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(Matr.-No., Signature)

Exam Fluid Dynamics

14 August 2020

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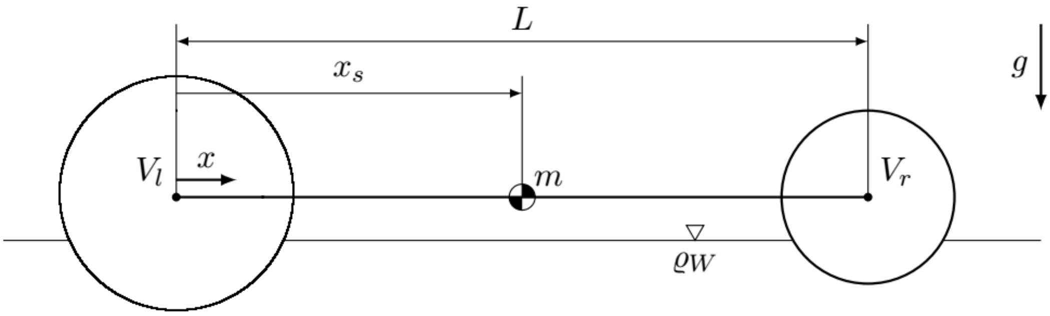
Problem 1 (8 Points)

A raft is modelled by two connected cylinders. The volumes of the cylinders are V_r and V_l and the center of gravity of the mass m is located at x_s. The stable position of the raft in the interval $x_{min} < x_s < x_{max}$ is ensured since both cylinders generate higher lift with increasing immersion depth.



a) Sketch position of the raft in the water for:

  I) $x_{min} < x_s < L/2$ and $V_l = V_r$

  II) $0 < x_s < x_{min}$

b) Determine the limits of the interval x_{min} and x_{max} for which the raft is in a stable position on the water.

c) Which status does $x_s = x_{min} = x_{max}$ describe? Describe a possible position of the raft in the water for this case.

Given:

$g, \quad V_l, \quad V_r, \quad m, \quad L, \quad \varrho_W$

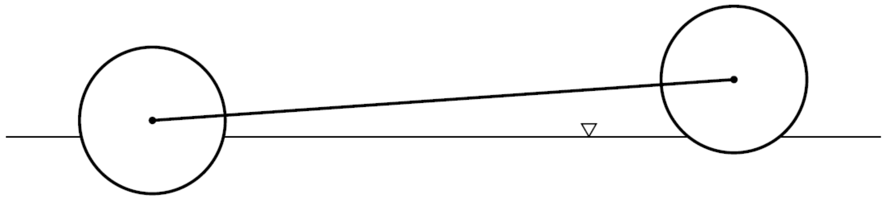
Hints:

- Neglect the influence of the surrounding air!
- Check the units and signs of your results!

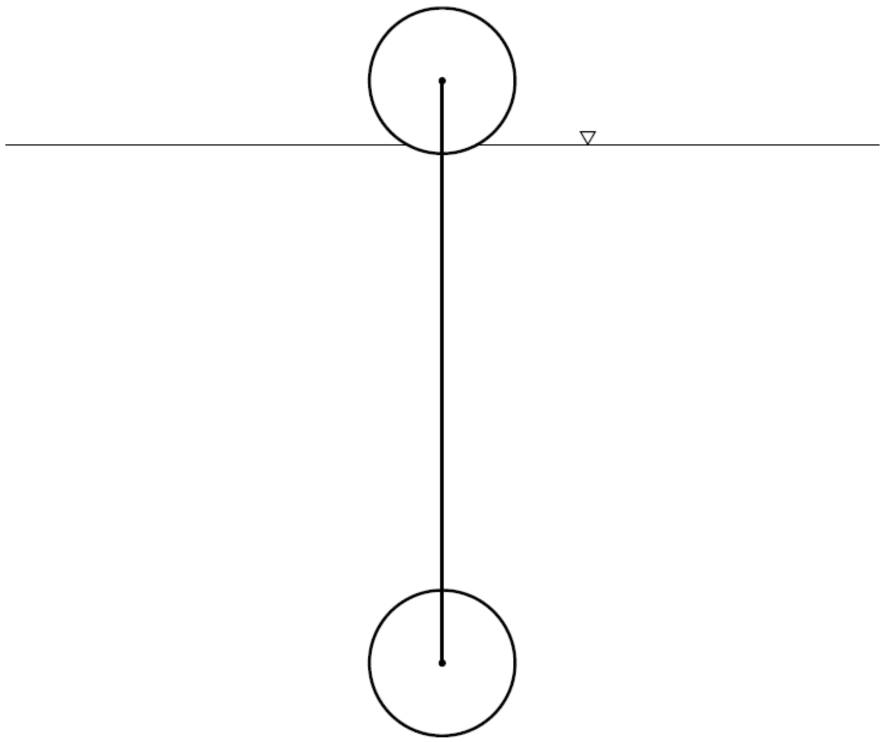
Problem 1

a) As long as the cylinder is not completely wetted, the buoyancy increases with the immersion depth.

   I) stable, because at $V_l = V_r$ the limit x_{max} is on the right side of $L/2$. $\rightarrow$ both cylinders are not completely wetted and the left cylinder is deeper in the water



   II) unstable $\rightarrow$ one float is completely wetted. The position of the raft is parallel to the acceleration of gravity.



b) Force balance of the raft considering the displaced volume of the cylinders V_l^* and V_r^*:

$$\sum F = 0 = -mg + V_l^* \varrho_W g + V_r^* \varrho_W g \tag{1}$$

Moment balance about the center of gravity when the raft is inclined by the angle α:

$$\sum M = 0 = -V_l^* \varrho_W g x_s \cos\alpha + V_r^* \varrho_W g (L - x_s) \cos\alpha \tag{2}$$

$$\rightarrow x_s = L V_r^* / (V_r^* + V_l^*)$$

The limits of the interval are determined by considering only one fully wetted cylinder. From the forces balance then follows

$$
\begin{aligned}
x_{min} : V_l^* = V_l \quad &\rightarrow \quad V_r^* = m/\varrho_W - V_l \\
x_{max} : V_r^* = V_r \quad &\rightarrow \quad V_l^* = m/\varrho_W - V_r \,.
\end{aligned}
$$

When plugged into the moment balance, the following results for the limits of the interval

$$
\begin{aligned}
x_{min} =& L(1 - \varrho_W V_l/m) \\
x_{max} =& L\varrho_W V_r/m
\end{aligned} \tag{3}
$$

c) From $x_{min} = x_{max}$ follows $m = \varrho_W(V_l + V_r)$ which corresponds to the force balance (1) with completely wetted cylinders $V_r^* = V_r$ and $V_l^* = V_l$. The raft is in the **floationg** state.

If additionally $x_s = x_{min} = x_{max}$, the center of gravity x_s fulfills the moment balance for $\cos\alpha \neq 0$.

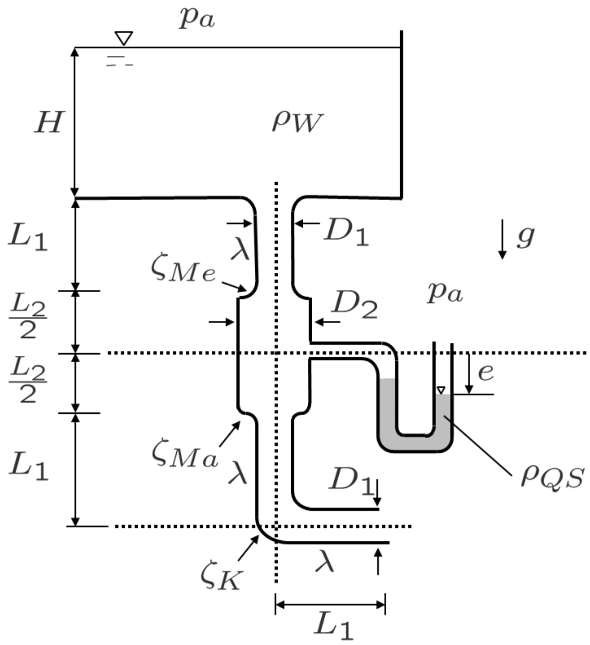
For example, using $m = \varrho(V_l + V_r)$ from (3)

$$
x_s = x_{max} = L\varrho_W V_r/(\varrho_w(V_r + V_l)) = LV_r/(V_r + V_l) \,. \tag{4}
$$

This corresponds to the moment balance (2) in the floating state $V_r^* = V_r$ and $V_l^* = V_l$ for all inclination angles α.

Problem 2 (12 Points)

Water flows from a large container through an angled, circular pipe into the open air. Inside the tube, there is a manometer filled with mercury to determine the flow rate. At the manometer and at the pipe elbow, the losses ζ_{Ma}, ζ_{Me} and ζ_K occur. In the pipe, there is the pipe friction coefficient λ. However,the friction for the pipe piece of length L_2 can be neglected. The losses in the well rounded inlet and the friction in the manometer body are also negligible.



a) Determine the mass flow through the pipe?

b) Determine the deflection e of the mercury column from the position at rest for the steady flow. To do this, first set up the force balance for the resting state and the deflected state. The velocity v_a at the outlet can be assumed to be known for this part of the task.

In the following, consider the system without mercury manometer. A throttle (ζ_D) is mounted at the outlet. When the throttle closes at $t > 0$ velocity at the outlet of the throttle is given by

$$v(t) = v_0 e^{\left(-\frac{t}{T_0}\right)}, \quad \text{with} \quad T_0 > 0 \quad \text{and} \quad v_0 = v(t \leq 0).$$

c) Specify the profile of the static pressure $p(t)$ at the outlet for $t \geq 0$.

Given: $D_1, \quad D_2, \quad L_1, \quad L_2, \quad \lambda, \quad g, \quad H, \quad p_a, \quad T_0, \quad \rho_W, \quad \rho_{QS},$

$\zeta_K, \quad \zeta_D, \quad \zeta_{Ma}, \quad \zeta_{Me}$

Hints:
- Check the units and signs of your results!

Problem 2

a) Mass flux $\dot{m}$:
Bernoulli from the upper edge of the container to the outlet:

$$p_a + \varrho_w g(H + 2L_1 + L_2) = p_a + \tfrac{\varrho_w}{2} v_a^2 \left(1 + \lambda \tfrac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma} \left(\tfrac{D_1}{D_2}\right)^4 + \zeta_k\right)$$

$$\Rightarrow v_a = \sqrt{\tfrac{2g(H+2L_1+L_2)}{1+\lambda\frac{3L_1}{D_1}+\zeta_{Me}+\zeta_{Ma}(\frac{D_1}{D_2})^4+\zeta_K}}$$

$$\underline{\dot{m} = \varrho_w v_a \tfrac{\pi D_1^2}{4} = \varrho_w \tfrac{\pi}{4} D_1^2 \sqrt{\tfrac{2g(H+2L_1+L_2)}{1+\lambda\frac{3L_1}{D_1}+\zeta_{Me}+\zeta_{Ma}(\frac{D_1}{D_2})^4+\zeta_K}}}$$

b) Deflection e

Balance of forces in the deflected state:

$$p_a + \varrho_{QS} g(h - e) = p_i + \varrho_w g(h - e) + \varrho_{QS} g e \tag{5}$$

$$\Leftrightarrow p_a - p_i + (\varrho_{QS} - \varrho_w) g h = e g(2\varrho_{QS} - \varrho_w) \tag{6}$$

Balance of forces at rest:

$$p_a + \varrho_w g(H + L_1 + \frac{L_2}{2}) + \varrho_w g h = p_a + \varrho_{QS} h \tag{7}$$

$$\Rightarrow h = \frac{\varrho_w}{\varrho_{QS} - \varrho_w}(H + L_1 + \frac{L_2}{2}) \tag{8}$$

Bernoulli from the top of the container to the manometer inlet:

$$p_a + \varrho_w g(H + L_1 + \frac{L_2}{2}) = p_i + \frac{\varrho_w}{2} v_i^2 + \zeta_{Me} \frac{\varrho_w}{2} v_a^2 + \lambda \frac{L_1}{D_1} \frac{\varrho_w}{2} v_a^2 \tag{9}$$

$$\Leftrightarrow p_i = p_a + \varrho_w g(H + L_1 + \frac{L_2}{2}) - \frac{\varrho_w}{2} v_a^2 \left[\left(\frac{D_1}{D_2}\right)^4 + \lambda \frac{L_1}{D_1} + \zeta_{Me}\right] \tag{10}$$

Using continuity: $v_i A_i = v_a A_a \Leftrightarrow v_i = v_a(\frac{D_1}{D_2})^2$

Inserting p_i and h and solving for e gives

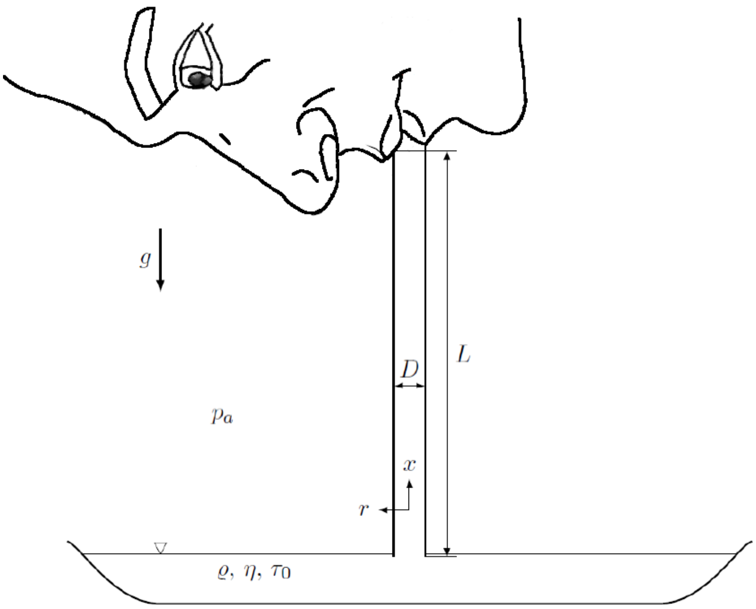
$$e = \frac{-\varrho_w g(H + 2L_1 + L_2) + \frac{\varrho_w}{2} v_a^2 \left[\left(\frac{D_1}{D_2}\right)^4 + \lambda \frac{L_1}{D_1} + \zeta_{Me}\right] + (\varrho_{QS} - \varrho_w) g(H + 2L_1 + L_2)\frac{\varrho_w}{\varrho_{QS}-\varrho_w}}{g(2\varrho_{QS} - \varrho_w)} \tag{11}$$

c) Unsteady Bernoulli from the upper container edge to the outlet:

$$p_a + \varrho_w g(H + 2L_1 + L_2) =$$
$$p(t) + \frac{\varrho_w}{2} v_a(t)^2 \left(1 + \lambda\frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma}\left(\frac{D_1}{D_2}\right)^4 + \zeta_K + \zeta_D\right) + \varrho_w \int_0^{aus} \frac{\partial v}{\partial t} ds$$
with$\varrho_w \int_0^{aus} \frac{\partial v}{\partial t} ds = \varrho_w \frac{dv_a(t)}{dt}\left(3L_1 + \left(\frac{D_1}{D_2}\right)^2 L_2\right), \frac{dv_a(t)}{dt} = -\frac{v_0}{T_0} e^{-\frac{t}{T_0}}$
$$\Rightarrow p(t) = p_a + \varrho_w g(H + 2L_1 + L_2)$$
$$-\frac{\varrho_w}{2} v_0^2 \left(e^{(-\frac{2t}{T_0})}\right)\left(1 + \lambda\frac{3L_1}{D_1} + \zeta_{Me} + \zeta_{Ma}\left(\frac{D_1}{D_2}\right)^4 + \zeta_K + \zeta_D\right) + \frac{v_0}{T_0} e^{-\frac{t}{T_0}} \left(3L_1 + \left(\frac{D_1}{D_2}\right)^2 L_2\right)$$

Problem 3 (7 Points)

The elastic limit τ_0 of a cheese sauce is of great importance for the efficient suction through a macaroni noodle. It is this quantity that is to be determined. The optimum elastic limit is reached when no sauce can be sucked in through the macaroni. To calculate the elastic limit, a macaroni with inner diameter D and length L between mouth and plate is considered. The macaroni is filled with sauce and stabilized by the mouth. For the mouth, the internal pressure p_i to suck in the sauce is known and at the lower end of the macaroni the ambient pressure p_a can be assumed. The cheese sauce is treated as Bingham fluid.



a) Derive the shear stress distribution in the cheese sauce and determine the elastic limit τ_0.

An error in preparation causes a negligible elastic limit and $\tau_0 = 0$ can be assumed. The sufficiently available sauce is now absorbed by the macaroni. The pressure difference between the inlet and outlet of the macaroni remains constant.

b) Determine the velocity distribution in the macaroni.

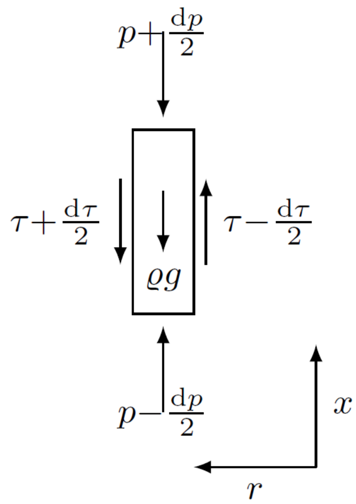
Given:

$g, \quad \rho, \quad \eta, \quad L, \quad D, \quad D \ll L, \quad p_i, \quad p_a$

Hints:

- Flow law for Bingham fluids: $\tau = \tau_0 + \eta \frac{du}{dr}$
- A macaroni is a pipe shaped noodle.
- Check the units and signs of your results!

Problem 3



a) Force balance in x-direction:

$$\begin{aligned}\sum F = 0 = &-\varrho g 2\pi r \mathrm{d}x\mathrm{d}r \\ &-(p+\mathrm{d}p/2)2\pi r\mathrm{d}r + (p-\mathrm{d}p/2)2\pi r\mathrm{d}r \\ &+(\tau-\mathrm{d}\tau/2)(r-\mathrm{d}r/2)2\pi\mathrm{d}x \\ &-(\tau+\mathrm{d}\tau/2)(r+\mathrm{d}r/2)2\pi\mathrm{d}x \\ 0 = &-\varrho g r - r\mathrm{d}p/\mathrm{d}x - \mathrm{d}(\tau r)/\mathrm{d}r\end{aligned}$$

Integrating from $r=0$ to r using the boundary condition $\tau(r=0)=0$:

$$\tau = -\frac{1}{2}\left(\varrho g + \frac{\mathrm{d}p}{\mathrm{d}x}\right) r$$

To prevent sauce from flowing through the macaroni, the elastic limit must be on the wall:

$$\tau_0 = -\frac{1}{2}\left(\varrho g + \frac{\mathrm{d}p}{\mathrm{d}x}\right)\frac{D}{2}$$

Using $\mathrm{d}p/\mathrm{d}x = (p_i - p_a)/L$ follows:

$$\tau_0 = -\frac{1}{2}\left(\varrho g + \frac{p_i - p_a}{L}\right)\frac{D}{2}\,.$$

b) For $\tau_0 = 0$, the Bingham fluid becomes a Newtonian fluid with $\tau = -\eta\mathrm{d}u/\mathrm{d}r$.

$$\mathrm{d}u = \frac{1}{2\eta}\left(\varrho g + \frac{p_i - p_a}{L}\right) r\mathrm{d}r$$

Using the boundary condition $u(r=D/2)=0$ yields

$$u(r) = \frac{1}{4\eta}\left(\varrho g + \frac{p_i - p_a}{L}\right)(r^2 - D^2/4)\,.$$

Problem 4 (11 Points)

In a parallel flow with the velocity u_0 in the x-direction, a source with the strength E is located at $(x, y) = (0, 0)$.

a) Determine the potential function $F(z)$. Sketch the flow field and mark clearly the contour streamline.

b) Determine the coordinates of the stagnation point in polar coordinates.

c) Determine y_{max} of the contour streamline.

d) A second source with the strength E is added to the flow field. It is located perpendicular to the parallel flow at a distance a above the first source. Sketch the contour streamline(s) for $a = \frac{y_{max}}{2}$ and $a = 4y_{max}$.

e) Determine is the velocity in the direction of the y-axis on the centerline between the two sources?

Given: u_0, $E > 0$, a

Hints:

- $z = x + iy = r \cdot e^{i\varphi} = r(\cos\varphi + i\sin\varphi)$
- Tabulated sin and cos values:

φ	0	$\frac{1}{8}\pi$	$\frac{1}{4}\pi$	$\frac{3}{8}\pi$	$\frac{1}{2}\pi$	$\frac{5}{8}\pi$	$\frac{3}{4}\pi$	$\frac{7}{8}\pi$	π
$\sin(\varphi)$	0	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	1	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	0
$\cos(\varphi)$	1	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	0	$-\frac{\sqrt{2-\sqrt{2}}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$	-1
$\tan(\varphi)$	0	$\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$	1	$\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$	∞	$-\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$	-1	$-\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}$	0

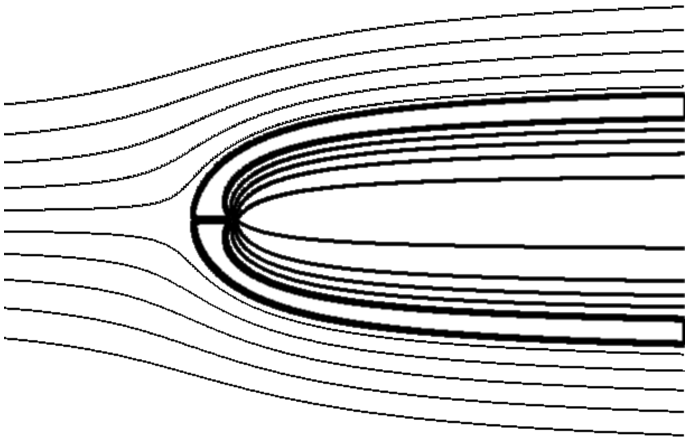
known complex potential functions:

Potential vortex:	$F(z) = -\frac{i\Gamma}{2\pi}\ln z$
Source/sink:	$F(z) = \frac{E}{2\pi}\ln z$
Dipole:	$F(z) = \frac{M}{2\pi z}$
Stagnation point:	$F(z) = \alpha z^2$
Parallel flow:	$F(z) = (u_0 - iv_0)z$

Problem 4

a)

$$F(z) = u_0 z + \frac{E}{2\pi} ln(z)$$



b)

$$F(z) = u_0(x + iy) + \frac{E}{2\pi} ln(re^{i\varphi})$$

The potential function reads:

$$\Phi = u_0 x + \frac{E}{2\pi} ln(\sqrt{x^2 + y^2})$$

This yoelds the velocities.

$$u = \frac{\partial \Phi}{\partial x} = u_0 + \frac{E}{2\pi} \frac{x}{x^2 + y^2}$$
$$v = \frac{\partial \Phi}{\partial y} = \frac{E}{2\pi} \frac{y}{x^2 + y^2}$$

At the stagnation point, both velocity components are 0. Thus, the stagnation point is at $x_S = -\frac{E}{2\pi u_0}$ and $y_S = 0$.
in polar coordinates:

$$r_S = \sqrt{x_S^2 + y_S^2} = \frac{E}{2\pi u_0}$$

$$\varphi_S = atan\left(\frac{y_S}{x_S}\right) = \pi$$

c) For the stream function, the following results are obtained:

$$\Psi = u_0 y + \frac{E}{2\pi}\varphi = u_0 r \sin\varphi + \frac{E}{2\pi}\varphi$$

From $x \to \infty$ and $y = y_{max}$ follows $\varphi \to 0$. The value of the stream function is:

$$\Psi_\infty = u_0 y_{max}$$

The value of the stream function along the contour of the half-body is constant. Therefore, the stream function is also evaluated at the stagnation point and equated with Ψ_∞.

$$\Psi_S = \Psi(r = \frac{E}{2\pi u_0}, \varphi = \pi) = u_0 \frac{E}{2\pi u_0} sin(\pi) + \frac{E}{2\pi}\pi$$

$$\Psi_s = \frac{E}{2}$$

$$\Psi_\infty = \Psi_s \to u_0 y_{max} = \frac{E}{2}$$

$$y_{max} = \frac{E}{2u_0}$$

d) Flow with two sources

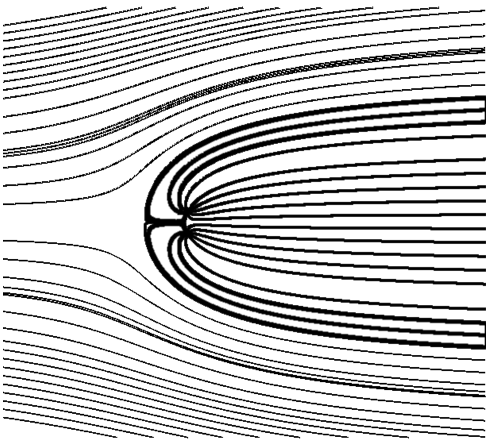


Abbildung 1: $a = \frac{y_{max}}{2}$

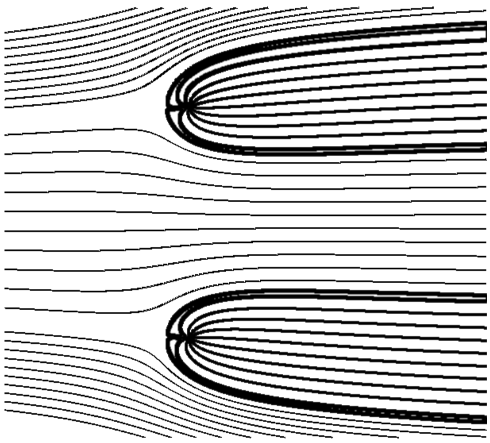
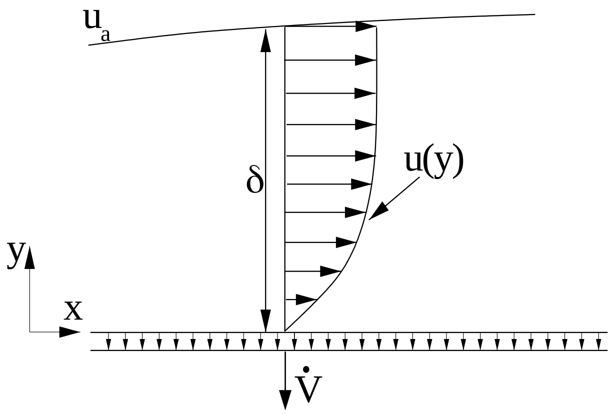


Abbildung 2: $a = 4y_{max}$

e) The velocity in the y-direction is 0.

Problem 5 (11 Points)



A laminar, incompressible boundary layer develops along a flat plate with span B. A constant volume flux $\dot{V}$ is sucked off over the length L through equidistantly distributed pressure holes in the plate. For the tangential velocity profile, the following polynomial approach applies:

$$\frac{u(y)}{u_a} = a_0 + a_1 \left(\frac{y}{\delta}\right) + a_2 \left(\frac{y}{\delta}\right)^2 + a_3 \left(\frac{y}{\delta}\right)^3$$

a) Determine the coefficients of the velocity profile for this boundary layer.

b) Sketch the graph of the laminar boundary layer thickness along the coordinate x starting at the leading edge of the plate. Draw the velocity distribution $u(0 \le y \le \delta)$ at three locations within the boundary layer as it develops.

Due to a technical defect, the suction device fails and the boundary layer can no longer be kept laminar.

c) What is the critical Reynolds number for the incompressible flow along a flat plate?

d) Assume that the boundary layer flow is turbulent right at the leading edge of the plate. Sketch again the distribution of the boundary layer thickness and the velocity profiles at three positions within the boundary layer. Specify two differences between the laminar and the turbulent boundary layer.

Given: $\eta, \rho, \delta, u_a = const., \dot{V}, B, L$

Hint:

Boundary layer equation (x momentum equation): $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\eta}{\rho}\frac{\partial^2 u}{\partial y^2}.$

Problem 5

a) Determination of the coefficients a_0, a_1, a_2, a_3

(I) no-slip condition: $u(y=0) = 0 \rightarrow a_0 = 0$

(II) at the edge of the boundary layer: $u(y=\delta) = u_a \rightarrow 1 = a_1 + a_2 = a_3$

(III) momentum equation in the x-direction at the wall : $-v\frac{\partial u}{\partial y} = \frac{\eta}{\rho}\frac{\partial^2 u}{\partial y^2}$

$$-\frac{\dot{V}}{BL}u_a a_1 = \frac{\eta}{\rho}\frac{2a_2}{\delta}$$
$$a_1 = \frac{-2BL\eta}{\rho\delta\dot{V}u_a}a_2$$

(IV) Smooth transition at the boundary layer edge: $\frac{\partial u}{\partial y}_{y=\delta} = 0 \rightarrow a_1 + 2a_2 + 3a_3 = 0$

Elimination of a_3 from (II) and (IV):

$$3 = 2a_1 + a_2$$

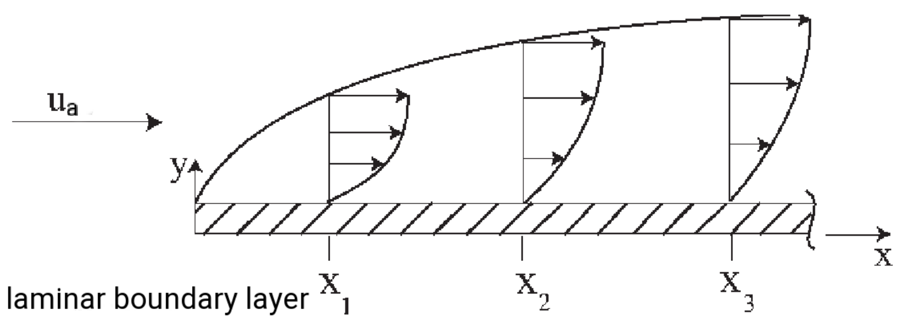
(III) plug in and transform:

$$a_2 = \frac{3\rho\delta\dot{V}u_a}{\rho\delta\dot{V}u_a - 4BL\eta}$$

This yields for the remaining coefficients

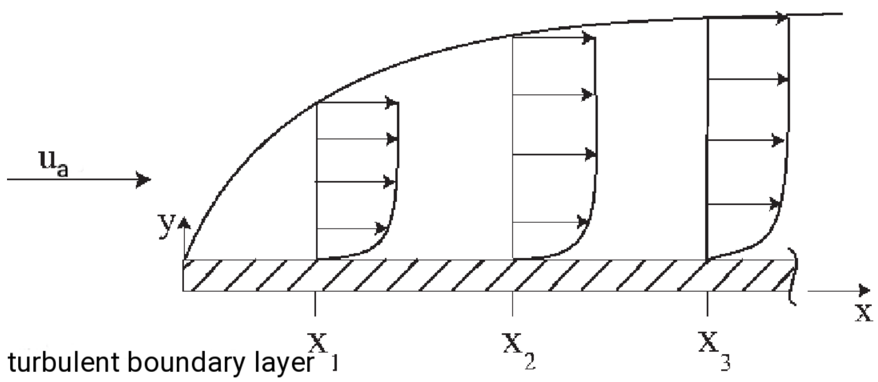
$$a_1 = \frac{-6BL\eta}{\rho\delta\dot{V}u_a - 4BL\eta}$$
$$a_3 = \frac{7BL\eta - 2\rho\delta\dot{V}u_a}{\rho\delta\dot{V}u_a - 4BL\eta}$$

b) Sketch of the laminar boundary layer thickness and the profile of the velocity $u(y)$



c) The critical Reynolds number for the flat plate is $5 \cdot 10^5$.

d)
- The velocity profile of the turbulent boundary layer is more bulky,
- the turbulent boundary layer is thicker,
- higher friction in the turbulent boundary layer ,
- In the laminar boundary layer the flow lines are almost parallel to each other (flow in layers)
- ...



turbulent boundary layer

Problem 6 (11 Points)

a) Show that the sum of molecular shear stress $\tau_{lam}(y)$ and turbulent apparent stress $\tau_{tur}(y)$ is constant.
Hint: $\tau_{tur} = -\varrho\overline{u'v'}$

b) Show with the Prandtl's mixing length approach that the time averaged velocity is proportional to the wall distance ($\overline{u} \sim y$) in the vicinity of the wall for $y \to 0$.
Hint: Prandtl's mixing length approach:

$$\tau_{tur} = \varrho l^2 \left|\frac{\partial \bar{u}}{\partial y}\right| \frac{\partial \bar{u}}{\partial y} \text{ with } l = \kappa y$$

Given: $\varrho, \eta, \overline{p}, u_0, \kappa$

c) Define the displacement thickness δ_1 and give the equation for $\delta_1 = f(\frac{u(y)}{u_a})$.

d) Determine the ratio of the momentum thickness to the boundary layer thickness in a turbulent boundary layer taking into account the $\frac{1}{7}$ velocity profile.

Problem 6

a)

a) $\eta, \varrho \neq f(y)$

$$0 = \frac{\partial}{\partial y}(\eta \frac{\partial \overline{u}}{\partial y}) - \frac{\partial \varrho \overline{u'v'}}{\partial y}$$
$$\Leftrightarrow 0 = \frac{\partial}{\partial y}(-\tau_{lam} + \tau_{tur})$$
$$\text{Intregation} \Rightarrow -\tau_{lam} + \tau_{tur} = \text{const.}$$

b)

$$\text{mit } \tau_{tur} = \varrho x^2 y^2 |\frac{\partial \overline{u}}{\partial y}| \frac{\partial \overline{u}}{\partial y}$$
$$\Rightarrow \lim_{y \to 0} [\eta \frac{\partial \overline{u}}{\partial y} + \varrho x^2 y^2 |\frac{\partial \overline{u}}{\partial y}| \frac{\partial \overline{u}}{\partial y}] = \eta \frac{\partial \overline{u}}{\partial y}$$
$$\eta \frac{\partial \overline{u}}{\partial y} = \text{const.}$$
$$\text{Integration } \Rightarrow \overline{u} = \frac{\text{const.}}{\eta} y + \text{const.}$$
$$\Leftrightarrow \overline{u} \sim y$$

c) The displacement thickness corresponds to the distance by which a body must be thickened in a hypothetical frictionless flow such that the same mass flow occurs as in the actual flow.

$$\delta_1 = \int_0^\infty (1 - \tfrac{u}{u_a}) dy$$

d) For the $\frac{1}{7}$ velocity profile, the power law gives the velocity distribution in the turbulent boundary layer:

$$\frac{\bar{u}}{u_a} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

with the definition of the momentum thickness

$$\delta_2 = \delta \int_0^1 \frac{\bar{u}}{u_a} \left(1 - \frac{\bar{u}}{u_a}\right) d\left(\frac{y}{\delta}\right)$$

the ratio of momentum thickness and boundary layer thickness is:

$$\frac{\delta_2}{\delta} = \int_0^1 \left(\left(\frac{y}{\delta}\right)^{\frac{1}{7}} - \left(\frac{y}{\delta}\right)^{\frac{2}{7}} \right) d\left(\frac{y}{\delta}\right)$$

$$\frac{\delta_2}{\delta} = \frac{7}{8}\left(\frac{y}{\delta}\right)^{\frac{8}{7}} - \frac{7}{9}\left(\frac{y}{\delta}\right)^{\frac{9}{7}} = \frac{7}{8} - \frac{7}{9}$$

$$\frac{\delta_2}{\delta} = \frac{7}{72}$$