beeinflusst. Die Einflüsse werden in der Durchflusszahl α zusammengefasst

(1) 5 $\rightarrow Q_{real} = \alpha m A_1 \sqrt{\frac{2\Delta p_w}{\rho}} = 0.124 \frac{m^3}{s}$ (1) 6

b) (1) 7



engsten Strahlquerschnitt

Impuls:

$-\rho v_e^2 A_e + \rho v_3^2 A_1 = (p_e - p_3) A_1$ (1) 8

Bernoulli bis in den engsten Strahlquerschnitt

$p_a = p_e + \frac{1}{2}\rho v_e^2$ (1) 9

$\dot{Q} = v_e A_e = v_e \frac{A_e}{A'} \frac{A'}{A_1} A_1 = v_e \, 4m A_1$

$\rightarrow v_e = \frac{\dot{Q}}{4m A_1} = 34{,}4 \; m/s$ (1) 10

(1) 11

$p_e = p_a - \frac{1}{2}\rho v_e^2 = 99288{,}2 \; N/m^2$

$v_3 = v_e \, A_e / A_1 = v_e \cdot 4m = 12.4 \; m/sec$

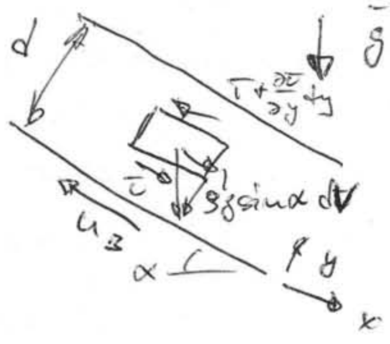
$p_3 = p_e + \rho v_e^2 \, 4m - \rho v_3^2 = 99614{,}9 \; N/m^2$ (1) 12

c) $P = \dot{Q} \, \Delta p_{0_{34}} = \dot{Q}(p_{04} - p_{03}) = \dot{Q}(p_4 - p_3) = \dot{Q}(p_e - p_3)$

(1) 13 $= 47{,}7 \; W$ (1) 14

(1) 15

A4 a)



$\vec{g}$ $0 = \tau \, dx\,dz - \left(\tau + \frac{\partial \tau}{\partial y} dy\right) dx\,dz + \rho g \sin\alpha \, dx\,dy\,dz$

$\Rightarrow \frac{\partial \tau}{\partial y} = \rho g \sin\alpha$ ①$_1$

Integr.

$\Rightarrow \tau = \rho g \sin\alpha \cdot y + C_1$ ①$_2$

RB: $\tau(y=d) = 0 \Rightarrow C_1 = -\rho g \sin\alpha \, d$

$\Rightarrow \tau(y) = \rho g \sin\alpha \, (y-d)$

$\tau = -K \left| \frac{du}{dy} \right| \frac{du}{dy}$ aus Hinweis

die Geschwindigkeit nimmt für $y > 0$ stetig zu $\Rightarrow \frac{du}{dy} > 0$ ①$_3$

also $\tau = -K \left(\frac{du}{dy}\right)^2 \Rightarrow \frac{du}{dy} = \sqrt{-\frac{1}{K} \cdot \rho g \sin\alpha \, (y-d)}$

Integrieren: $u(y) = \int_y \sqrt{\frac{\rho g \sin\alpha}{K} (d-y)} \, dy$ ①$_4$

$= -\frac{2}{3} \sqrt{\frac{\rho g \sin\alpha}{K}} (d-y)^{3/2} + C_2$

RB: $u(y=0) = u_B$ ①$_5$ $\Rightarrow C_2 = u_B + \sqrt{\frac{\rho g \sin\alpha}{K}} \cdot \frac{2}{3} d^{3/2}$

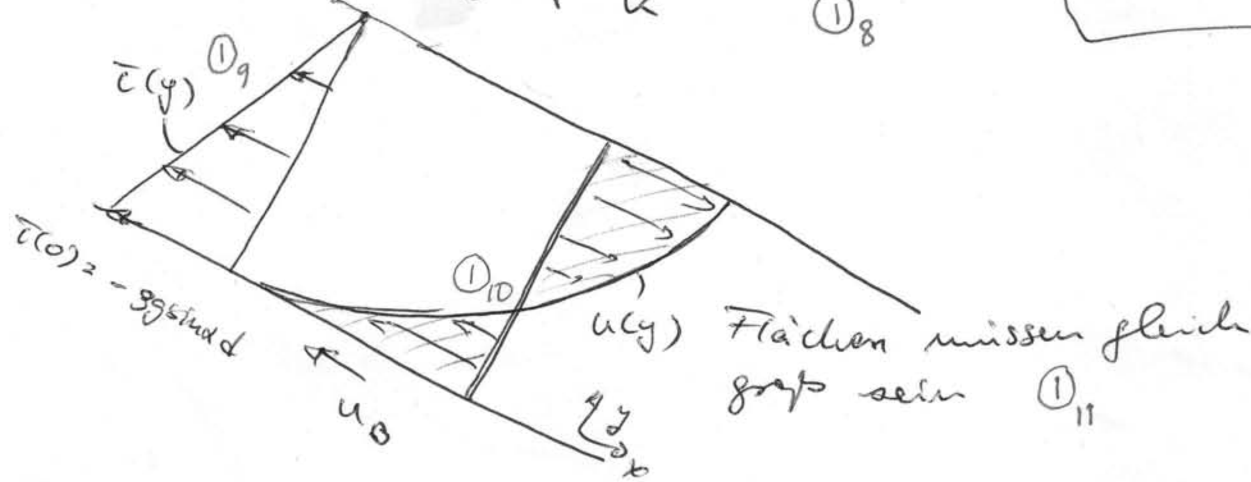
$\Rightarrow u(y) = \frac{2}{3} \sqrt{\frac{\rho g \sin\alpha}{K}} \left[d^{3/2} - (d-y)^{3/2} \right] + u_B$ ①$_6$

b) $\dot{Q} = \int_0^d u(y) \, dy \stackrel{!}{=} 0$ ①$_7$ $= \frac{2}{3} \sqrt{\frac{\rho g \sin\alpha}{K}} \left[d^{3/2} y - \frac{2}{5}(-1)(d-y)^{5/2} \right]_0^d + \ldots$

$\ldots + u_B y \Big|_0^d$

$\Rightarrow u_B = -\frac{2}{5} \sqrt{\frac{\rho g \sin\alpha}{K}} \, d^{3/2}$ ①$_8$

c)



A5

a) Stat.: $\frac{\partial(v_r)}{\partial t} = \frac{\partial(v_z)}{\partial t} = 0$ (1); ausgeb.: (1) $\frac{\partial v_z}{\partial z} = 0$; aus Konti: $\frac{\partial(r v_r)}{\partial r} = 0$

mit Randbed. $\Rightarrow v_r = 0$ (1)

$g_r = 0;\ g_z = g$ (1)

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \cancel{\frac{\partial v_z}{\partial z}}^{=0} = 0$$

$$\rho\left(\cancel{\frac{\partial v_r}{\partial t}}^{stat} + \cancel{v_r}\frac{\partial v_r}{\partial r}^{0(v_r=0)} + v_z \cancel{\frac{\partial v_r}{\partial z}}^{0(v_r=0)}\right) = \rho \cancel{g_r}^{0} - \frac{\partial p}{\partial r} + \eta\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\cancel{\frac{\partial}{\partial r}(r v_r)}^{0(Konti)}\right) + \cancel{\frac{\partial^2 v_r}{\partial z^2}}^{v_r=0}\right]$$

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}^{stat} + \cancel{v_r}^{v_r=0}\frac{\partial v_z}{\partial r} + v_z \cancel{\frac{\partial v_z}{\partial z}}^{=0}\right) = \rho g_z - \frac{\partial p}{\partial z} + \eta\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{\partial^2 v_z}{\partial z^2}}^{=0}\right]$$

$$\Rightarrow \quad \frac{\partial p}{\partial r} = 0$$

$$-\frac{\partial p}{\partial r} + \rho g + \frac{\eta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = 0 \quad (1)$$

b) $\bar r = \frac{r}{R};\ \bar z = \frac{z}{L};\ v_z = \frac{v_z}{u_m};\ \bar p = \frac{p}{p_{ref}};\ g = g_{ref};\ \rho = \rho_{ref};\ \bar\eta = \frac{\eta}{\eta_{ref}}$ (1)

$$\Rightarrow -\frac{p_{ref}}{L}\frac{\partial \bar p}{\partial \bar z} + \rho g + \frac{u_m \eta_{ref}}{R^2}\frac{\bar\eta}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar v_z}{\partial \bar r}\right) = 0 \quad (1)$$

$$\frac{\partial \bar p}{\partial \bar z} - \underbrace{\frac{L}{p_{ref}}\rho\cdot g}_{K_{1,I}^*} - \underbrace{\frac{u_m\cdot\eta_{ref}\cdot L}{R^2\cdot p_{ref}}}_{K_{2,I}^*}\frac{\bar\eta}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar v_z}{\partial \bar r}\right) = 0 \quad (1)$$

Alternativ mit $p_{ref} = \rho u_m^2$

$$\frac{\partial \bar p}{\partial \bar z} - \underbrace{\frac{L\cdot g}{u_m^2}}_{K_{1,II}^*} - \underbrace{\frac{\eta_{ref}\cdot L}{R^2\rho u_m}}_{K_{2,II}^*}\frac{\bar\eta}{\bar r}\frac{\partial}{\partial \bar r}\left(\bar r\frac{\partial \bar v_z}{\partial \bar r}\right) = 0$$

c)$_I$ $\Delta p, \rho, u_m, \eta, L, R, g$ (1)

$K = \Delta p\cdot \rho^\beta\cdot u_m^\gamma D^\delta$ (1) $\quad \left[\frac{kg}{s^2 m}\right]\cdot\left[\frac{kg}{m^3}\right]\cdot\left[\frac{m}{s}\right][m]$

Δp: kg: $0 = 1+\beta$
m: $0 = -1-3\beta+\gamma+\delta$
s: $0 = -2-\gamma$

$\beta = -1;\ \gamma = -2;\ \delta = 0$

$K_1 = \frac{\Delta p}{\rho u_m^2} = Eu$ (1)

η: kg: $0 = 1+\beta$
m: $0 = -1-3\beta+\gamma+\delta$
s: $0 = -1-\gamma$

$\beta = -1;\ \gamma = -1;\ \delta = -1$

$K_2 = \frac{\eta}{\rho u_m D} = \frac{1}{Re}$ (1)

L: kg: $0 = \beta$
m: $0 = 1-3\beta+\gamma+\delta$
s: $0 = -\gamma$

$\beta = \gamma = 0;\ \delta = -1$

$K_3 = \frac{l}{D}$ (1)

$$g: \begin{array}{l} kg: 0 = \beta \\ m: 0 = 1 - 3\beta + \gamma + \delta \\ s: 0 = -2 - \gamma \end{array} \Bigg\} \begin{array}{l} \beta = 0;\ \gamma = -2;\ \delta = 1 \\ K_4 = \dfrac{g \cdot D}{u_m^2} = \dfrac{1}{Fr^2} \end{array} \qquad 1$$

Alternativ $p_{ref} = \rho u_m^2$

L, u_m, η, R, ρ, g

$K = \eta\, \rho^{\beta} \cdot u_m^{\gamma} D^{\delta}$

$$\eta: \begin{array}{l} kg: 0 = 1 + \beta \\ m: 0 = -1 - 3\beta + \gamma + \delta \\ s: 0 = -1 - \gamma \end{array} \Bigg\} \begin{array}{l} \beta = -1;\ \gamma = -1;\ \delta = -1 \\ K_{1,II} = \dfrac{\eta}{\rho \cdot u_m \cdot D} = \dfrac{1}{Re} \end{array}$$

$$L: \begin{array}{l} kg: 0 = \beta \\ m: 0 = 1 - 3\beta + \gamma + \delta \\ s: 0 = -\gamma \end{array} \Bigg\} \begin{array}{l} \beta = 0;\ \gamma = 0;\ \delta = -1 \\ K_{2,II} = \dfrac{L}{D} \end{array}$$

$$g: \begin{array}{l} kg: 0 = \beta \\ m: 0 = 1 - 3\beta + \gamma + \delta \\ s: 0 = -2 - \gamma \end{array} \Bigg\} \begin{array}{l} \beta = 0;\ \gamma = -2;\ \delta = 1 \\ K_{3,III} = \dfrac{g \cdot D}{u_m^2} = \dfrac{1}{Fr^2} \end{array}$$

d) $$K_1^* = \frac{L \cdot \rho \cdot g}{p_{ref}} = \frac{\rho \cdot u_m^2}{p_{ref}} \cdot \frac{g \cdot D}{u_m^2} \cdot \frac{L}{D} = \frac{1}{Eu} \cdot \frac{1}{Fr^2} \cdot \frac{L}{D} \qquad 1$$

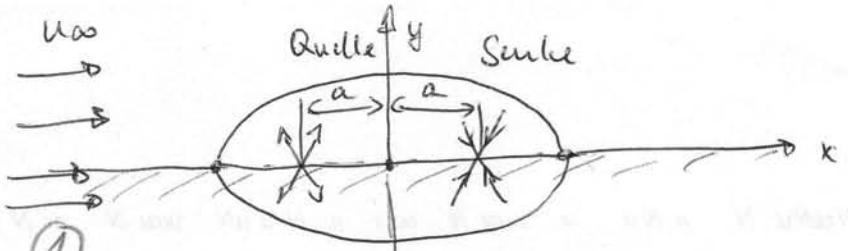
$$K_2^* = \frac{\eta_{ref} \cdot L \cdot u_m}{R^2 \cdot p_{ref}} = \frac{\rho \cdot u_m^2}{p_{ref}} \cdot \frac{\eta_{ref}}{\rho \cdot u_m R} \cdot \frac{L}{D} = \frac{1}{Eu} \cdot \frac{1}{Re} \cdot \frac{L}{D} \qquad 1$$

Alt.

$$K_{1,II}^* = \frac{g \cdot L}{u_m^2} = \frac{g \cdot D}{u_m^2} \cdot \frac{L}{D} = \frac{1}{Fr^2} \cdot \frac{L}{D}$$

$$K_{2,II}^* = \frac{\eta_{ref} \cdot L}{\rho u_m \cdot R^2} = \frac{\eta_{ref}}{\rho u_m \cdot D} \frac{L}{D} = \frac{1}{Re} \cdot \frac{L}{D}$$

A6 a)



(1)

$$F(z) = u_\infty z + \underbrace{\frac{E}{2\pi}\ln(z+a) - \frac{E}{2\pi}\ln(z-a)}_{(1)}$$

(1)

b) $$\bar{w} = \frac{dF}{dz} = u - iv$$

$$= u_\infty + \frac{E}{2\pi}\left(\frac{1}{z+a} - \frac{1}{z-a}\right) \quad (1)$$

$$= u_\infty + \frac{E}{2\pi}\left(\frac{x+a-iy}{(x+a)^2+y^2} - \frac{x-a-iy}{(x-a)^2+y^2}\right) \quad (1)$$

$$\Rightarrow \left.\begin{aligned} u(x,y) &= u_\infty + \frac{E}{2\pi}\left(\frac{x+a}{(x+a)^2+y^2} - \frac{x-a}{(x-a)^2+y^2}\right) \\ v(x,y) &= \frac{E}{2\pi}\left[\frac{y}{(x+a)^2+y^2} - \frac{y}{(x-a)^2+y^2}\right] \end{aligned}\right\} \quad (1)$$

c) Konturstromlinie = Staupunktstromlinie

Staupunkte: $u = v = 0$

Symmetrie: $y_S = 0$ (1) $\Rightarrow v(x, y_S) = 0$

aus $u(x_S, y_S = 0) = 0 \Rightarrow 0 = u_\infty + \frac{E}{2\pi}\left(\frac{x_S+a}{(x_S+a)^2+\cancel{y_S^2}} - \frac{x_S-a}{(x_S-a)^2+\cancel{y_S^2}}\right)$ (1)

$$\Rightarrow x_S = \pm\sqrt{\frac{aE}{\pi u_\infty} + a^2} \quad (1)$$

Konturstromlinie: $\Psi(x,y) = \Psi_K = \mathrm{Im}[F(z)]$

$F(z) = \Phi(x,y) + i\Psi(x,y)$ (1)

$$= u_\infty(x+iy) + \frac{E}{2\pi}\left[\ln r_1 e^{i\varphi_1} - \ln r_2 e^{i\varphi_2}\right]$$

mit $r_1 = \sqrt{(x+a)^2+y^2}$ $\qquad r_2 = \sqrt{(x-a)^2+y^2}$

$\varphi_1 = \arctan\frac{y}{x+a}$ $\qquad \varphi_2 = \arctan\frac{y}{x-a}$

$$\Rightarrow \Psi(x,y) = u_\infty y + \frac{E}{2\pi}\left[\arctan\frac{y}{x+a} - \arctan\frac{y}{x-a}\right] \quad (1)$$

7. Aufgabe a) Bernoulli: $\frac{dp}{dx} = -\rho\, u \frac{du}{dx} \rightarrow \int_{p_0}^{p} dp = \int_{x_0}^{x} -\rho \frac{u_0^2}{x_0^2}\, x\, dx$ 1_1

$\rightarrow p(x) = p_0 - \frac{1}{2}\rho \frac{u_0^2}{x_0^2}(x^2 - x_0^2)$ 1_2

Kont.: $u_0 H_0 = u(x) \cdot H(x)$ 1_3 $\rightarrow H(x) = \frac{u_0}{u(x)} H_0 = H_0 \frac{x_0}{x}$ 1_4

b) R.B.: Haftbed.: $u(0) = 0 \rightarrow a_0 = 0$

$u(\delta) = U \rightarrow a_1 + a_2 = 1$ 1_5

Wandbindung: $\frac{dp}{dx} = \eta \frac{\partial^2 u}{\partial y^2}\Big|_{y=0}$ 1_6 $\rightarrow -\rho \frac{u_0^2}{x_0^2} x = \eta \frac{2a_2}{\delta^2} \frac{u_0}{x_0} x$

$\rightarrow a_2 = -\frac{\rho}{2} \frac{u_0}{x_0} \frac{\delta^2}{\eta} \rightarrow a_1 = 1 + \frac{\rho}{2} \frac{u_0}{x_0} \frac{\delta^2}{\eta}$ 1_7

c) $\delta_1 = \delta \int_0^1 \left[1 - \left(1 + \frac{\rho}{2} \frac{u_0}{x_0} \frac{\delta^2}{\eta}\right) \frac{y}{\delta} + \frac{\rho}{2} \frac{u_0}{x_0} \frac{\delta^2}{\eta} \left(\frac{y}{\delta}\right)^2\right] d\frac{y}{\delta}$ 1_8

$= \delta\left[1 - \frac{1}{2}\left(1 + \frac{\rho}{2} \frac{u_0}{x_0} \frac{\delta^2}{\eta}\right) + \frac{1}{6}\rho \frac{u_0}{x_0} \frac{\delta^2}{\eta}\right]$

$= \delta\left[\frac{1}{2} - \frac{1}{12}\rho \frac{u_0}{x_0} \frac{\delta^2}{\eta}\right]$ 1_9

$\delta_2 = \delta \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\frac{y}{\delta}$ 1_{10} $= \delta \int_0^1 \left(a_1 \frac{y}{\delta} + a_2 \left(\frac{y}{\delta}\right)^2\right)\left(1 - a_1 \frac{y}{\delta} - a_2 \left(\frac{y}{\delta}\right)^2\right) d\frac{y}{\delta}$

$= \delta \int_0^1 a_1 \frac{y}{\delta} + (a_2 - a_1^2)\left(\frac{y}{\delta}\right)^2 - 2a_1 a_2 \left(\frac{y}{\delta}\right)^3 - a_2^2 \left(\frac{y}{\delta}\right)^4 d\left(\frac{y}{\delta}\right)$

$= \delta\left[\frac{1}{2} a_1 + \frac{1}{3}(a_2 - a_1^2) - \frac{1}{2} a_1 a_2 - \frac{1}{5} a_2^2\right]$

$= \delta\left[\frac{1}{2}(1 - a_2) + \frac{1}{3} a_2 - \frac{1}{3}(1 - a_2)^2 - \frac{1}{2}(1 - a_2) a_2 - \frac{1}{5} a_2^2\right]$

$= \delta\left[\frac{1}{6} + \left(-\frac{1}{2} + \frac{1}{3} + \frac{2}{3} - \frac{1}{2}\right) a_2 + \left(-\frac{1}{3} + \frac{1}{2} - \frac{1}{5}\right) a_2^2\right] = \delta\left(\frac{1}{6} - \frac{1}{30} a_2^2\right)$

$= \frac{1}{6} - \frac{1}{120}\left(\rho \frac{u_0}{x_0} \frac{\delta^2}{\eta}\right)^2$ 1_{11}

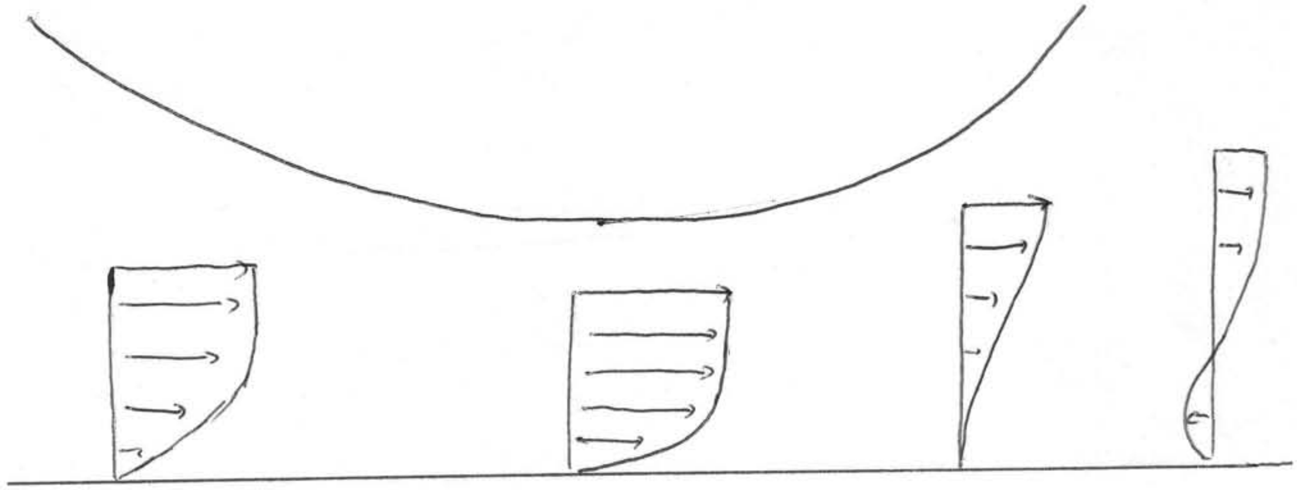
d) $$\frac{x_0}{u_0 x}\frac{u_0}{x_0}\left[\delta\left(\frac{1}{3}-\frac{1}{4}g\frac{u_0}{x_0}\frac{\delta^2}{\xi}\right)-\frac{17}{120}\left(g\frac{u_0}{x_0}\frac{\delta^2}{\xi}\right)^2+\frac{1}{2}-\frac{1}{12}g\frac{u_0}{x_0}\frac{\delta^2}{\xi}\right]$$

$$=\frac{\tau_w x_0^2}{g u_0^2 x^2} \Rightarrow k_1\cdot\frac{1}{x}=\tau_w\cdot k_2\cdot\frac{1}{x^2}$$ 1₁₂ für $\tau=-\tau_w$ 1₁₃

$$\rightarrow \tau_w=\frac{k_1}{k_2}\cdot x \rightarrow \tau \text{ wächst linear mit } x$$

e) Die betrachtete Grenzschicht ist beschleunigt und kann daher nicht ablösen. 1₁₄

f)



1₁₅ 1₁₆ 1₁₇

8. Aufgabe Stoß $\Rightarrow A_1 = A_2$ Konti: $\varrho_1 u_1 = \varrho_2 u_2$

a) Dichteverhältnis $\frac{\varrho_2}{\varrho_1} \overset{(1)}{=} \frac{u_1}{u_2} = \frac{u_1^2}{u_1 u_2} = \frac{u_1^2}{c^{*2}} = \frac{\gamma+1}{\gamma-1+\frac{2}{Ma^2}}$ (1)

Druckverhältnis $\frac{p_2}{p_1}$ $\lim_{M\to\infty} \frac{\varrho_2}{\varrho_1} = \frac{\gamma+1}{\gamma-1}$

Imp.: $\varrho_2 u_2^2 - \varrho_1 u_1^2 = p_1 - p_2$ (1) $/ : p_1$

$$\frac{p_2}{p_1} = 1 - \frac{\varrho_2 u_2^2}{p_1} + \frac{\varrho_1 u_1^2}{p_1} = 1 - \frac{\varrho_2 u_2^2}{\varrho_1 R T_1} + \frac{u_1^2}{R T_1}\cdot\frac{\gamma}{\gamma}$$

$$= 1 + M_1^2 \gamma - \frac{u_1}{u_2}\,\frac{u_2^2}{R T_1}\cdot\frac{u_1}{u_1} = 1 + M_1^2 \gamma\left(1 - \frac{u_2}{u_1}\right)$$

(1)

$$= 1 + \gamma M_1^2\left(1 - \frac{(\gamma-1)M_1^2+2}{(\gamma+1)M_1^2}\right) = 1 + \gamma M_1^2\left(\frac{2(M_1^2-1)}{(\gamma+1)M_1^2}\right) \quad (2)$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1) \quad ; \lim_{M\to\infty} \frac{p_2}{p_1} \to +\infty$$

b) σ_{max}: $\perp$ VS' $(90°)$ (1); σ_{min}: Mach'sche Linie $\left(\alpha = \arcsin\frac{1}{M}\right)$ (1)

c) $\varrho_1 u_1 A_1 = \varrho_2 u_2 A_2 = \varrho^* u^* A^*$ (1)

$$\frac{A^*}{A_1} = \frac{\varrho_1}{\varrho^*}\,\frac{u_1}{u^*} = \frac{\varrho_1}{\varrho_0}\,\frac{\varrho_0}{\varrho^*}\cdot\frac{u_1}{c_0}\cdot\frac{c_0}{u^*}$$

$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$ (1); $\frac{T_0}{T^*} = \frac{\gamma+1}{2} = \left(\frac{\varrho_0}{\varrho^*}\right)^{\gamma-1}$ (1); $\frac{c_0}{u^*} = \sqrt{\frac{\gamma R T_0}{\gamma R T^*}} = \sqrt{\frac{T_0}{T^*}}$

$\Rightarrow \frac{\varrho_0}{\varrho^*} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}$; $\frac{c_0}{u^*} = \sqrt{\frac{\gamma+1}{2}}$; $\frac{\varrho_1}{\varrho_0} = \left(\frac{T_1}{T_0}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\frac{1}{\gamma-1}}$

$\frac{u_1}{c_0} = M_1 \cdot \frac{c_1}{c_0} = M_1\sqrt{\frac{T_1}{T_0}} = M_1\left(1 + \frac{\gamma-1}{2}M_1^2\right)^{-\frac{1}{2}}$ (1)

Einsetzen ergibt:

$$A_2 = A_1 \cdot \frac{M_1}{\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \quad (1)$$

d) $\sigma = 90°$; abgelöster VS
(1)

(1)