Musterlösung H02

1. Aufgabe

a) Barometrische Höhenformel:

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho(z)g = -\frac{p(z)}{R_L T(z)}g \quad \text{mit} \quad \rho(z) = \frac{p(z)}{R_L T(z)}$$

$$\Rightarrow \frac{1}{p}\mathrm{d}p = -\frac{g}{R_L}\frac{\mathrm{d}z}{(T_0 - az)}$$

$$\Rightarrow \int_{p(z=0)}^{p(z)} \frac{1}{p}\mathrm{d}p = -\frac{g}{R_L}\int_{z=0}^{z}\frac{1}{T_0 - az}\mathrm{d}z = \ln\left(\frac{T_0 - az}{T_0}\right)^{\frac{g}{R_L a}} \quad \text{mit} \quad p(z=0) = p_a$$

$$\Rightarrow p(z) = p_a\left(\frac{T_0 - az}{T_0}\right)^{\frac{g}{R_L a}}$$

b) Kräftegleichgewicht für $z = 0$:

$$\sum_i F_i = 0 \Rightarrow -(m_z g + F_s + m_{He}g) + V_{AK}\rho_L(z=0)g = 0$$

$$\Rightarrow m_z g + F_s = V_{AK}\left(\rho_L(z=0) - \rho_{He}(z=0)\right)g$$

$$\Rightarrow m_z g + F_s = V_{AK}\left(\frac{p_a}{R_L T_0} - \frac{p_a}{R_{He}T_0}\right)g$$

$$V_{AK} = (m_z g + F_s)\frac{T_0}{p_a g}\left(\frac{R_L R_{He}}{R_{He} - R_L}\right)$$

$$m_{He} = \rho_{He}V_{Ges} = \frac{m_z g + F_s}{g}\left(\frac{R_L}{R_{He} - R_L}\right)$$

c) Gewichtskraft der verbleibenden Löschwassermenge: $0,1F_s$
Kräftegleichgewicht im Schwebezustand z_{max}

$$\sum_i F_i = 0 \Rightarrow \rho_L(z_{max})gV_{AK}^* = 0,1F_s + m_z g + m_{He}^* g$$

$$\text{mit} \quad \rho_L(z) = \rho_0\left(\frac{T_0 - az}{T_0}\right)^{\frac{g}{R_L a}-1} \quad \text{folgt}$$

$$\frac{p_a}{R_L T_0}\left(\frac{T_0 - az_{max}}{T_0}\right)^{\frac{g}{R_L a}-1} V_{AK}^* g = 0,1F_s + m_z g + m_{He}^* g$$

$$\Rightarrow z_{max} = \frac{T_0}{a} - \frac{T_0}{a}\left(\frac{R_L T_0}{p_a V_{AK}^* g}\left(0,1F_s + m_z g + m_{He}^* g\right)\right)^{\frac{1}{\frac{g}{R_L a}-1}}$$

2. Aufgabe

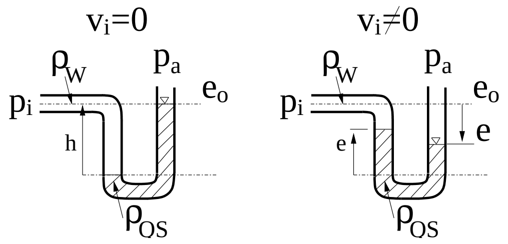
a) Massenstrom $\dot{m}$:
Bernoulli vom oberen Beckenrand zum Austritt:

$$p_a + \rho_W g(H + 2L_1 + L_2) = p_a + \frac{\rho_W}{2} v_a^2 \left(1 + \lambda \frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma} \left(\frac{D_1}{D_2}\right)^4 + \zeta_K\right)$$

$$\Rightarrow v_a = \sqrt{\frac{2g(H + 2L_1 + L_2)}{1 + \lambda \frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma} \left(\frac{D_1}{D_2}\right)^4 + \zeta_K}}$$

$$\dot{m} = \rho_W v_a \frac{\pi D_1^2}{4} = \rho_W \frac{\pi}{4} D_1^2 \sqrt{\frac{2g(H + 2L_1 + L_2)}{1 + \lambda \frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma} \left(\frac{D_1}{D_2}\right)^4 + \zeta_K}}$$

b) Auslenkung e



$$v_i = 0: \quad p_a + \rho_W g(H + L_1 + \frac{L_2}{2} + h) = p_a + \rho_{QS} g h$$

$$\Rightarrow h = (H + L_1 + \frac{L_2}{2}) \frac{\rho_W}{\rho_{QS} - \rho_W} := K_1$$

$$v_i \neq 0: \quad p_i + \rho_W g(h - e) + \rho_{QS} e g = p_a + \rho_{QS} g(h - e)$$

$$\Rightarrow p_i = p_a + \rho_{QS} g(h - 2e) - \rho_W g(h - e) \quad (1)$$

Konti: $\quad v_i A_i = v_a A_a \leftrightarrow v_i = v_a \frac{D_1^2}{D_2^2}$

Bernoulli vom oberen Beckenrand zum Eintritt Manometer:

$$p_a + \rho_W g(H + L_1 + \frac{L_2}{2}) = p_i + \frac{\rho_W}{2} v_i^2 + \zeta_{Me} \frac{\rho_W}{2} v_a^2 + \lambda \frac{L_1}{D_1} \frac{\rho_W}{2} v_a^2$$

$$\Rightarrow \rho_W g(H + L_1 + \frac{L_2}{2}) - \frac{\rho_W}{2} v_a^2 \left[\left(\frac{D_1}{D_2}\right)^4 + \lambda \frac{L_1}{D_1} + \zeta_{Me}\right] := K_2 = p_i - p_a$$

mit $\quad p_i - p_a = h(\rho_{QS} - \rho_W) g - e(2\rho_{QS} - \rho_W) g \quad$ aus $\quad (1)$

$$\Rightarrow \qquad e = \frac{K_1(\rho_{QS} - \rho_W) g - K_2}{(2\rho_{QS} - \rho_W) g}$$

c) Mit $D_2 \downarrow$ wird $v_i \uparrow \rightarrow p_i \downarrow \Rightarrow e$ wird größer

d)

$$p_a + \rho_W g(H + 2L_1 + L_2) =$$

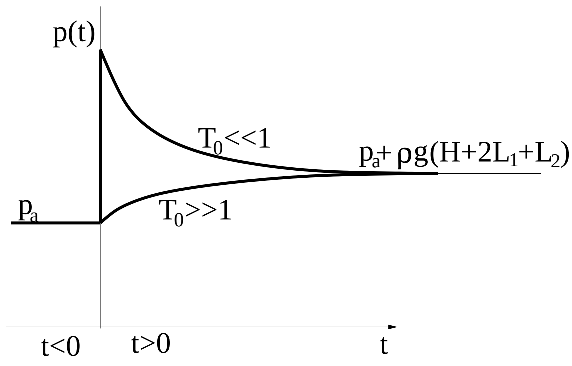
$$p(t) + \frac{\rho_W}{2} v_a(t)^2 \left(1 + \lambda \frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma} \left(\frac{D_1}{D_2}\right)^4 + \zeta_K + \zeta_D\right) + \rho_W \int_0^{Aus} \frac{\partial v}{\partial t} \mathrm{d}s$$

$$\text{mit} \quad \rho_W \int_0^{Aus} \frac{\partial v}{\partial t} \mathrm{d}s = \rho_W \frac{\mathrm{d}v_a(t)}{\mathrm{d}t} \left(3L_1 + \left(\frac{D_1}{D_2}\right)^2 L_2\right), \quad \frac{\mathrm{d}v_a(t)}{\mathrm{d}t} = -\frac{v_0}{T_0} e^{\left(\frac{-t}{T_0}\right)}$$

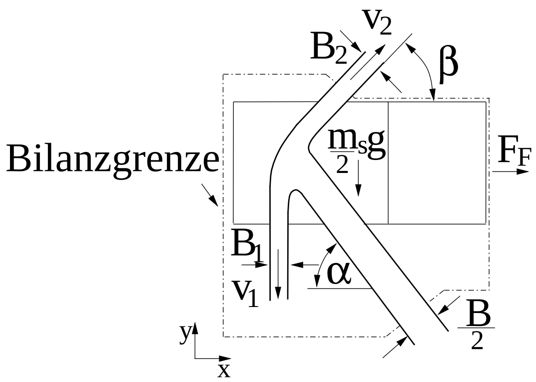
$$\Rightarrow \quad p(t) = p_a + \rho_W g(H + 2L_1 + L_2) -$$

$$\frac{\rho_W}{2} v_0^2 \left(e^{\left(\frac{-2t}{T_0}\right)}\right) \left(1 + \lambda \frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma} \left(\frac{D_1}{D_2}\right)^4 + \zeta_K + \zeta_D\right) +$$

$$\frac{v_0}{T_0} e^{\left(\frac{-t}{T_0}\right)} \left(3L_1 + \left(\frac{D_1}{D_2}\right)^2 L_2\right)$$



3. Aufgabe



a) Breite B_1 und B_2 der Teilstrahlen

Bernoulli: $\Rightarrow v_1 = v_2 = v_0$

Konti: $\Rightarrow B_1 v_1 + B_2 v_2 = \frac{B}{2} v_0$

y-Impuls: $-\rho v_0^2 \frac{B}{2} H \sin\alpha + \rho v_0^2 B_2 H \sin\beta - \rho v_0^2 B_1 H = -\frac{1}{2} m_s g$

$\Rightarrow -\frac{B}{2}\sin\alpha + (\frac{B}{2} - B_1)\sin\beta - B_1 = -\frac{m_s g}{2\rho v_0^2 H}$

$\Rightarrow \frac{B}{2}(\sin\beta - \sin\alpha) - B_1(\sin\beta + 1) = -\frac{m_s g}{2\rho v_0^2 H}$

$\Rightarrow B_1 = \frac{B}{2}(\frac{\sin\beta - \sin\alpha}{1+\sin\beta}) + \frac{m_s g}{2\rho v_0^2 H(1+\sin\beta)}$

$\Rightarrow B_2 = \frac{B}{2} - B_1 = \frac{B}{2}\left(\frac{1+\sin\alpha}{1+\sin\beta}\right) - \frac{m_s g}{2\rho v_0^2 H(1+\sin\beta)}$

b) Federkraft F_F:

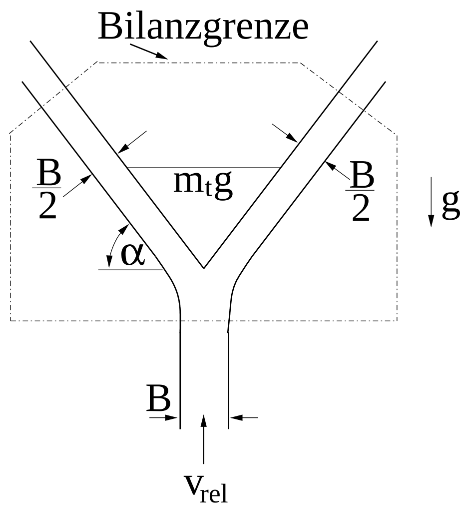
x-Impuls: $\Rightarrow \rho\frac{B}{2} H v_0^2 \cos\alpha + \rho H B_2 v_0^2 \cos\beta = F_F$

$\Rightarrow F_F = \rho H v_0^2 \left(\frac{B}{2}\cos\alpha + \left[\frac{B}{2}\frac{1+\sin\alpha}{1+\sin\beta} - \frac{m_s g}{2\rho v_0^2 H(1+\sin\beta)}\right]\cos\beta\right)$

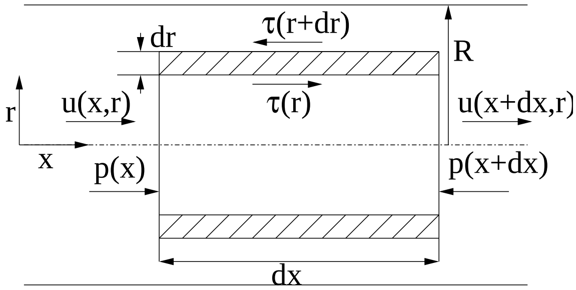
c) Masse des Strahlteilers m_t:

y-Impuls im Relativsystem: $\Rightarrow -\rho v_{rel}^2 BH + 2\rho\frac{B}{2} H v_{rel}^2 \sin\alpha = -m_t g$

mit $\quad v_{rel} = v_0 + v_s = \frac{3}{2} v_0 \Rightarrow m_t = \frac{\rho v_{rel}^2 BH}{g}(1-\sin\alpha) = \frac{9}{4}\frac{\rho v_0^2 BH}{g}(1-\sin\alpha)$


Bilanzgrenze
B
2
m_t g
B
2
g
α
B
v_rel

4. Aufgabe



a) Impulssatz am ringförmigen Element:
ausgebildete Strömung $\Rightarrow u(x,r) = u(x+\mathrm{d}x, r)$
$\Rightarrow$ eintr. Impulstrom $\dot{I}_{Ein}$= austr. Impulsstrom $\dot{I}_{Aus}$

$$\Rightarrow \dot{I}_{Ein} - \dot{I}_{Aus} = \quad 2\pi r p \mathrm{d}r - 2\pi r(p + \frac{\mathrm{d}p}{\mathrm{d}x}\mathrm{d}x)\mathrm{d}r + 2\pi r \mathrm{d}x\tau - 2\pi(r+\mathrm{d}r)\mathrm{d}x(\tau + \frac{\mathrm{d}\tau}{\mathrm{d}r}\mathrm{d}r) = 0$$

$$\Rightarrow \quad -\frac{\mathrm{d}p}{\mathrm{d}x} - \frac{\mathrm{d}\tau}{\mathrm{d}r} - \frac{\tau}{r} - \frac{\mathrm{d}\tau}{r} = 0, \quad \text{wobei } \frac{\mathrm{d}\tau}{r} \text{ von höherer Ordnung klein ist}$$

$$\Rightarrow \quad \frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{1}{r}\frac{\mathrm{d}(\tau r)}{\mathrm{d}r}$$

$$\Rightarrow \quad \text{1. Integration:} \quad \tau = -\frac{r}{2}\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{C_1}{r}, \quad \text{mit Rdb.:} \quad \tau(r=0) = 0 \Rightarrow C_1 = 0$$

$$\text{Newton'sches Fließgesetz: } \tau = -\eta\frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{r}{2}\frac{\mathrm{d}p}{\mathrm{d}x} \quad \text{mit} \quad -\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{p_1 - p_2}{L}$$

$$\Rightarrow \quad \text{2. Integration: } u(r) = \frac{1}{2\eta}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{r^2}{2} + C_2, \quad \text{mit Rdb.: } u(r=R) = 0$$

$$\Rightarrow \quad C_2 = -\frac{1}{2\eta}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{R^2}{2}$$

$$\Rightarrow \quad u(r) = \frac{R^2}{4\eta}\frac{p_1 - p_2}{L}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

b) Wandschubspannung τ_w und mittlere Geschwindigkeit u_m

$$\tau_w = -\tau(r=R) = -\frac{R}{2}\frac{(p_1 - p_2)}{L}, \quad \text{mit} \quad u_m = \frac{\dot{Q}}{A}, \quad A = \pi R^2$$

$$\dot{Q} = \int_0^R u(r) 2\pi r \mathrm{d}r = \frac{\pi R^4}{2\eta}\frac{p_1 - p_2}{L}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^R = \frac{\pi R^4}{8\eta}\frac{p_1 - p_2}{L}$$

$$\Rightarrow \quad u_m = \frac{R^2}{8\eta}\frac{p_1 - p_2}{L}$$

c) Rohrreibungsbeiwert λ

$$\Delta p = \lambda \frac{L}{D}\frac{\rho}{2}u_m^2, \quad \text{mit} \quad \Delta p = p_1 - p_2$$

$$u_m = \frac{R^2}{8\eta}\frac{p_1 - p_2}{L} \Longleftrightarrow \frac{p_1 - p_2}{\frac{\rho}{2}u_m^2} = \frac{16\eta L}{\rho u_m R^2} = \frac{64\eta}{\rho u_m D}\frac{L}{D} \Rightarrow \lambda = \frac{64}{Re_D}$$

5. Aufgabe

a) Variable des Systems: η, ρ, H, D, F_W, f, u_∞ mit

$$[\eta] = \left[\frac{\text{kg}}{\text{ms}}\right], \quad [\rho] = \left[\frac{\text{kg}}{\text{m}^3}\right], \quad [H] = [m], \quad [D] = [m],$$
$$[F_W] = \left[\frac{\text{kgm}}{\text{s}^2}\right], \quad [f] = \left[\frac{1}{\text{s}}\right], \quad [u_\infty] = \left[\frac{\text{m}}{\text{s}}\right]$$

$\Rightarrow$ sieben Einflußgrößen, drei Grunddimensionen $\rightarrow$ vier Kennzahlen
wähle drei wiederkehrende Variable, z.B. : ρ, u_∞, D

$$K_1 = \eta \cdot u_\infty^{\alpha_1} \cdot \rho^{\beta_1} \cdot D^{\gamma_1}$$
$$\begin{array}{lccccccc} \text{kg}: & 1 & + & 0 & + & \beta_1 & + & 0 = 0 \Rightarrow \beta_1 = -1 \\ \text{m}: & -1 & + & \alpha_1 & - & 3\beta_1 & + & \gamma_1 = 0 \Rightarrow \gamma_1 = -1 \\ \text{s}: & -1 & - & \alpha_1 & + & 0 & + & 0 = 0 \Rightarrow \alpha_1 = -1 \end{array}$$
$$\Rightarrow K_1 = \frac{\eta}{\rho u_\infty D} = \frac{1}{Re}$$

$$K_2 = F_W \cdot u_\infty^{\alpha_2} \cdot \rho^{\beta_2} \cdot D^{\gamma_2}$$
$$\begin{array}{lccccccc} \text{kg}: & 1 & + & 0 & + & \beta_2 & + & 0 = 0 \Rightarrow \beta_2 = -1 \\ \text{m}: & 1 & + & \alpha_2 & - & 3\beta_2 & + & \gamma_2 = 0 \Rightarrow \gamma_2 = -2 \\ \text{s}: & -2 & - & \alpha_2 & + & 0 & + & 0 = 0 \Rightarrow \alpha_2 = -2 \end{array}$$
$$\Rightarrow K_2 = \frac{F_W}{\rho u_\infty^2 D^2} = \bar{c_w}$$

$$K_3 = H \cdot u_\infty^{\alpha_3} \cdot \rho^{\beta_3} \cdot D^{\gamma_3}$$
$$\begin{array}{lccccccc} \text{kg}: & 0 & + & 0 & + & \beta_3 & + & 0 = 0 \Rightarrow \beta_3 = 0 \\ \text{m}: & 1 & + & \alpha_3 & + & 0 & + & \gamma_1 = 0 \Rightarrow \gamma_3 = -1 \\ \text{s}: & 0 & - & \alpha_3 & + & 0 & + & 0 = 0 \Rightarrow \alpha_3 = 0 \end{array}$$
$$\Rightarrow K_3 = \frac{H}{D} = geometr.$$

$$K_4 = f \cdot u_\infty^{\alpha_4} \cdot \rho^{\beta_4} \cdot D^{\gamma_4}$$
$$\begin{array}{lccccccc} \text{kg}: & 0 & + & 0 & + & \beta_4 & + & 0 = 0 \Rightarrow \beta_4 = 0 \\ \text{m}: & 0 & - & \alpha_4 & - & 3\beta_4 & + & \gamma_4 = 0 \Rightarrow \gamma_4 = 1 \\ \text{s}: & -1 & - & \alpha_4 & + & 0 & + & 0 = 0 \Rightarrow \alpha_4 = -1 \end{array}$$
$$\Rightarrow K_4 = \frac{fD}{u_\infty} = Sr$$

b)

$$Re_{Real} = Re_{Kryo} \Rightarrow \frac{u_{\infty Kryo}}{u_{\infty Real}} = \frac{\eta_{Kryo}}{\eta_{Real}} \frac{\rho_{Real}}{\rho_{Kryo}} \frac{D_{Real}}{D_{Kryo}} = \left(\frac{T_{Kryo}}{T_{Real}}\right)^{1{,}72} \frac{D_{Real}}{D_{Kryo}}$$

mit $\quad \dfrac{\eta_{Kryo}}{\eta_{Real}} = \left(\dfrac{T_{Kryo}}{T_{Real}}\right)^{0{,}72}$

und id. Gas $\quad \dfrac{p}{\rho} = RT \rightarrow \dfrac{\rho_{Real}}{\rho_{Kryo}} = \dfrac{T_{Kryo}}{T_{Real}}, \quad$ da $\quad R_{Real} = R_{Kryo}$

$$Sr_{Real} = Sr_{Kryo} \Rightarrow \frac{f_{Kryo}}{f_{Real}} = \frac{D_{Real}}{D_{Kryo}} \frac{u_{\infty Kryo}}{u_{\infty Real}} = \left(\frac{D_{Real}}{D_{Kryo}}\right)^2 \left(\frac{T_{Kryo}}{T_{Real}}\right)^{1{,}72} = 37,24$$

c)

$$M_{Kryo} \leq \frac{u_{\infty Kryo}}{\sqrt{\gamma R T_{Kryo}}} = \left(\frac{T_{Kryo}}{T_{Real}}\right)^{1{,}72} \frac{D_{Real}}{D_{Kryo}} \frac{1}{\sqrt{\gamma R T_{Kryo}}} u_{\infty Real} = 0,0145 u_{\infty Real} \cdot \left[\frac{\mathrm{s}}{\mathrm{m}}\right]$$

mit $\quad M_{Kryo,max} \leq 0,3 \quad \Rightarrow u_{\infty Real,max} \leq 20 \dfrac{\mathrm{m}}{\mathrm{s}}$

6. Aufgabe

a) Randbedingungen:

1. R.B.: $y/\delta = 0$: $u/U = 0$
2. R.B.: $y/\delta = 1$: $u/U = 1$
3. R.B.: $y/\delta = 0$: $\frac{\partial^2 u}{\partial y^2} = 0$ (Wandbindung und $\frac{\mathrm{d}p}{\mathrm{d}x} = 0$ für ebene Platte)
4. R.B.: $y/\delta = 1$: $\frac{\partial u}{\partial y} = 0$

$$\Rightarrow \quad \left| \begin{array}{llllll} 0 & = a_0 & & & \\ 1 & = a_0 & +a_1 & +a_2 & +a_3 \\ 0 & = & & 2a_2 & \\ 0 & = & a_1 & +2a_2 & +3a_3 \end{array} \right|$$

$$\ldots \quad \Rightarrow \quad \left| \begin{array}{lll} a_0 & = & 0 \\ a_1 & = & \frac{3}{2} \\ a_2 & = & 0 \\ a_3 & = & -\frac{1}{2} \end{array} \right|$$

$$\Rightarrow \quad \frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

b) Grenzschichtdicke $\delta(x)$:

$$\delta_2 \ := \ \delta \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) \mathrm{d}\left(\frac{y}{\delta}\right)$$

$$\Rightarrow \quad \delta_2 = \delta \int_0^1 \left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 - \left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)^2\right) d\left(\frac{y}{\delta}\right) = \frac{39}{280}\delta$$

$\Rightarrow$ aus von Kármánscher Integralbeziehung: $\frac{39}{280}\frac{\mathrm{d}\delta}{\mathrm{d}x} = \frac{3}{2}\frac{\eta}{\rho U \delta}$

$$\text{mit} \quad \tau_w = \eta \frac{\mathrm{d}u}{\mathrm{d}y}\Big|_{y=0} = \eta \frac{3}{2}\frac{U}{\delta} \quad \text{und} \quad \frac{\mathrm{d}U}{\mathrm{d}x} = 0$$

$$\Rightarrow \quad \frac{39}{280}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}\delta^2\right) = \frac{3}{2}\frac{\eta}{\rho U}$$

$$\Rightarrow \quad \delta^2 = \frac{280}{13}\frac{\eta}{\rho U}x + C_1, \quad \text{mit Rbd.:} \quad \delta(x=0) = 0 \Rightarrow C_1 = 0$$

$$\Rightarrow \quad \delta = \sqrt{\frac{280}{13}\frac{\eta}{\rho U x}}x \approx \frac{4.64}{\sqrt{Re_x}}x, \quad \text{mit} \quad Re_x = \frac{\rho U x}{\eta}$$

c) Widerstandsbeiwert c_W der Platte

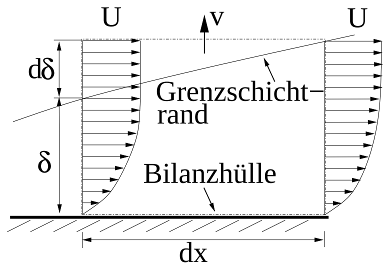
$$c_W = \frac{F_W}{\frac{1}{2}\rho U^2 LB}$$

$$F_W = B\int_0^L \tau_w \mathrm{d}x = B\int_0^L \frac{3}{2}\eta\frac{U}{\delta}\mathrm{d}x = B\frac{3}{2}\sqrt{\frac{13}{280}\frac{\rho U}{\eta}}\eta U 2\sqrt{L}$$

$$= 3\sqrt{\frac{13}{280}}\sqrt{\frac{\eta}{\rho U}L}\rho U^2 BL = 6\sqrt{\frac{13}{280}}\frac{1}{\sqrt{Re_L}}\frac{1}{2}\rho U^2 BL$$

$$\Rightarrow \qquad c_W = 6\sqrt{\frac{13}{280}}\frac{1}{\sqrt{Re_L}} = 1,293\cdot\frac{1}{\sqrt{Re_L}}$$

d) $\frac{v(x)}{U}$:



Konti für ein differentielles Element dx :

$$-\rho\int_0^\delta u(y)\mathrm{d}y - \rho U\mathrm{d}\delta + \rho v\mathrm{d}x + \rho\int_0^{\delta+\mathrm{d}\delta} u(y)dy = 0$$

$$\Rightarrow \qquad -\rho U\delta\int_0^1 \frac{u}{U}\mathrm{d}\left(\frac{y}{\delta}\right) - \rho U\mathrm{d}\delta + \rho v\mathrm{d}x + \rho U(\delta+\mathrm{d}\delta)\int_0^1 \frac{u}{U}\mathrm{d}\left(\frac{y}{\delta}\right) = 0$$

$$\Rightarrow \qquad -\rho U\delta\left(\frac{3}{4}-\frac{1}{8}\right) - \rho U\mathrm{d}\delta + \rho v\mathrm{d}x + \rho U(\delta+\mathrm{d}\delta)\left(\frac{3}{4}-\frac{1}{8}\right) = 0$$

$$\Rightarrow \qquad \frac{v}{U} = \frac{3}{8}\frac{\mathrm{d}\delta}{\mathrm{d}x} \quad \text{mit} \quad \frac{\mathrm{d}\delta}{\mathrm{d}x} = \frac{4,64}{2}\frac{1}{\sqrt{Re_x}} \quad \text{aus b)}$$

$$\Rightarrow \qquad \frac{v}{U} = \frac{0,87}{\sqrt{Re_x}}$$

7. Aufgabe

a) Die komplexe Potentialfunktion $F(z)$ setzt sich (ohne Wandeinfluß) aus Potentialwirbel (Zirkulation Γ) und Senke (Schluckvermögen E) zusammen. Der Wandeinfluß kann durch Spiegelung (Abstand jeweils a) simuliert werden (Drehsinn des gespiegelten Wirbels entgegengesetzt zum ersten). Damit ergibt sich:

$$F(z) = -\frac{E}{2\pi}\left[\ln(z+a) + \ln(z-a)\right] + \frac{i\Gamma}{2\pi}\left[\ln(z+a) - \ln(z-a)\right]$$

b) kart. Geschwindigkeitskomponenten $u(x,y), v(x,y)$

$$\text{konjugiert komplexe Geschwindigkeit:} \quad \frac{\mathrm{d}F}{\mathrm{d}z} = \bar{w} = u - iv$$

$$= -\frac{E}{2\pi}\left[\frac{1}{z+a} + \frac{1}{z-a}\right] + \frac{i\Gamma}{2\pi}\left[\frac{1}{z+a} - \frac{1}{z-a}\right]$$

$$= -\frac{E}{\pi}\frac{z}{z^2-a^2} - \frac{i\Gamma}{\pi}\frac{a}{z^2-a^2}$$

$$\text{mit} \quad z^2 - a^2 = (x+iy)^2 - a^2 = b + 2ixy \quad \text{und} \quad b = x^2 - y^2 - a^2$$

$$\Rightarrow \bar{w} \quad = -\frac{E}{\pi}\frac{(x+iy)(b-2ixy)}{(b+2ixy)(b-2ixy)} - \frac{i\Gamma}{\pi}\frac{a(b-2ixy)}{(b+2ixy)(b-2ixy)}$$

$$= -\frac{E}{\pi}\frac{bx + 2xy^2 + i(by - 2x^2y)}{(b^2+4x^2y^2)} - \frac{i\Gamma}{\pi}\frac{ab - 2ixya}{(b^2+4x^2y^2)}$$

$$= \frac{1}{\pi(b^2+4x^2y^2)}\left[-E(bx+2xy^2) - 2\Gamma axy + i(-E(by-2x^2y) - \Gamma ab)\right]$$

$$\Rightarrow u \quad = -\frac{E(bx+2xy^2) + 2\Gamma axy}{\pi(b^2+4x^2y^2)}$$

$$\Rightarrow v \quad = \frac{E(by-2x^2y) + \Gamma ab}{\pi(b^2+4x^2y^2)}$$

c) Staupunkt: $(x_s, y_s) = (0, -a)$

$$\text{mit} \quad x_s = 0 \Rightarrow b = -(y^2+a^2) \quad \text{und} \quad u_s \equiv 0$$

$$v_s = \frac{E(-y^2-a^2)y + \Gamma a(-y^2-a^2)}{\pi(-y^2-a^2)^2} = 0$$

$$\text{Nenner:} \pi(y^2+a^2)^2 \neq 0 \quad \forall \quad y \in R$$

$$\text{Zähler:} -E(y^2+a^2)y - \Gamma a(y^2+a^2) = 0$$

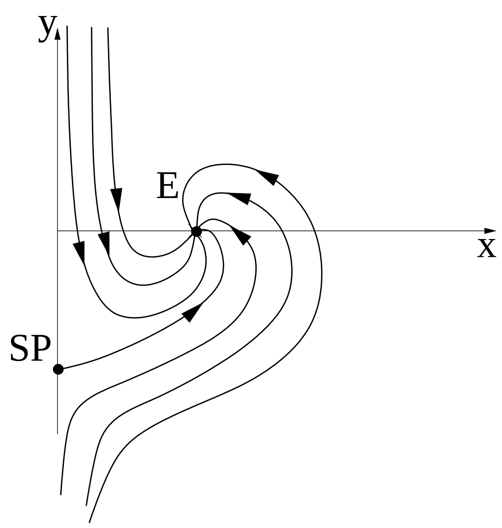
$$\Rightarrow \quad y_s = -\frac{\Gamma}{E}a = -a \Rightarrow E = \Gamma$$

$$\text{Geschwindigkeit im Ursprung}(0,0) = |\vec{v}_0| = -v(0,0), \quad \text{da} \quad u(0,0) = 0$$

$$\Rightarrow \quad v_0 = \frac{\Gamma ab}{\pi b^2} = \frac{\Gamma a}{\pi(-a^2)} \Rightarrow \Gamma = -v_0\pi a$$

$$\Rightarrow \quad E = \Gamma = -v_0\pi a$$

d) Skizze



8. Aufgabe

a) Temperaturverhältnis $\frac{T}{T_0}$

$$h_0 = h + \frac{1}{2}u^2 \quad \Rightarrow \quad c_p T_0 = c_p T + \frac{1}{2}\gamma R T M^2$$

$$\text{mit} \quad h = c_p T, \quad c_p = \frac{\gamma R}{\gamma - 1} \quad \text{und} \quad u = M\sqrt{\gamma R T}$$

$$\Rightarrow \quad \frac{T}{T_0} = (1 + \frac{(\gamma - 1)}{2}M^2)^{-1}$$

Temperatur T am Austritt

$$T = T^* = \frac{2}{\gamma + 1}T_0 = 244,17\,\mathrm{K} \quad \text{mit} \quad M = 1 \quad (\text{engster Querschnitt und } \frac{p_a}{p_0} = \frac{1}{5} < 0,528)$$

Geschwindigkeit u am Austritt

$$u = c^* = \sqrt{\gamma R T^*} = 313,22\,\mathrm{m/s}$$

b) Massenstrom $\dot{m}$

$$\dot{m} = \rho^* c^* A^*$$

$$\Rightarrow \quad \dot{m} = \frac{\rho^*(t)}{\rho_0(t)}\rho_0(t)\sqrt{\gamma R \frac{T^*}{T_0}T_0}A_{aus} = \frac{p^*}{p_0(t)}\frac{p_0(t)}{RT^*}\sqrt{\gamma R \frac{T^*}{T_0}T_0}A_{aus} =$$

$$\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} p_0(t)\sqrt{\frac{\gamma}{RT_0}}\left(\frac{2}{\gamma+1}\right)^{-\frac{1}{2}}A_{aus}$$

$$\Rightarrow \dot{m} = C_1 p_0(t) \quad \text{mit} \quad C_1 = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\sqrt{\frac{\gamma}{RT_0}}A_{aus} = 2,36 \cdot 10^{-7}$$

c) austretende Masse Δm

$$\Delta m = -V(\rho_0(t_e) - \rho_0(t_0))$$

$$\Rightarrow \quad \Delta m = \frac{V}{RT_0}(p_0(t_0) - p_0(t_e)) \quad \text{mit} \quad \rho_0 = \frac{p_0}{RT_0} \quad \text{und} \quad T_0 = \text{konstant}$$

$$p_0(t_e) = \frac{p_0(t_e)}{p_a}p_a = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}p_a \quad (\text{kritisches Druckverhältnis})$$

$$\Rightarrow \quad \Delta m = \frac{V}{RT_0}\left(p_0(t_0) - \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}p_a\right) = 0,37\,\mathrm{kg}$$

d) zeitlicher Verlauf des Druckes $p_0(t)$ in der Flasche

$$\dot{m}(t) = -V\frac{\mathrm{d}\rho_0(t)}{\mathrm{d}t} = -V\frac{d}{dt}\left(\frac{p_0(t)}{RT_0}\right)$$

$$\Rightarrow \quad -\frac{V}{RT_0}\frac{dp_0(t)}{dt} = C_1 p_0(t) \quad \text{aus b)}$$

$$\Rightarrow \quad -\frac{C_1 RT_0}{V}p_0(t) = \frac{\mathrm{d}p_0(t)}{\mathrm{d}t} \Rightarrow -\frac{C_1 RT_0}{V}\mathrm{d}t = \frac{1}{p_0(t)}\mathrm{d}p_0(t)$$

$$\Rightarrow \quad p_0(t) = p_0(t_0)e^{(\frac{-C_1 RT_0 t}{V})}$$