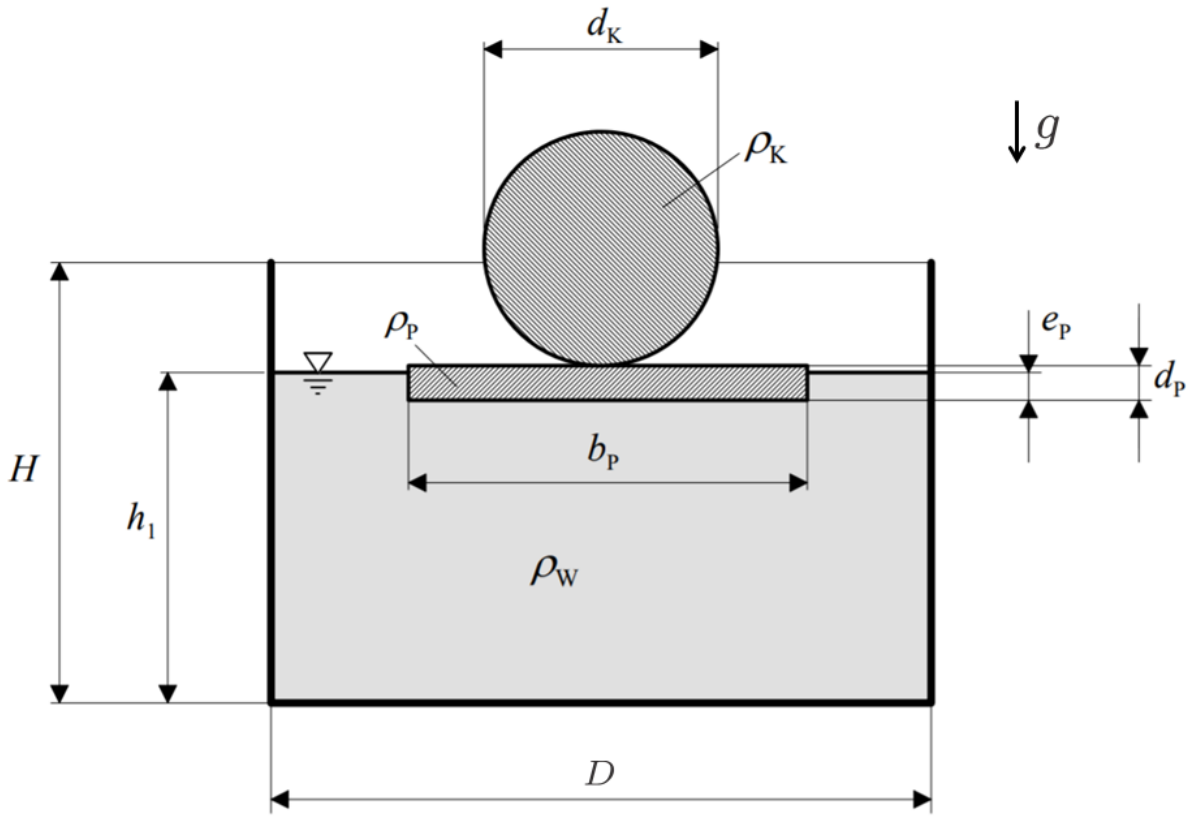
Problem 1 (9 Points)

A rectangular plate (length L_P, width b_P, thickness d_P) floats in a circular basin with diameter D. A sphere with diameter d_K is located in the center of the plate. The water level in the basin is h_1.



a) Determine the immersion depth e_P as a function of the given quantities.

Mow, the plate is removed and the sphere sinks to the ground at constant speed c.

b) Determine the new water level h_2 as a function of the given quantities.

c) Determine the steady-state sinking speed c as a function of the given quantities. Assume the total drag of the sphere $C_{W,ges}$ to be known.

Given:

$h_1, \quad D, \quad b_P, \quad L_P, \quad d_P, \quad d_K, \quad \rho_W, \quad \rho_P, \quad \rho_K, \quad g$

Hints:

- Neglect the influence of the surrounding air!
- The volume of a sphere is: $V_K = \frac{4}{3}\pi r^3$
- The drag force of a body related to its projectedarea A_{proj} is determined by

$$F_W = C_W \frac{\rho}{2} v^2 A_{proj}$$

- Check the units and signs of your results!

Problem 1

a)

$$\begin{aligned} F_G &= F_A \\ \rho_K V_K g + \rho_P V_P g &= \rho_W V_{ein} g \\ \rho_K \frac{\pi}{6} d_K^3 g + \rho_P L_P b_P d_P g &= \rho_W L_P b_P e_P g \\ e_P &= \frac{\rho_K \frac{\pi}{6} d_K^3 + \rho_P L_P b_P d_P}{\rho_W L_P b_P} \end{aligned}$$

b) Volumen of the water:

$$V_W = \pi \frac{D^2}{4} h_1 - L_P b_P e_P$$

New water level h_2:

$$\begin{aligned} V_{ges} = V_W + V_K &= \pi \frac{D^2}{4} h_2 \\ \pi \frac{D^2}{4} h_1 - L_P b_P e_P + \frac{\pi}{6} d_K^3 &= \pi \frac{D^2}{4} h_2 \\ h_2 &= \frac{\pi \frac{D^2}{4} h_1 - L_P b_P e_P + \frac{\pi}{6} d_K^3}{\pi \frac{D^2}{4}} \end{aligned}$$

c) Equilibrium of the forces:

$$\begin{aligned} F_G &= F_A + F_W \\ \rho_K \frac{\pi}{6} d_K^3 g &= \rho_W \frac{\pi}{6} d_K^3 g + C_{W,ges} \frac{\rho_W}{2} c^2 \frac{\pi}{4} d_K^2 \\ c^2 &= \frac{\rho_K \frac{\pi}{6} d_K^3 g - \rho_W \frac{\pi}{6} d_K^3 g}{C_{W,ges} \frac{\rho_W}{2} \frac{\pi}{4} d_K^2} \\ c &= \sqrt{\frac{4(\rho_K - \rho_W) d_K g}{3 C_{W,ges} \rho_W}} \end{aligned}$$