....
(Name, Matr.-Nr, Unterschrift)

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1. Aufgabe

a) $G = F_A$

$$G = \varrho_W (V_B + V_{L_1}) \cdot g$$

$$G = \varrho_W \left(\frac{G}{g \cdot \varrho_B} + V_{L_1} \right) \cdot g$$

$$V_{L_1} = \frac{G}{g} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right)$$

$$\varrho_{L_1} = \frac{p_1}{R \cdot T_W} = \frac{p_a + \varrho_W g(H+h)}{R \cdot T_W}$$

$$m_{L_1} = \varrho_{L_1} \cdot V_{L_1}$$

$$m_{L_1} = \frac{G}{g} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right) \cdot \frac{p_a + \varrho_W g(H+h)}{R \cdot T_W}$$

b) $G = F_A \quad \wedge \quad V_{L_1} = V_{L_2} = V_L$

$$G = \varrho_W (V_B + V_L) \cdot g$$

$$G = \varrho_W \left(\frac{G}{g \cdot \varrho_B} + A \cdot h \right) \cdot g$$

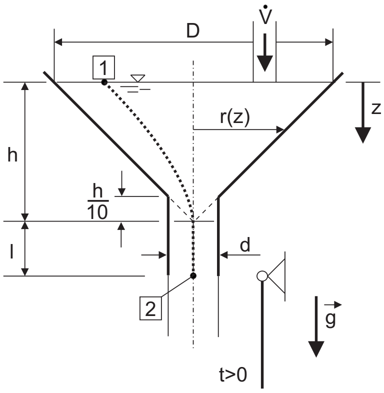
$$h = \frac{G}{g \cdot A} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right)$$

$$\varrho_{L_2} = \frac{p_a + \varrho_W g h}{R \cdot T_W}$$

$$m_{L_2} = \varrho_{L_2} \cdot A \cdot h = \frac{p_a + \varrho_w g h}{R T_W} \frac{G}{g} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right)$$

$$\Delta m = m_{L_1} - m_{L_2} = G \left(\frac{1}{\varrho_B} - \frac{1}{\varrho_W} \right) \frac{\varrho_W H}{R T_W}$$

<u>2. Aufgabe</u>



a) Bernoulli von $\boxed{1}$ nach $\boxed{2}$:

$$p_a + \frac{\varrho}{2}v_1^2 = p_a - \varrho g(h+l) + \frac{\varrho}{2}v_2^2 + \varrho\int_1^2 \frac{\partial v}{\partial t}ds$$

$$\frac{A_1}{A_2} = \left(\frac{D}{d}\right)^2 = 100 \quad \text{und} \quad v_1 = 0$$

Konti: $\quad vA = v_2 A_2 \quad \Longrightarrow \quad v = v_2\frac{A_2}{A(z)}$

wobei: $\quad \dfrac{A_2}{A(z)} = \dfrac{\frac{\pi}{4}d^2}{\pi r(z)^2} = \left(\dfrac{d}{D}\right)^2 \dfrac{1}{\left(1-\frac{z}{h}\right)^2} = \left(\dfrac{d}{D}\right)^2 \dfrac{h^2}{(h-z)^2}$

$$\Longrightarrow \quad \varrho\int_1^2 \frac{\partial v}{\partial t}ds = \varrho\int_1^2 \frac{\partial}{\partial t}\left(v_2\frac{A_2}{A(z)}\right)ds = \varrho\frac{dv_2}{dt}\int_1^2 \frac{A_2}{A(z)}ds$$

mit: $\quad \displaystyle\int_1^2 = \int_0^{\frac{9}{10}h} + \int_{\frac{9}{10}h}^{h+l}$

$$= \varrho\frac{dv_2}{dt}\int_0^{\frac{9}{10}h}\frac{A_2}{A(z)}dz + \varrho\frac{dv_2}{dt}\left(l + \frac{h}{10}\right)$$

$$\Longrightarrow \int_0^{\frac{9}{10}h}\frac{A_2}{A(z)}dz = \left(\frac{d\cdot h}{D}\right)^2 \cdot \left.\frac{1}{h-z}\right|_0^{\frac{9}{10}h} = 0,09h$$

$$\Longrightarrow \varrho g(h+l) = \frac{\varrho}{2}v_2^2(t) + \varrho\frac{dv_2}{dt}(l + 0,19h)$$

für t = 0: $\quad v_2 = 0 \quad \Longrightarrow \quad \left.\dfrac{dv_2}{dt}\right|_{t=0} = \dfrac{g(h+l)}{l+0,19h}$

b) aus a) folgt:

$$g(h+l) - \frac{v_2^2}{2} = \frac{dv_2}{dt}(l+0,19h) \quad \Longrightarrow \quad dt = (l+0,19)\frac{2\cdot dv_2}{2g(h+l)-v_2^2}$$

stationäre Endgeschwindigkeit: $v_2(t\to\infty) = \sqrt{2g(h+l)}$

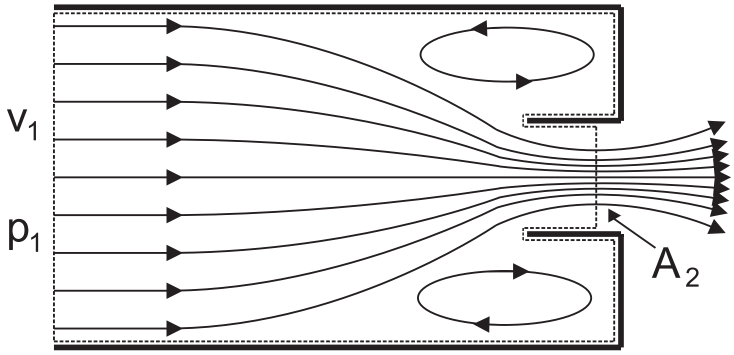
da $v_2(t) < v_2(t\to\infty) \quad \Longrightarrow \quad$ Integral für $|x| < a$

$$\Longrightarrow \Delta T = \frac{l+0,19h}{\sqrt{2g(h+l)}}\ln\left.\frac{\sqrt{2g(h+l)}+v_2}{\sqrt{2g(h+l)}-v_2}\right|_0^{0,9\sqrt{2g(h+l)}}$$

$$\Longrightarrow \Delta T = \frac{l+0,19h}{\sqrt{2g(h+l)}}\ln(19)$$

3. Aufgabe

a) Strömungsfeld im Stutzen



b) IES in x-Richtung:

$$-\varrho v_1^2 A_1 + \varrho v_2^2 A_2 = p_1 A_1 - p_{tot}(A_1 - A_0) - p_a A_0 \qquad (1)$$

Konti: $v_1 A_1 = v_2 A_2 \quad \Longrightarrow \quad v_1 = \frac{A_2}{A_1} v_2 \qquad (2)$

Bernoulli: $p_1 + \frac{\varrho}{2} v_1^2 = p_a + \frac{\varrho}{2} v_2^2 = p_{tot} \qquad (3)$

(2) und (3) in (1): $\varrho v_2^2 \left(A_2 - \frac{A_2^2}{A_1} \right) = p_1 A_1 - \left(p_a + \frac{\varrho}{2} v_2^2 \right)(A_1 - A_0) - p_a A_0 \qquad (4)$

(2) in (3): $p_1 = p_a + \frac{\varrho}{2} v_2^2 \left(1 - \left(\frac{A_2}{A_1} \right)^2 \right) \qquad (5)$

(5) in (4):

$$\varrho v_2^2 \left(A_2 - \frac{A_2^2}{A_1} \right) = p_a A_1 + \frac{\varrho}{2} v_2^2 \left(A_1 - \frac{A_2^2}{A_1} \right) - p_a A_1 - \frac{\varrho}{2} v_2^2 A_1 + p_a A_0 + \frac{\varrho}{2} v_2^2 A_0 - p_a A_0$$

$$A_2 - \frac{A_2^2}{A_1} = -\frac{1}{2} \frac{A_2^2}{A_1} + \frac{1}{2} A_0$$

$$2A_2 - \frac{A_2^2}{A_1} = A_0$$

$$2A_2 A_1 - A_2^2 = A_0 A_1$$

$$A_2^2 - 2A_2 A_1 + A_1^2 = -A_0 A_1 + A_1^2$$

$$(A_2 - A_1)^2 = A_1^2 - A_0 A_1$$

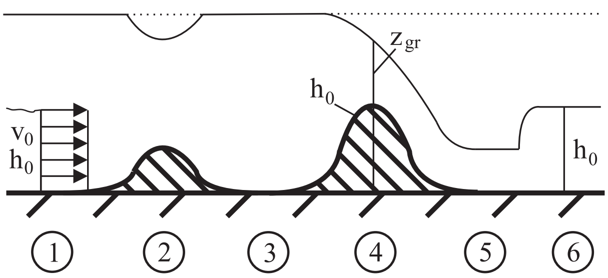
$$A_2 = A_1 \pm \sqrt{A_1^2 - A_0 A_1}$$

nur neg. Vorzeichen physikalisch sinnvoll

$$A_2 = A_1 - \sqrt{A_1^2 - A_0 A_1}$$

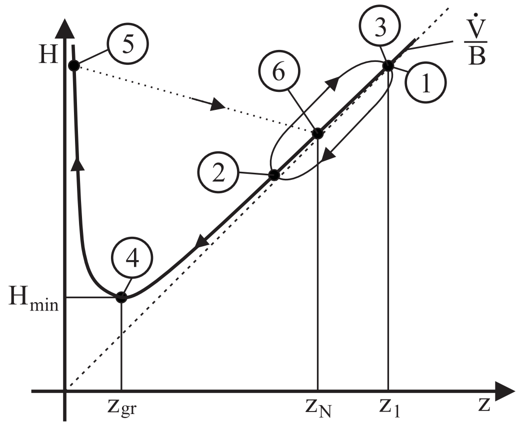
4. Aufgabe

a) Skizze



Aufstau bei konstantem

$$\frac{\dot{V}}{B} = v_0 h_0$$



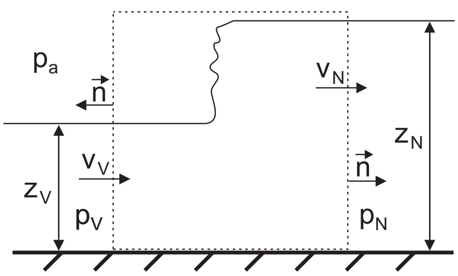
$$H_{ges} = H_{fluid} + y = konst$$

b) $H_V + y_V = H_4 + y_4$ mit $H_4 = H_{min}$, $y_4 = h_0$ und $y_V = 0$

$$H_V = h_0 + z_{gr} + \frac{\dot{V}^2}{2g z_{gr}^2 B^2}$$

$$\Longrightarrow H_V = h_0 + \left(\frac{\dot{V}^2}{gB^2}\right)^{\frac{1}{3}} + \frac{1}{2}\left(\frac{\dot{V}^2}{gB^2}\right)^{1-\frac{2}{3}} = h_0 + \frac{3}{2}\left(\frac{v_0^2 h_0^2}{g}\right)^{\frac{1}{3}}$$

c) Energieverlust



Konti: $v_N z_N = v_V z_V$

IES: Herleitung Skript S.126

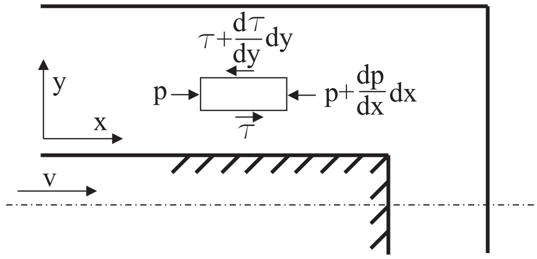
$$\Longrightarrow \quad -\varrho v_V^2 z_V + \varrho v_N^2 z_N = \varrho g\left(\frac{z_V^2}{2} - \frac{z_N^2}{2}\right)$$

$$\Delta H = H_N - H_V = z_N - z_V + \frac{\dot{V}^2}{2gB^2}\left(\frac{1}{z_N^2} - \frac{1}{z_V^2}\right)$$

mit IES und Konti: $\dfrac{\dot{V}^2}{gB^2} = \dfrac{z_V z_N (z_V + z_N)}{2}$

$$\Longrightarrow \Delta H = z_N - z_V + \frac{1}{4}\left(\frac{1}{z_N^2} - \frac{1}{z_V^2}\right) z_V z_N (z_V + z_N)$$

5. Aufgabe



a) Bilanzierung an infinitesimal kleinem Element

$$\frac{dp}{dx} = -\frac{d\tau}{dy}$$

$$\tau = -\eta \frac{du}{dy} \quad \Longrightarrow \quad \frac{d^2u}{dy^2} = \frac{1}{\eta}\frac{dp}{dx}$$

1. Integration: $\dfrac{du}{dy} = \dfrac{1}{\eta}\dfrac{dp}{dx}y + C_1 \quad \wedge \quad$ 2. Integration: $u = \dfrac{1}{2\eta}\dfrac{dp}{dx}y^2 + C_1 y + C_2$

R.B.:

$$\left.\begin{array}{ll} 1) & y = 0: \quad u = v \\ 2) & y = \dfrac{b_2 - b_1}{2} \equiv H: \quad u = 0 \end{array}\right\} \text{Haftbedingung}$$

aus 1) $\quad C_2 = v$

aus 2) $\quad 0 = \dfrac{1}{2\eta}\dfrac{dp}{dx}H^2 + C_1 H + v = 0 \quad \Longleftrightarrow \quad C_1 = -\dfrac{1}{2\eta}\dfrac{dp}{dx}H - \dfrac{v}{H}$

$$\Longrightarrow \quad u(y) = \frac{1}{2\eta}\frac{dp}{dx}H^2 \cdot \left(\frac{y^2}{H^2} - \frac{y}{H}\right) + v \cdot \left(1 - \frac{y}{H}\right)$$

$$\Longrightarrow \quad u(y) = \frac{1}{4\eta}\frac{dp}{dx}y \cdot (b_2 - b_1)\left(\frac{y}{b_2 - b_1} - 1\right) + v \cdot \left(1 - \frac{2y}{b_2 - b_1}\right)$$

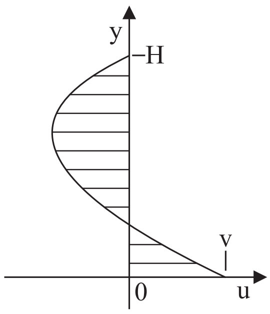
b) Volumenstrombilanz: $\quad \dfrac{\dot{V}}{B} = 2 \cdot \displaystyle\int_0^H u(y)dy = -b_1 \cdot v$

$$\int_0^H u(y)dy = \frac{1}{2\eta}\frac{dp}{dx}H^2\left(\frac{1}{3}\frac{y^3}{H^2} - \frac{1}{2}\frac{y^2}{H}\right) + v \cdot \left(y - \frac{1}{2}\frac{y^2}{H}\right)\Bigg|_0^H$$

$$= -\frac{1}{12\eta}\frac{dp}{dx}H^3 + v\frac{H}{2} = -b_1\frac{v}{2} \Longleftrightarrow v \cdot \left(\frac{b_1}{2} + \frac{H}{2}\right) = \frac{1}{12\eta}\frac{dp}{dx}H^3$$

$$\Longleftrightarrow \quad \frac{1}{2\eta}\frac{dp}{dx}H^2 = 3\frac{v}{H}(H + b_1) \Longleftrightarrow \frac{dp}{dx} = \frac{6\eta v}{H^2} \cdot \left(1 + \frac{b_1}{H}\right)$$

$$\Longrightarrow \quad u(y) = 3v \cdot \left(1 + \frac{b_1}{H}\right)\left(\frac{y^2}{H^2} - \frac{y}{H}\right) + v \cdot \left(1 - \frac{y}{H}\right)$$

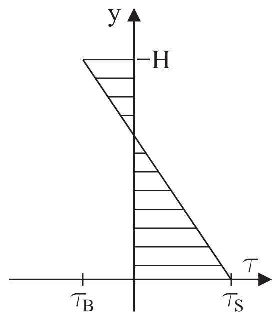


$$\Longrightarrow \quad u(y) = 6v \cdot \left(1 + \frac{2b_1}{b_2 - b_1}\right)\left(\frac{2y^2}{(b_2 - b_1)^2} - \frac{y}{b_2 - b_1}\right) + v \cdot \left(1 - \frac{2y}{b_2 - b_1}\right)$$

c) $\tau = -\eta \dfrac{du}{dy} = -\eta \left(3v \left(1 + \dfrac{b_1}{H}\right) \left(\dfrac{2y}{H^2} - \dfrac{1}{H}\right) - \dfrac{v}{H}\right)$

Stempel $y = 0$: $\quad \tau_S = 3\eta \dfrac{v}{H} \left(1 + \dfrac{b_1}{H}\right) + \eta \dfrac{v}{H}$

B-Wand $y = H$: $\quad \tau_B = -3\eta \dfrac{v}{H} \left(1 + \dfrac{b_1}{H}\right) + \eta \dfrac{v}{H}$



6. Aufgabe

a) Rohrvibration, Oberflächenbeschaffenheit, Geometrie des Einlaufs, etc.

b) Reynoldszahl, mit dem Rohrdurchmesser bzw. den hydraulischen Durchmesser.

$$Re_D = \frac{\varrho u_\infty D}{\eta}$$

c) Die Reynoldssche Mittelung beschreibt die Strömungsgröße f als Summe aus zeitlicher Mittelwert $\overline{f}$ und Schwankungsanteil f'.

d) Skizze