....
(Name, Matr.-Nr, Unterschrift)

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1. Aufgabe

a) $G = F_A$

$$G = \varrho_W (V_B + V_{L_1}) \cdot g$$

$$G = \varrho_W \left(\frac{G}{g \cdot \varrho_B} + V_{L_1} \right) \cdot g$$

$$V_{L_1} = \frac{G}{g} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right)$$

$$\varrho_{L_1} = \frac{p_1}{R \cdot T_W} = \frac{p_a + \varrho_W g (H + h)}{R \cdot T_W}$$

$$m_{L_1} = \varrho_{L_1} \cdot V_{L_1}$$

$$m_{L_1} = \frac{G}{g} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right) \cdot \frac{p_a + \varrho_W g (H + h)}{R \cdot T_W}$$

b) $G = F_A \quad \wedge \quad V_{L_1} = V_{L_2} = V_L$

$$G = \varrho_W (V_B + V_L) \cdot g$$

$$G = \varrho_W \left(\frac{G}{g \cdot \varrho_B} + A \cdot h \right) \cdot g$$

$$h = \frac{G}{g \cdot A} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right)$$

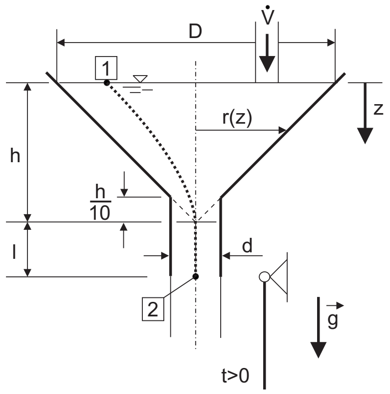
$$\varrho_{L_2} = \frac{p_a + \varrho_W g h}{R \cdot T_W}$$

$$m_{L_2} = \varrho_{L_2} \cdot A \cdot h = \frac{p_a + \varrho_w g h}{R T_W} \frac{G}{g} \left(\frac{1}{\varrho_W} - \frac{1}{\varrho_B} \right)$$

$$\Delta m = m_{L_1} - m_{L_2} = \frac{G}{g} \left(\frac{1}{\varrho_B} - \frac{1}{\varrho_W} \right) \frac{\varrho_W g H}{R T_W}$$

c) kalte Luft $\Rightarrow$ höhere Luftdichte $\Rightarrow$ weniger verdrängtes Volumen $\Rightarrow$ weniger Auftrieb $\Rightarrow$ Behälter sinkt

2. Aufgabe



a) Bernoulli von $\boxed{1}$ nach $\boxed{2}$:

$$p_a + \frac{\varrho}{2}v_1^2 = p_a - \varrho g(h+l) + \frac{\varrho}{2}v_2^2 + \varrho\int_1^2 \frac{\partial v}{\partial t}ds$$

$$\frac{A_1}{A_2} = \left(\frac{D}{d}\right)^2 = 100 \quad \text{und} \quad v_1 = 0$$

Konti: $\quad vA = v_2 A_2 \quad \Longrightarrow \quad v = v_2\dfrac{A_2}{A(z)}$

$$\Longrightarrow \quad \varrho\int_1^2 \frac{\partial v}{\partial t}ds = \varrho\int_1^2 \frac{\partial}{\partial t}\left(v_2\frac{A_2}{A(z)}\right)ds = \varrho\frac{dv_2}{dt}\int_1^2 \frac{A_2}{A(z)}ds$$

mit: $\displaystyle \int_1^2 = \int_0^{\frac{9}{10}h} + \int_{\frac{9}{10}h}^{h+l}$

$$= \varrho\frac{dv_2}{dt}\int_0^{\frac{9}{10}h}\frac{A_2}{A(z)}dz + \varrho\frac{dv_2}{dt}\left(l + \frac{h}{10}\right)$$

wobei: $\displaystyle \frac{A_2}{A(z)} = \frac{\frac{\pi}{4}d^2}{\pi r(z)^2} = \left(\frac{d}{D}\right)^2\frac{1}{(1-\frac{z}{h})^2} = \left(\frac{d}{D}\right)^2\frac{h^2}{(h-z)^2}$

$$\Longrightarrow \int_0^{\frac{9}{10}h}\frac{A_2}{A(z)}dz = \left(\frac{d\cdot h}{D}\right)^2\cdot\frac{1}{h-z}\bigg|_0^{\frac{9}{10}h} = 0,09h$$

$$\Longrightarrow \varrho g(h+l) = \frac{\varrho}{2}v_2^2(t) + \varrho\frac{dv_2}{dt}(l+0,19h)$$

für t = 0: $\quad v_2 = 0 \quad \Longrightarrow \quad \left.\dfrac{dv_2}{dt}\right|_{t=0} = \dfrac{g(h+l)}{l+0,19h}$

b) aus a) folgt:

$$g(h+l) - \frac{v_2^2}{2} = \frac{dv_2}{dt}(l+0,19h) \quad \Longrightarrow \quad dt = (l+0,19)\frac{2\cdot dv_2}{2g(h+l)-v_2^2}$$

stationäre Endgeschwindigkeit: $v_2(t\to\infty) = \sqrt{2g(h+l)}$

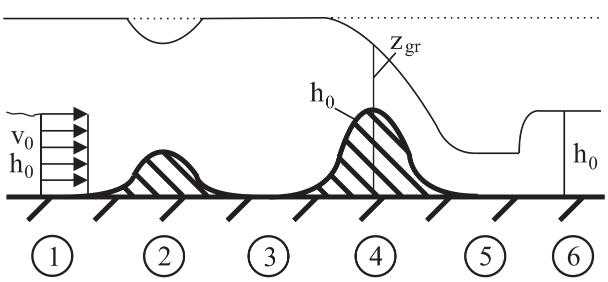
da $v_2(t) < v_2(t\to\infty) \quad \Longrightarrow \quad$ Integral für $|x| < a$

$$\Longrightarrow \Delta T = \frac{l+0,19h}{\sqrt{2g(h+l)}}\ln\frac{\sqrt{2g(h+l)}+v_2}{\sqrt{2g(h+l)}-v_2}\Bigg|_0^{0,9\sqrt{2g(h+l)}}$$

$$\Longrightarrow \Delta T = \frac{l+0,19h}{\sqrt{2g(h+l)}}\ln(19)$$

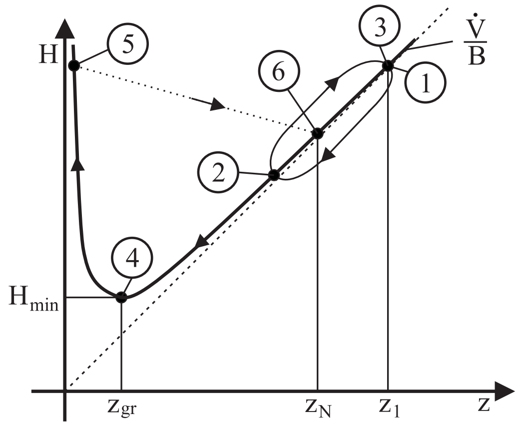
3. Aufgabe

a) Skizze



Aufstau bei konstantem

$$\frac{\dot{V}}{B} = v_0 h_0$$



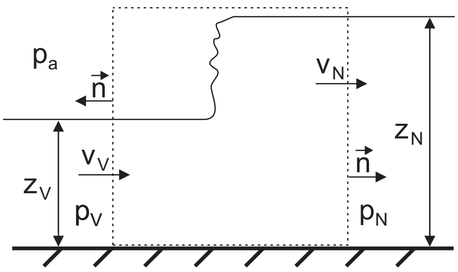
$$H_{ges} = H_{fluid} + y = konst$$

b) $H_V + y_V = H_4 + y_4$ mit $H_4 = H_{min}$, $y_4 = h_0$ und $y_V = 0$

$$H_V = h_0 + z_{gr} + \frac{\dot{V}^2}{2g z_{gr}^2 B^2}$$

$$\Longrightarrow H_V = h_0 + \left(\frac{\dot{V}^2}{gB^2}\right)^{\frac{1}{3}} + \frac{1}{2}\left(\frac{\dot{V}^2}{gB^2}\right)^{1-\frac{2}{3}} = h_0 + \frac{3}{2}\left(\frac{v_0^2 h_0^2}{g}\right)^{\frac{1}{3}}$$

c) Energieverlust



Konti: $v_N z_N = v_V z_V$

IES: Herleitung Skript S.126

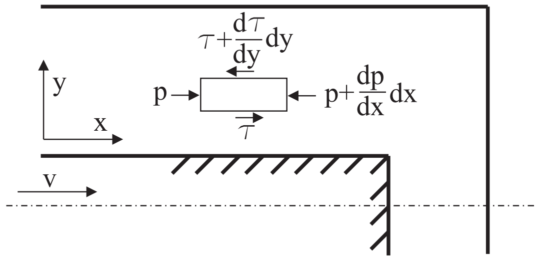
$$\Longrightarrow \quad -\varrho v_V^2 z_V + \varrho v_N^2 z_N = \varrho g \left(\frac{z_V^2}{2} - \frac{z_N^2}{2}\right)$$

$$\Delta H = H_N - H_V = z_N - z_V + \frac{\dot{V}^2}{2gB^2}\left(\frac{1}{z_N^2} - \frac{1}{z_V^2}\right)$$

mit IES und Konti: $\dfrac{\dot{V}^2}{gB^2} = \dfrac{z_V z_N (z_V + z_N)}{2}$

$$\Longrightarrow \Delta H = z_N - z_V + \frac{1}{4}\left(\frac{1}{z_N^2} - \frac{1}{z_V^2}\right) z_V z_N (z_V + z_N)$$

4. Aufgabe



a) Bilanzierung an infinitesimal kleinem Element

$$\frac{dp}{dx} = -\frac{d\tau}{dy}$$

$$\tau = -\eta \frac{du}{dy} \quad \Longrightarrow \quad \frac{d^2u}{dy^2} = \frac{1}{\eta}\frac{dp}{dx}$$

1. Integration: $\frac{du}{dy} = \frac{1}{\eta}\frac{dp}{dx} y + C_1 \quad \wedge$ 2. Integration: $u = \frac{1}{2\eta}\frac{dp}{dx} y^2 + C_1 y + C_2$

R.B.:

$$\left.\begin{array}{ll} 1) & y = 0: \quad u = v \\ 2) & y = \frac{b_2 - b_1}{2} \equiv H: \quad u = 0 \end{array}\right\} \text{Haftbedingung}$$

aus 1) $C_2 = v$

aus 2) $0 = \frac{1}{2\eta}\frac{dp}{dx} H^2 + C_1 H + v = 0 \iff C_1 = -\frac{1}{2\eta}\frac{dp}{dx} H - \frac{v}{H}$

$$\Longrightarrow \quad u(y) = \frac{1}{2\eta}\frac{dp}{dx} H^2 \cdot \left(\frac{y^2}{H^2} - \frac{y}{H}\right) + v \cdot \left(1 - \frac{y}{H}\right)$$

$$\Longrightarrow \quad u(y) = \frac{1}{4\eta}\frac{dp}{dx} y \cdot \left(\frac{y}{b_2 - b_1} - 1\right) + v \cdot \left(1 - \frac{2y}{b_2 - b_1}\right)$$

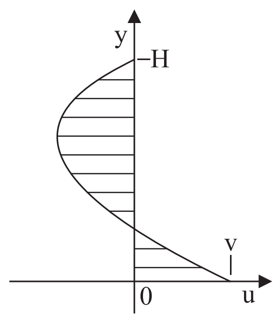
b) Volumenstrombilanz: $\frac{\dot{V}}{B} = 2 \cdot \int_0^H u(y)dy = -b_1 \cdot v$

$$\int_0^H u(y)dy = \frac{1}{2\eta}\frac{dp}{dx} H^2 \left(\frac{1}{3}\frac{y^3}{H^2} - \frac{1}{2}\frac{y^2}{H}\right) + v \cdot \left(y - \frac{1}{2}\frac{y^2}{H}\right)\Bigg|_0^H$$

$$= -\frac{1}{12\eta}\frac{dp}{dx} H^3 + v\frac{H}{2} = -b_1 \frac{v}{2} \iff v \cdot \left(\frac{b_1}{2} + \frac{H}{2}\right) = \frac{1}{12\eta}\frac{dp}{dx} H^3$$

$$\iff \quad \frac{1}{2\eta}\frac{dp}{dx} H^2 = 3\frac{v}{H}(H + b_1) \iff \frac{dp}{dx} = \frac{6\eta v}{H^2} \cdot \left(1 + \frac{b_1}{H}\right)$$

$$\Longrightarrow \quad u(y) = 3v \cdot \left(1 + \frac{b_1}{H}\right)\left(\frac{y^2}{H^2} - \frac{y}{H}\right) + v \cdot \left(1 - \frac{y}{H}\right)$$

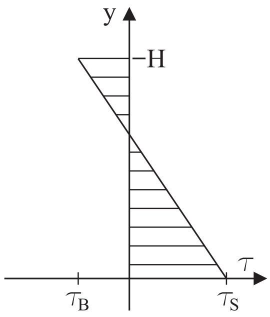


$$\Longrightarrow \quad u(y) = 6v \cdot \left(1 + \frac{2b_1}{b_2 - b_1}\right)\left(\frac{2y^2}{(b_2 - b_1)^2} - \frac{y}{b_2 - b_1}\right) + v \cdot \left(1 - \frac{2y}{b_2 - b_1}\right)$$

c) $\tau = -\eta \dfrac{du}{dy} = -\eta \left(3v \left(1 + \dfrac{b_1}{H}\right) \left(\dfrac{2y}{H^2} - \dfrac{1}{H}\right) - \dfrac{v}{H} \right)$

Stempel $y = 0$: $\quad \tau_S = 3\eta \dfrac{v}{H} \left(1 + \dfrac{b_1}{H}\right) + \eta \dfrac{v}{H}$

B-Wand $y = H$: $\quad \tau_B = -3\eta \dfrac{v}{H} \left(1 + \dfrac{b_1}{H}\right) + \eta \dfrac{v}{H}$



5. Aufgabe

a) Referenzgrößen: u_{ref}, L, T_a

$$\bar{u} = \frac{u}{u_{ref}}, \bar{v} = \frac{v}{u_{ref}}, \bar{x} = \frac{x}{L}, \bar{y} = \frac{y}{L}, \bar{T} = \frac{T}{T_a}$$

$$\begin{aligned}
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} &= 0 \\
\frac{u_{ref}^2}{L}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) &= -g\frac{1}{\bar{T}}(\bar{T}-1) + \nu\frac{u_{ref}}{L^2}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \\
u_{ref}\frac{T_a}{L}\left(\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right) &= a\frac{T_a}{L^2}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}
\end{aligned}$$

$$\begin{aligned}
\Pi_1 &= \frac{gL}{u_{ref}^2} \\
\Pi_2 &= \frac{\nu}{u_{ref}L} \\
\Pi_3 &= \frac{a}{Lu_{ref}}
\end{aligned}$$

b)

$$\begin{aligned}
\Pi_1 &= \frac{1}{Fr^2} \\
\Pi_2 &= \frac{1}{Re} \\
\Pi_3 &= \frac{\lambda}{\rho c_p L u_{ref}} = \frac{\lambda\eta}{\rho c_p L u_{ref}\eta} = \frac{1}{Re}\frac{1}{Pr}
\end{aligned}$$

c)
- Froudezahl : Verhältnis von Trägheits- zu Schwerekräften
- Reynoldszahl : Verhältnis von Trägheits- zu Reibungskräften
- Prandtlzahl : Verhältnis von Reibungswärme zu abgeleiteter Wärme

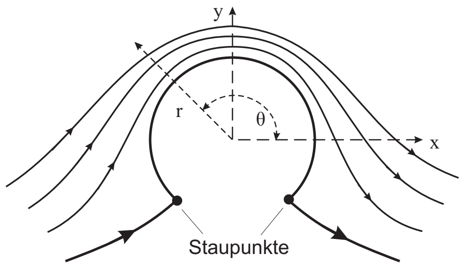
6. Aufgabe

a) $|\Gamma| < |\Gamma_{\text{gr}}|$: Zwei Staupunkte auf der Oberfläche des Zylinders. Staupunkte liegen auf der Unterseite des Zylinders. Strömung auf der Oberseite beschleunigt.
$|\Gamma| = |\Gamma_{\text{gr}}|$: Staupunkte laufen in einem Punkt auf der Zylinderoberfläche zusammen. Dieser Punkt liegt auf der y-Achse auf der Unteseite des Zylinders.
$|\Gamma| > |\Gamma_{\text{gr}}|$: Der Staupunkt entfernt sich entlang der y-Achse von der Zylinderoberfläche.

Anm. für Korrektur: eindeutige Skizzen sind auch o.k.

b) Parallelströmung in x-Richtung + Dipol + Potentialwirbel:

$$F(z) = u_a z + \frac{M}{2\pi z} - \frac{i\Gamma}{2\pi}\ln z =$$
$$u_a r e^{i\theta} + \frac{M}{2\pi r}e^{-i\theta} - \frac{i\Gamma}{2\pi}(\ln r + i\theta)$$



c) $$F(z) = u_a r(\cos\theta + i\sin\theta) + \frac{M}{2\pi r}(\cos\theta - i\sin\theta) - \frac{\Gamma}{2\pi}(i\ln r - \theta)$$

$$\Rightarrow \quad \phi = \text{Re}(F(z)) = u_a r\cos\theta + \frac{M}{2\pi r}\cos\theta + \frac{\Gamma}{2\pi}\theta \qquad \text{oder}$$

$$\Rightarrow \quad \psi = \text{Im}(F(z)) = u_a r\sin\theta - \frac{M}{2\pi r}\sin\theta - \frac{\Gamma}{2\pi}\ln r$$

Geschwindigkeitskomponenten:

$$v_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = u_a\cos\theta - \frac{M}{2\pi r^2}\cos\theta$$

Kontur des Vorsprungs: $\quad v_r(R) = 0 \quad \Rightarrow \quad M = 2\pi R^2 u_a$

Staupunkt: $\quad v_\theta\left(R, \theta_S = \frac{3}{2}\pi \pm \frac{\alpha}{2}\right) = 0$

$$v_\theta(R,\theta_S) = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r} = -u_a\sin\theta_S - \frac{2\pi R^2 u_a}{2\pi R^2}\sin\theta_S + \frac{\Gamma}{2\pi R}$$

$$= -2u_a\sin\theta_S + \frac{\Gamma}{2\pi R} = 0 \quad \Rightarrow \quad \Gamma = 4\pi R u_a\sin\theta_S = 4\pi R u_a \sin\left(\frac{3}{2}\pi \pm \frac{\alpha}{2}\right)$$

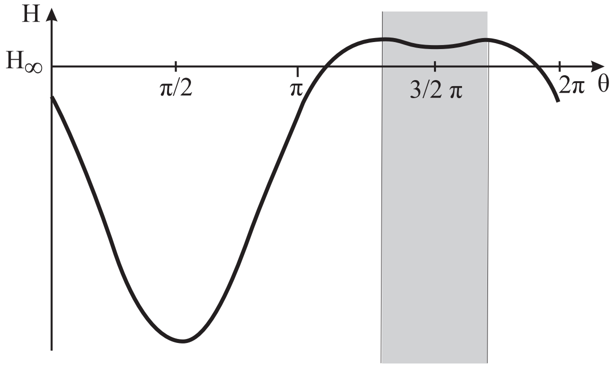
$$\alpha = \frac{\pi}{2} \quad \Rightarrow \quad \sin\left(\frac{3}{2}\pi \pm \frac{\pi}{4}\right) = -\frac{1}{2}\sqrt{2} \quad \Rightarrow \quad \Gamma = -2\sqrt{2}\pi R u_a$$

d) Bernoulli: $\quad p_\infty + \frac{1}{2}\rho u_a^2 + \rho g H_\infty = p_\infty + \frac{1}{2}\rho u^2(R,\theta) + \rho g H(R,\theta)$

mit $u(R,\theta) = v_\theta(R)$:

$$u^2(R,\theta) = \left(-2u_a\sin\theta - \frac{2\sqrt{2}R\pi u_a}{2\pi R}\right)^2 = u_a^2\left(2\sin\theta + \sqrt{2}\right)^2$$

$$H(R,\theta) = H_\infty + \frac{u_a - u^2(R,\theta)}{2g} = H_\infty + \frac{u_a^2}{2g}\left[1 - \left(2\sin\theta + \sqrt{2}\right)^2\right]$$



H
H∞
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7. Aufgabe

a)

$$\text{Haftbedingung:} \quad \frac{y}{\delta}=0 \rightarrow \frac{u}{u_a}=0 \quad \Rightarrow \quad a_0=0$$

$$\text{Grenzschichtrand:} \quad \frac{y}{\delta}=1 \rightarrow \frac{u}{u_a}=1 \quad \Rightarrow \quad a_1=1$$

$$\Rightarrow \quad \frac{u}{u_a}=\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

b) $p(x=0)=p_0 \quad \Rightarrow \quad$ Bernoulli:

$$p(x)+\frac{\rho}{2}u_a^2=p_0-C\frac{x^2}{2}+\frac{\rho}{2}u_a^2=p_0$$

$$u_a=x\sqrt{\frac{C}{\rho}}$$

c)
$$\delta_1=\delta\int_0^1\left(1-\frac{u}{u_a}\right)d\left(\frac{y}{\delta}\right)=\delta\int_0^1\left(1-\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right)d\left(\frac{y}{\delta}\right)=\delta\left[\frac{y}{\delta}+\frac{2}{\pi}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]_0^1$$

$$\Rightarrow \delta_1=\left(1-\frac{2}{\pi}\right)\delta=k_1\delta$$

$$\delta_2=\delta\int_0^1\frac{u}{u_a}\left(1-\frac{u}{u_a}\right)d\left(\frac{y}{\delta}\right)=\delta\int_0^1\left(\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)-\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right)d\left(\frac{y}{\delta}\right)$$

$$=\delta\left[-\frac{2}{\pi}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)-\frac{1}{2}\frac{y}{\delta}+\frac{1}{2\pi}\sin\left(\frac{\pi y}{\delta}\right)\right]_0^1$$

$$\Rightarrow \delta_2=\left(\frac{2}{\pi}-\frac{1}{2}\right)\delta=k_2\delta$$

$$\tau(y=0)=-\eta\left.\frac{du}{dy}\right|_{y=0}=-\frac{\eta\,u_a}{\delta}\left.\frac{d\left(\frac{u}{u_a}\right)}{d\left(\frac{y}{\delta}\right)}\right|_{\frac{y}{\delta}=0}=-\frac{\eta\,u_a}{\delta}\frac{\pi}{2}$$

d) Einsetzen in die von Kármánsche Integralbeziehung:

$$k_2\frac{d\delta}{dx}+\sqrt{\frac{\rho}{C}}\frac{1}{x}\sqrt{\frac{C}{\rho}}\,(2k_2+k_1)\,\delta-\frac{\eta\,x\,\pi\,\rho}{2\,\delta\,\rho\,x^2\,C}\sqrt{\frac{C}{\rho}}=0$$

Umformen:

$$\frac{d\delta}{dx}+\underbrace{\frac{2\,k_2+k_1}{k_2}}\frac{\delta}{x}-\underbrace{\frac{\eta\,\pi}{2k_2\sqrt{C\,\rho}}}\frac{1}{\delta\,x}=0$$

$$\frac{d\delta}{dx}+\Gamma\frac{\delta}{x}-\Omega\frac{1}{\delta\,x}=0$$

Differentialgleichung lösen:

$$x\frac{d\delta}{dx}=\Omega\frac{1}{\delta}-\Gamma\delta=\frac{\Omega-\Gamma\delta^2}{\delta}=\Gamma\left(\frac{\frac{\Omega}{\Gamma}-\delta^2}{\delta}\right)$$

$$\Rightarrow \quad \frac{1}{\Gamma}\left(\frac{\delta}{\frac{\Omega}{\Gamma}-\delta^2}\right)d\delta=\frac{1}{x}dx$$

$$\Rightarrow \qquad -\frac{1}{2\Gamma} \ln\left(\frac{\Omega}{\Gamma} - \delta^2\right)\Bigg|_{\delta_0}^{\delta} = \ln x|_{x_0}^{x}$$

$$\Rightarrow \qquad -\frac{1}{2\Gamma}\left[\ln \frac{\frac{\Omega}{\Gamma} - \delta^2}{\frac{\Omega}{\Gamma} - \delta_0^2}\right] = \ln\left(\frac{x}{x_0}\right)$$

Auflösen nach $\delta(x)$:

$$\left(\frac{\frac{\Omega}{\Gamma} - \delta^2}{\frac{\Omega}{\Gamma} - \delta_0^2}\right)^{-\frac{1}{2\Gamma}} = \frac{x}{x_0}$$

$$\Rightarrow \frac{\Omega}{\Gamma} - \delta^2 = \left(\frac{x}{x_0}\right)^{-2\Gamma}\left(\frac{\Omega}{\Gamma} - \delta_0^2\right)$$

$$\Rightarrow \delta = \sqrt{\frac{\Omega}{\Gamma} - \left(\frac{x}{x_0}\right)^{-2\Gamma}\left(\frac{\Omega}{\Gamma} - \delta_0^2\right)}$$

mit

$$\Omega = \frac{\eta\,\pi}{2\,k_2\sqrt{C\,\rho}} = \frac{\eta\,\pi}{2\left(\frac{\pi}{2} - \frac{1}{2}\right)\sqrt{C\,\rho}}, \qquad \Gamma = \frac{2k_2 + k_1}{k_2} = \frac{2\left(\frac{pi}{2} - \frac{1}{2}\right) + \left(1 - \frac{2}{\pi}\right)}{\left(\frac{\pi}{2} - \frac{1}{2}\right)} = \frac{4}{4 - \pi}$$

8. Aufgabe

a) Kondensationstemperatur:

$$\frac{p_K}{p_B} = \frac{T_K}{T_B} \quad \Rightarrow \quad T_K = T_B \frac{p_K}{p_B}$$

Kondensation im Austrittsquerschnitt $\Rightarrow p_K = p_a = p_\infty \quad \Rightarrow \quad T_a = T_K = T_B \frac{p_\infty}{p_B}$

Isentrope Zustandsänderung: $\frac{T_0}{T_a} = 1 + \frac{\gamma - 1}{2} M_a^2$

$$\frac{p_0}{p_a} = \left(\frac{T_0}{T_a}\right)^{\frac{\gamma}{\gamma-1}} \quad \Rightarrow \quad p_0 = p_\infty \left(\frac{T_0}{T_B}\frac{p_B}{p_\infty}\right)^{\frac{\gamma}{\gamma-1}}$$

Machzahl: $M_a = \sqrt{\left(\frac{T_0}{T_a} - 1\right)\frac{2}{\gamma - 1}} = \sqrt{\left(\frac{T_0}{T_B}\frac{p_B}{p_\infty} - 1\right)\frac{2}{\gamma - 1}}$

b) $\dot{m} = \rho_a u_a A_a \quad \Rightarrow \quad A_a = \frac{\dot{m}}{\rho_a u_a}$

$$\rho_a = \frac{p_\infty}{RT_a}$$

mit $R = \frac{\gamma - 1}{\gamma} c_p \quad \Rightarrow \quad \rho_a = \frac{p_\infty \gamma p_B}{(\gamma - 1) c_p T_B p_\infty} = \frac{\gamma p_B}{(\gamma - 1) c_p T_B}$

$$u_a = M_a \sqrt{\gamma R T_a} = M_a \sqrt{(\gamma - 1) c_p T_B \frac{p_\infty}{p_B}}$$

$$\Rightarrow A_a = \frac{\dot{m}\,(\gamma - 1) c_p T_B}{\gamma p_B M_a \sqrt{(\gamma - 1) c_p T_B \frac{p_\infty}{p_B}}}$$

Kritischer Querschnitt: $\dot{m} = \rho^* u^* A^* \quad \Rightarrow \quad A^* = \frac{\dot{m}}{\rho^* u^*}$

Kritische Größen: $\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma-1}}$ mit $\rho_0 = \frac{p_0}{RT_0} = \frac{p_0 \gamma}{(\gamma - 1) c_p T_0}$

$$T^* = T_0 \left(\frac{2}{\gamma + 1}\right) \quad \Rightarrow \quad u^* = c^* = \sqrt{\gamma R T^*} = \sqrt{\frac{\gamma - 1}{\gamma + 1} 2 c_p T_0}$$

$$\Rightarrow \quad A^* = \frac{\dot{m}(\gamma - 1) c_p T_0}{p_0 \gamma \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \sqrt{\frac{\gamma-1}{\gamma+1} 2 c_p T_0}}$$

c) Stoßbeziehung: $u_1 u_2 = c^{*2} \quad \Rightarrow \quad u_2 = \frac{c^{*2}}{u_1}$

Stoß im Austrittsquerschnitt, also gilt $u_1 = u_a, \quad u_2 = u_{a2}$

$$u_{a2} = \frac{\frac{\gamma-1}{\gamma+1} 2 c_p T_0}{M_a \sqrt{(\gamma - 1) c_p T_B \frac{p_\infty}{p_B}}}$$