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(Name, Matr.-Nr, Unterschrift)

Klausur „Strömungsmechanik I“ (Bachelor) & „Technische Strömungslehre“ (Diplom)

03. 08. 2012

1. Aufgabe

a) Kräftegleichgewicht zur Auftriebsberechnung: $\sum F = 0$ => $F_A = m_U \cdot g + G_{Ws}$

$$G_{Ws} = A \cdot g \int_0^H \rho_w(z)dz + p_a A = A \cdot g \left[\rho_{W_0} z + \Delta\rho \frac{z^2}{2H}\right]_0^H + p_a A$$

$$G_{Ws} = A \cdot g \cdot H \left(\rho_{W_0} + \frac{\Delta\rho}{2}\right) + p_a A$$

$$F_A = \rho_W(H) \cdot g \cdot V_L$$

$$(\rho_{W_0} + \Delta\rho) \cdot g \cdot V_L = m_u \cdot g + A \cdot g \cdot H \left(\rho_{W_0} + \frac{\Delta\rho}{2}\right) + p_a A$$

$$V_L = \frac{(m_U + A \cdot H \left(\rho_{W_0} + \frac{\Delta\rho}{2}\right)}{(\rho_{W_0} + \Delta\rho)} + \frac{p_a \cdot A}{g(\rho_{W_0} + \Delta\rho)} \qquad [m^3]$$

b) $\sum F = 0, \quad m_u g = F_A$

$$m_L = \rho_L \cdot V_L$$

$$V_L \cdot \rho_W(z) \cdot g = m_U \cdot g$$

$$V_L = \frac{m_U}{\rho_{W_0} + \Delta\rho \frac{z}{H}}$$

$$m_L = \rho_L \cdot V_L, \quad \rho_L = \frac{p(z)}{R_L \cdot T_W}$$

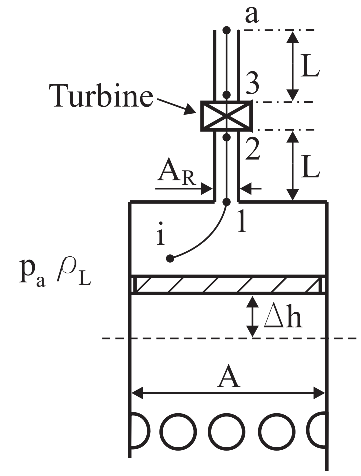
$$p(z) = p_a + \int_0^z \rho(z) g dz = p_a + g \int_0^z (\rho_{W_0} + \Delta\rho \frac{z}{H}) dz = p_a + \rho_{W_0} \cdot gz + \Delta\rho g \frac{z^2}{2H}$$

$$\rho_L = \frac{p_a + \rho_{W_0} \cdot gz + \Delta\rho g \frac{z^2}{2H}}{R_L T_W}$$

$$m_L = \frac{p_a + \rho_{W_0} \cdot gz + \Delta\rho g \frac{z^2}{2H}}{R_L T_W} \cdot \frac{m_U}{\left(\rho_{W_0} + \Delta\rho \frac{z}{H}\right)} \qquad [kg]$$

2. Aufgabe

a) Leistung: $P = \dot{V}\Delta p_{0,T} = \dot{V}\Delta p_T \Rightarrow \Delta p_T = \frac{P}{\dot{V}} = \frac{P}{Av_i}$



Bernoulli $\boxed{i} \rightarrow \boxed{a}$:

$$p_i(\omega,t) + \frac{\rho_L}{2}v_i^2 = p_a + \rho_L g \cdot 2L + \frac{\rho_L}{2}v_R^2 + \Delta p_T + \rho_L \int_i^a \frac{\partial v_R}{\partial t} ds$$

Konti: $v_i A = v_R A_R \Rightarrow v_R = v_i \frac{A}{A_R}$

Auslenkung (A.S.): $h(t) = h_i(t) = h_{max} \cdot \sin(\omega t)$

Geschwindigkeit: $\frac{dh_i}{dt} = v_i(t) = h_{max}\,\omega \cdot \cos(\omega t)$

Beschleunigung: $\frac{dv_i}{dt} = -h_{max}\,\omega^2 \cdot \sin(\omega t)$

$$\Rightarrow p_i(\omega,t) + \frac{\rho_L}{2}v_i^2 = p_a + \rho_L g \cdot 2L + \frac{\rho_L}{2}\left(\frac{A}{A_R}\right)^2 v_i^2 + \frac{P}{Av_i} + \rho_L\, 2L \cdot \frac{dv_R}{dt}$$

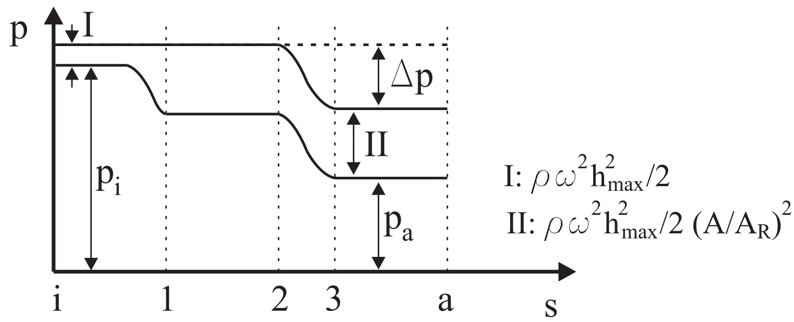
$$\Rightarrow \frac{P}{Av_i} = p_i(\omega,t) - p_a - \rho_L g \cdot 2L + \frac{\rho_L}{2}v_i^2\left(1 - \left(\frac{A}{A_R}\right)^2\right) - \rho_L\, 2L \cdot \frac{dv_R}{dt}$$

$$\Rightarrow P = Av_i\left(p_i(\omega,t) - p_a - \rho_L g \cdot 2L + \frac{\rho_L}{2}v_i^2\left(1 - \left(\frac{A}{A_R}\right)^2\right) - \rho_L\, 2L \cdot \frac{dv_R}{dt}\right)$$

$$P = A \cdot h_{max}\,\omega \cdot \cos(\omega t)\left(p_i(\omega,t) - p_a - \rho_L g \cdot 2L + \frac{\rho_L}{2}(h_{max}\,\omega \cdot \cos(\omega t))^2\left(1 - \left(\frac{A}{A_R}\right)^2\right)\right.$$

$$\left. + \rho_L\, 2L \cdot h_{max}\,\omega^2 \cdot \sin(\omega t)\frac{A}{A_R}\right) \qquad \left[m^2 m \frac{1}{s}(\frac{N}{m^2})\right] = \left[\frac{Nm}{s}\right] = [W]$$

b) Druckverlauf entlang der Stromlinie $\boxed{i} \rightarrow \boxed{a}$



3. Aufgabe

a) Impulssatz in X-Richtung:

$$\sum \vec{F}_i = \int \vec{v}(\vec{v}\cdot\vec{n})dA$$

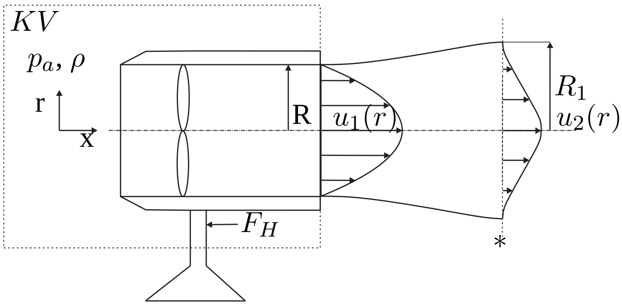
$$F_H = 2\pi\rho \int_0^R u_1(r)^2 r dr$$

$$= 2\pi\rho u_{max_1}^2 \int_0^R \left(1 - 2\frac{r^2}{R^2} + \frac{r^4}{R^4}\right) r dr$$

$$F_H = 2\pi\rho u_{max_1}^2 \left(\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4}\right)$$

$$= \pi\rho u_{max_1}^2 \frac{R^2}{3}$$

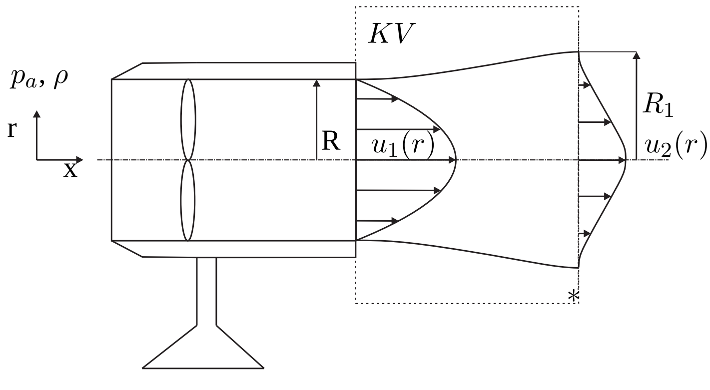
$$P = F_H \cdot u_m$$



$$u_m = \frac{2}{R^2}\int_0^R u_{max_1}\left(1 - \frac{r^2}{R^2}\right) r dr = 2\frac{u_{max_1}}{R^2}\left(\frac{R^2}{2} - \frac{R^2}{4}\right) = \frac{u_{max_1}}{2}$$

$$\Rightarrow \quad P = \pi\rho u_{max_1}^3 \frac{R^2}{6} \qquad [W]$$

b)



Kontinuitätsgleichung:

$$2\pi \int_0^R u_{max_1}\left(1 - \frac{r^2}{R^2}\right) r dr = 2\pi \int_0^{R_1} u_{max_2}\left(1 - \frac{r^2}{4R_1^2}\right) r dr$$

$$u_{max_1}\left(\frac{R^2}{2} - \frac{R^2}{4}\right) = u_{max_2}\int_0^{R_1}\left(r - \frac{r^3}{4R_1^2}\right) dr$$

$$u_{max_1}\frac{R^2}{4} = u_{max_2}\left[\frac{r^2}{2} - \frac{r^4}{16R_1^2}\right]_0^{R_1} = \frac{7}{16}R_1^2 u_{max_2}$$

$$u_{max_1} = \frac{7}{4}\frac{R_1^2}{R^2}u_{max_2}$$

Impulssatz in x-Richtung:

$$2\pi\rho \int_0^R u_{max_1}^2 \left(1 - \frac{r^2}{R^2}\right)^2 r dr = 2\pi\rho \int_0^{R_1} u_{max_2}^2 \left(1 - \frac{r^2}{4R_1^2}\right)^2 r dr$$

$$u_{max_1}^2 \left(\frac{R^2}{2} - \frac{R^2}{2} + \frac{R^2}{6}\right) = u_{max_2}^2 \int_0^{R_1} \left(r - \frac{r^3}{2R_1^2} + \frac{r^5}{16R_1^4}\right) dr$$

$$u_{max_1}^2 \frac{R^2}{6} = u_{max_2}^2 \left(\frac{R_1^2}{2} - \frac{R_1^2}{8} + \frac{R_1^2}{96}\right)$$

$$\frac{R^2}{6} u_{max_1}^2 = \frac{37}{96} R_1^2 u_{max_2}^2$$

$$u_{max_1} = \frac{\sqrt{37}}{4} \frac{R_1}{R} u_{max_2}$$

Einsetzen in Kontinuitätsgleichung:

$$\frac{\sqrt{37}}{4} \frac{R_1}{R} u_{max_2} = \frac{7R_1^2}{4R^2} u_{max_2}$$

$$\Rightarrow \quad R_1 = \frac{\sqrt{37}}{7} R \qquad [m]$$

4. Aufgabe

a) $Fr < 1$ Die Spiegelhöhe nach dem Wehr ist niedriger als die der Anströmung.

b) Bernoulli:

$$\rho g z + \rho g y + \frac{\rho}{2} v^2 = const$$

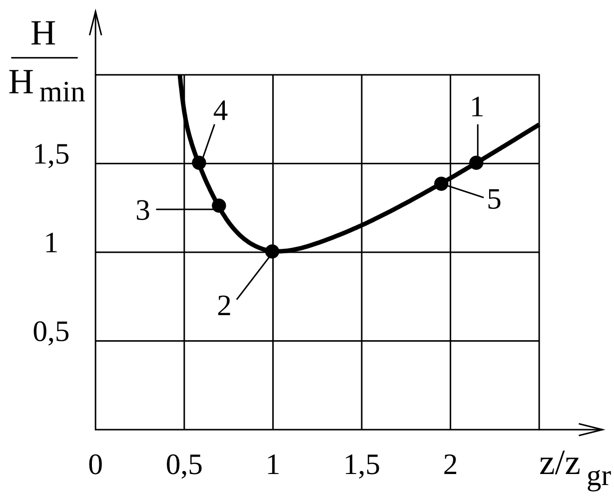
$$\Rightarrow \quad z + \frac{v^2}{2g} + y = const \quad \text{mit} \quad H = z + \frac{v^2}{2g} \quad \text{und} \quad v = \frac{\dot{V}}{Bz}$$

$$\Rightarrow \quad H = z + \frac{\dot{V}^2}{2gB^2z^2}$$

$$H_{min} : \frac{\partial H}{\partial z} = 0 \quad \Rightarrow \quad z_{gr} = \sqrt[3]{\frac{\dot{V}^2}{gB^2}} \qquad \left[\left(\frac{m^6}{s^2} \frac{s^2}{m} \frac{1}{m^2} \right)^{1/3} \right] = [m]$$

$$\Rightarrow \quad H_{min} = z_{gr} + \frac{\dot{V}^2}{2gB^2 z_{gr}^2} = \frac{3}{2} \sqrt[3]{\frac{\dot{V}^2}{gB^2}} \qquad \left[\sqrt[3]{\frac{m^6}{s^2} \frac{s^2}{m} \frac{1}{m^2}} \right] = [m]$$

c) Skizze:



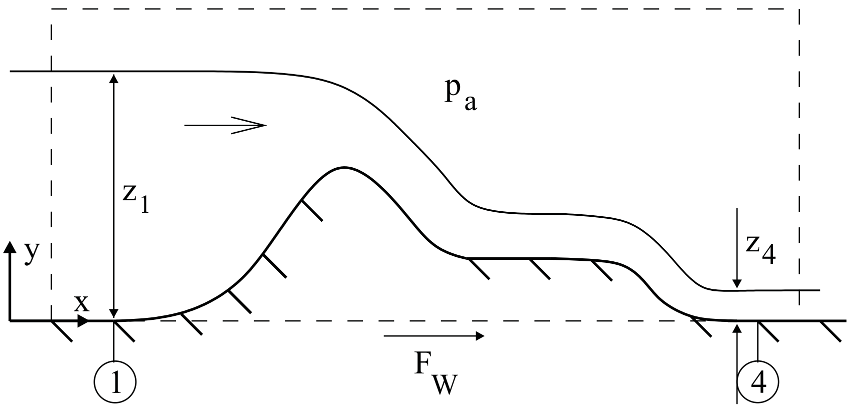
d) Konti:

$$v_4 z_4 B = v_1 z_1 B \quad \Rightarrow \quad v_4 = v_1 \frac{z_1}{z_4} = 4 v_1$$

$$\Rightarrow \quad Fr_4 = \frac{v_4}{\sqrt{g z_4}} = \frac{v_1}{\sqrt{g \frac{z_4}{z_1} z_1}} \frac{z_1}{z_4} = 8 \frac{v_1}{\sqrt{g z_1}} \quad \text{mit} \quad v_1 = \frac{\dot{V}}{B z_1}$$

$$\Rightarrow \quad Fr_4 = 8 \frac{\dot{V}}{B z_1 \sqrt{g z_1}} \qquad [.]$$

e) Impulserhaltungssatz (Bilanzhülle):



$$-\rho v_1^2 z_1 B - B\int_0^{z_1} p_1(z)\mathrm{d}z + \rho v_4^2 z_4 B + B\int_0^{z_4} p_4(z)\mathrm{d}z + p_a B(z_1 - z_4) + F_W = 0$$

mit $\quad p_1(z) = p_a + \rho g(z_1 - z)$ $\quad$ und $\quad p_4(z) = p_a + \rho g(z_4 - z)$

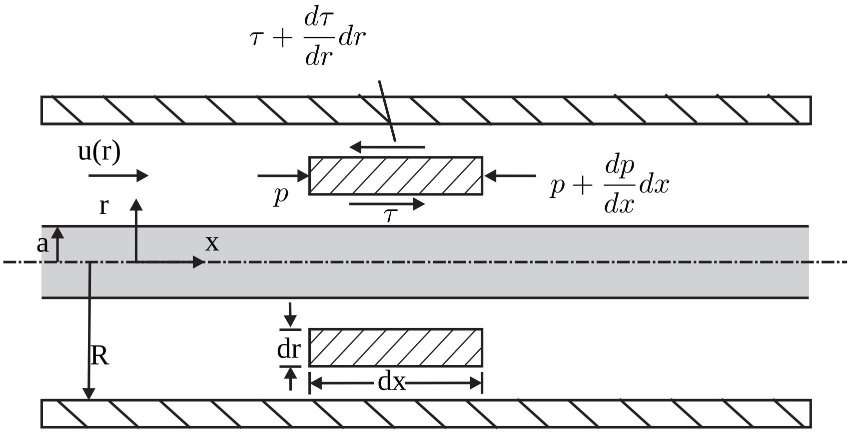
$$\Rightarrow \quad F_W = \rho v_1^2 z_1 B - \rho v_4^2 z_4 B + B\rho g\frac{z_1^2}{2} - B\rho g\frac{z_4^2}{2}$$

$$\Rightarrow \quad F_W = \rho B\left(v_1^2 z_1 - 4v_1^2 z_1 + g\frac{z_1^2}{2} - g\frac{z_1^2}{32}\right) = \rho B\left(\frac{15}{32}\, g z_1^2 - 3\, v_1^2 z_1\right)$$

$$\Rightarrow \quad F_W = \rho B\left(\frac{15}{32}\, g z_1^2 - \frac{3}{64}\, g z_1^2\right)$$

$$\Rightarrow \quad F_W = \frac{27}{64}\,\rho B g\, z_1^2 \qquad \left[\frac{kg}{m^3}\frac{m}{s^2}mm^2\right] = [N]$$

5. Aufgabe



a) $\tau = \tau(r) \quad \frac{\partial p}{\partial y} = 0 \rightarrow \frac{\partial p}{\partial x} = \frac{dp}{dx}$

Kräftegleichgewicht $\sum F_x = 0$:

$$p \cdot 2\pi r dr - \left(p + \frac{dp}{dx}dx\right) 2\pi r dr + \tau 2\pi r dx - \left(\tau + \frac{d\tau}{dr}dr\right) 2\pi(r+dr)dx = 0$$

$$\frac{dp}{dx}dr + \frac{d\tau}{dr}dr + \frac{\tau}{r}dr + \frac{1}{r}\frac{d\tau}{dr}dr^2 = 0$$

Vernachlässigung von Termen höherer Ordnung:

$$\frac{dp}{dx} + \frac{d\tau}{dr} + \frac{\tau}{r} = 0 \qquad \Longrightarrow \qquad \frac{dp}{dx} + \frac{1}{r}\frac{d(r\tau)}{dr} = 0$$

Newton'sches Fluidmodell: $\tau = -\eta\frac{du}{dr}$

$$\frac{\eta}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) - \frac{dp}{dx} = 0$$

b) Randbedingungen:

$$F_R|_{r=a} = 0 \quad \Leftrightarrow \quad \tau|_{r=a} = 0 \quad \Leftrightarrow \quad \left.\frac{du}{dr}\right|_{r=a} = 0 \qquad (1)$$

$$u|_{r=R} = 0 \qquad (2)$$

aus a): $\quad \frac{dp}{dx} r dr = \eta \cdot d\left(r\frac{du}{dr}\right)$

$$\frac{dp}{dx} \cdot \frac{r^2}{2} = \eta \cdot r \cdot \frac{du}{dr} + C_1 \qquad \Longrightarrow \qquad \frac{dp}{dx} \cdot \frac{r}{2} = \eta \cdot \frac{du}{dr} + \frac{C_1}{r}$$

$$\frac{dp}{dx} \cdot \frac{r^2}{4} = \eta \cdot u(r) + C_1 \cdot ln(r) + C_2$$

mit (1): $\quad \frac{dp}{dx} \cdot \frac{a^2}{2} = C_1 \quad$ mit (2): $\quad \frac{dp}{dx} \cdot \frac{R^2}{4} - C_1 \cdot ln(R) = C_2$

$$u(r) = \frac{1}{\eta}\frac{dp}{dx}\left(\frac{r^2 - R^2}{4} + \frac{a^2}{2}ln\left(\frac{R}{r}\right)\right)$$

$$u_a = u|_{r=a} = \frac{1}{\eta}\frac{dp}{dx}\left(\frac{a^2 - R^2}{4} + \frac{a^2}{2}ln\frac{R}{a}\right) \qquad \left[\frac{s}{kgm}\frac{kgm}{s^2}\frac{1}{m}m^2\right] = \left[\frac{m}{s}\right]$$

6. Aufgabe

a) $\tau_t = \rho l^2 |\frac{d\bar{u}}{dy}|\frac{d\bar{u}}{dy}$

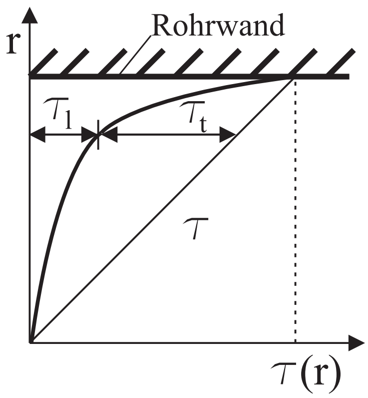
b) Der Mischungsweg ist die Strecke in Richtung der Normalen, die ein sich mit seiner ursprünglichen Geschwindigkeit bewegender Turbulenzballen zurücklegen muss, damit die Differenz zwischen seiner Geschwindigkeit und der Geschwindigkeit der neuen Schicht der gemittelten absoluten SchwankungsgröSSe entspricht.

c) Das universelle Wandgesetz von Prandtl ist für hydraulisch glatte und kreisförmige Querschnitte definiert.

d) $\lambda = \frac{0{,}316}{\sqrt[4]{Re}}$

e)

$$\tau = \tau_t + \tau_l = -\rho\overline{u'v'} + \eta\frac{d\overline{u}}{dr}$$



f)

$$\begin{aligned}
\overline{f\cdot g} &= \frac{1}{T}\int_T fg dt = \frac{1}{T}\int_T (\overline{f}+f')(\overline{g}+g')dt \\
&= \frac{1}{T}\int_T (\overline{f}\overline{g} + f'\overline{g} + \overline{f}g' + f'g')dt \\
&= \overline{f}\overline{g} + \overline{g}\underbrace{\frac{1}{T}\int_T f'dt}_{=0} + \overline{f}\underbrace{\int_T g'dt}_{=0} + \overline{f'g'} \\
&= \overline{f}\overline{g} + \overline{f'g'}
\end{aligned}$$