1 Aufgabe | $z = H$. $G = G_{HL} + G_B = F_A$

$$G_B + \rho_{HL_0}\, g \cdot V_0 = \rho_L\, g\, V_{HL}$$

$$p = \rho R T \rightarrow \rho_L = \frac{\rho_{L_0} T_{L_0} p_L}{p_{L_0} T_L}$$

$$pV = RT \rightarrow V_{HL} = \frac{p_{L_0} T_{HL}}{p_L T_{HL_0}} V_0$$

schlaffe Hülle $\rightarrow p_L = p_{HL} \rightarrow \rho_{HL_0} = \rho_0 \dfrac{T_{L_0}}{T_{HL_0}} = \dfrac{\rho_{L_0}}{1{,}25}$

$$\rho_L V_{HL} = \rho_{L_0} V_0 \frac{1{,}25 - 10^{-5} z/z_0}{1{,}25}$$

$$G_B = \rho_{L_0} V_0 g \frac{1{,}25 - 10^{-5} H/z_0}{1{,}25} - \rho_{L_0} \frac{g V_0}{1{,}25}$$

$$= \rho_{L_0} V_0 g \frac{0{,}25 - 10^{-5} H/z_0}{1{,}25} = 300\,N$$

$\rho_B \gg \rho_{L_0} \rightarrow$ Auftrieb von Ballonhülle + Nutzlast

darf vernachlässigt werden

2. Aufgabe

a) $u_{a,stat} = \sqrt{2gH} = 6{,}32 \ m/s$

b) $u(x) = \dfrac{u_2 h_2}{h(x)} = \dfrac{u_2 h_2}{h_2 + (h_1 - h_2)\dfrac{L_1 - x}{L_1}}$

$$p_a + \rho g H = p_a + \frac{1}{2}\rho u_a^2 + \int_1^2 \rho \frac{\partial u(x)}{\partial t} ds + \int_2^a \rho \frac{\partial u_a}{\partial t} ds$$

$$\rho \int_1^2 \frac{\partial u(x)}{\partial t} ds = \rho L_1 \frac{\partial u_a}{\partial t} \int_1^2 \frac{dx}{2L_1 - x}$$

$$= -\rho L_1 \frac{\partial u_a}{\partial t} \ln(2L_1 - x)\Big|_1^2 = 0{,}693 \rho L_1 \frac{\partial u_a}{\partial t}$$

$$\rho \int_2^a \frac{\partial u}{\partial t} ds = L_2 \rho \frac{\partial u_a}{\partial t}$$

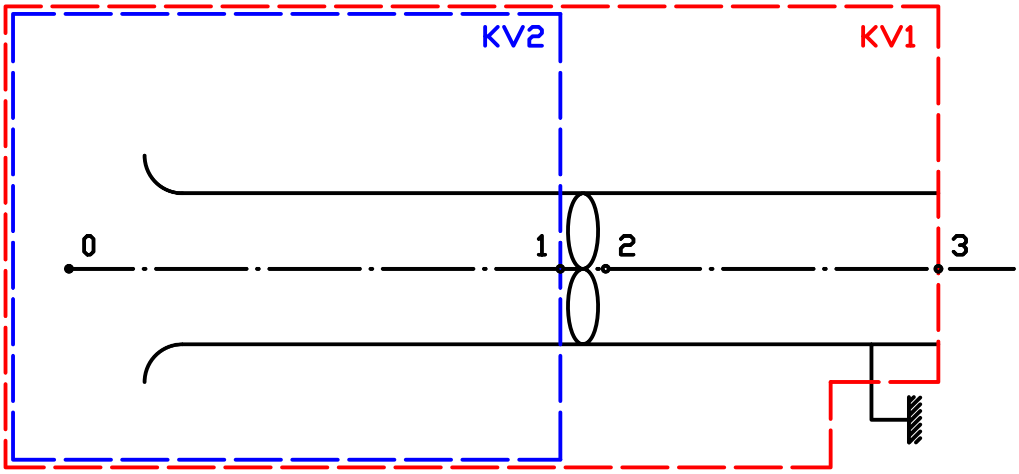
$$\rightarrow gH = \frac{1}{2} u_a^2 + \frac{du_a}{dt}(0.693 L_1 + L_2)$$

$$T = \int dt = \int_0^{0{,}99 u_{a,stat}} \frac{0{,}693 L_1 + L_2}{gH - \frac{1}{2}u_a^2} du_a = \int_0^{0{,}99 u_{stat}} \frac{2(0{,}693 L_1 + L_2)}{2gH - u_a^2} du_a$$

$$= \frac{0{,}693 L_1 + L_2}{\sqrt{2gH}} \ln \frac{\sqrt{2gH} + u_a}{\sqrt{2gH} - u_a}\Bigg|_0^{0{,}99 u_{a,stat}} = 5{,}34 \ sec$$

3. Aufgabe

I



I a)

Bernoulli $0 \rightarrow 1$:

$$p_a = p_1 + \frac{\rho}{2}v^2$$

Bernoulli $2 \rightarrow 3$:

$$p_2 + \frac{\rho}{2}v^2 = p_3 + \frac{\rho}{2}v^2 = p_a + \frac{\rho}{2}v^2$$

$$\Rightarrow \quad \Delta p = p_2 - p_1 = p_a - p_1 = \frac{\rho}{2}v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2\Delta p}{\rho}}$$

$$\Rightarrow \quad \dot{Q} = vA = \sqrt{\frac{2\Delta p}{\rho}}\,\frac{\pi d^2}{4}$$

I b)

$$P = \dot{Q}\Delta p = \sqrt{\frac{2\Delta p}{\rho}}\,\frac{\pi d^2}{4}\Delta p$$

I c)

Impulssatz KV 1:

$$F_1 = \rho v^2 A = 2\Delta p\,\frac{\pi d^2}{4}$$

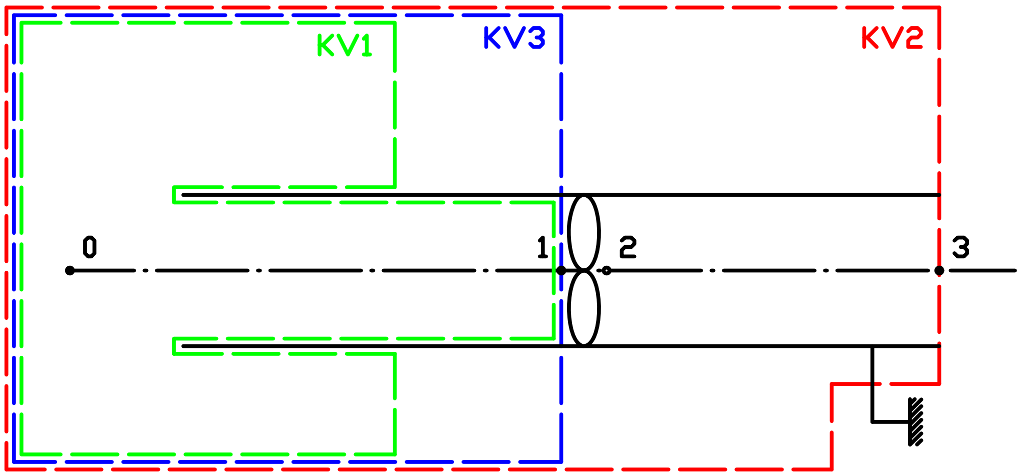
I d)

Impulssatz KV 2:

$$\rho v^2 A = (p_a - p_1)A + F_2$$

$$\Rightarrow \quad F_2 = \Delta p\,\frac{\pi d^2}{4}$$

II



II a)

Bernoulli 2 $\rightarrow$ 3:

$$p_2 + \frac{\rho}{2}v^2 = p_3 + \frac{\rho}{2}v^2(1 + \lambda\frac{L}{2d}) = p_a + \frac{\rho}{2}v^2(1 + \lambda\frac{L}{2d})$$

$$\Rightarrow \quad p_a = p_2 - \frac{\rho v^2}{4}\lambda\frac{L}{d}$$

Impulssatz KV 1

$$\rho v^2 A = -\tau_w \pi d \frac{L}{2} + (p_a - p_1)A$$

mit $\lambda = \frac{8\tau_w}{\rho v^2}$

$$\begin{aligned}
\rho v^2 A &= -\frac{\lambda \rho v^2}{8}\pi d \frac{L}{2} + (p_a - p_1)A \\
&= -\lambda \frac{L}{d}\frac{\rho v^2}{4}A - \frac{\rho v^2}{4}\lambda\frac{L}{d}A + (p_2 - p_1)A \\
&= -\lambda \frac{L}{d}\frac{\rho v^2}{2}A + \Delta p A
\end{aligned}$$

$$\Rightarrow \quad v = \sqrt{\frac{2\Delta p}{\rho(2 + \lambda\frac{L}{d})}}$$

$$\Rightarrow \quad \dot{Q} = \sqrt{\frac{2\Delta p}{\rho(2 + \lambda\frac{L}{d})}}\ \frac{\pi d^2}{4}$$

II b)

$$P = \dot{Q}\Delta p = \sqrt{\frac{2\Delta p^3}{\rho(2 + \lambda\frac{L}{d})}}\ \frac{\pi d^2}{4}$$

II c)

Impulssatz KV 2:

$$F_1 = \rho v^2 A = \frac{2\Delta p}{(2 + \lambda\frac{L}{d})}\ \frac{\pi d^2}{4}$$

II d)

Impulssatz KV 3:

$$\rho v^2 A = (p_a - p_1)A + F_2$$

$$\begin{aligned}
\Rightarrow F_2 &= \left(\rho v^2 - (p_a - p_1)\right) \frac{\pi d^2}{4} \\
&= \left(\rho v^2 + \frac{\rho}{2} v^2 \lambda \frac{L}{2d} - (p_2 - p_1)\right) \frac{\pi d^2}{4} \\
&= -\frac{\Delta p \ \lambda \ L \ \pi \ d}{8(2 + \lambda \frac{L}{d})}
\end{aligned}$$

4. Aufgabe

Masse: $\pi R_1^2 u_1 = \int_0^R u_{max} \left(1 - \frac{r^2}{R^2}\right) 2\pi r\, dr$

$$= 2\pi u_{max} \left(\frac{1}{2} R^2 - \frac{1}{4} R^2\right)$$

$$\rightarrow R_1^2 u_1 = R^2 \frac{u_{max}}{2}$$

Impuls: $u_1^2 \pi R_1^2 = \int_0^R 2\pi u_{max}^2 \left(1 - \frac{r^2}{R^2}\right)^2 r\, dr$

$$= 2\pi u_{max}^2 \left(\frac{R^2}{2} - 2\frac{R^2}{4} + \frac{1}{6} R^2\right)$$

$$\rightarrow u_1^2 R_1^2 = u_{max}^2 \cdot \frac{1}{3} R^2 \rightarrow u_1 = u_{max} \cdot \frac{R}{R_1} \cdot \frac{1}{\sqrt{3}}$$

$$R_1^2 u_{max} \cdot \frac{R}{R_1} \cdot \frac{1}{\sqrt{3}} = R^2 \frac{u_{max}}{2}$$

$$\rightarrow \frac{R_1}{R} = \frac{1}{2}\sqrt{3}$$

5. Aufgabe | a) $\bar{T} = T/T_R \qquad \bar{Y} = Y/l \qquad \bar{u} = u/u_R$

$$\rightarrow \frac{\lambda T_R}{l^2} \frac{\partial^2 \bar{T}}{\partial \bar{Y}^2} + \eta \frac{u_R^2}{l^2} \left(\frac{\partial \bar{u}}{\partial \bar{Y}}\right)^2 = 0 \qquad \Big| : \frac{\lambda T_R}{l^2}$$

$$\frac{\partial^2 \bar{T}}{\partial \bar{Y}^2} + \frac{\eta u_R^2}{\lambda T_R} = 0 \rightarrow k = \frac{\eta u_R^2}{\lambda T_R}$$

b) $k = k(\lambda, \eta, T_R, u_R, l)$

$$= \lambda^\alpha{}_R\, \eta^\beta\, T_R^\gamma\, u_R^\delta\, l^\varepsilon \qquad \text{Wähle } \alpha = 1$$

$$\begin{aligned} kg&: 0 = 1 + \beta & \beta &= -1 \\ m&: 0 = 1 - \beta + \delta + \varepsilon & \varepsilon &= 0 \\ s&: 0 = -3 - \beta - \delta \quad \rightarrow & \delta &= -2 \\ K&: 0 = -1 + \gamma & \gamma &= 1 \end{aligned}$$

$$\rightarrow k = \frac{\lambda T_R}{\eta u_R^2}$$

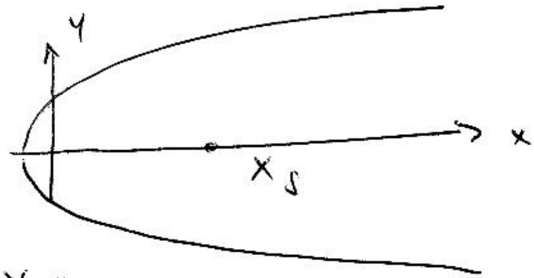
c) $$k = \frac{\lambda T_R c_p}{\eta u_R^2 c_p} = \frac{\lambda}{\eta c_p} \frac{T_R c_p}{u_R^2} = \frac{1}{Pr} \frac{T_R \gamma}{(\gamma - 1) R u_R^2}$$

$$= \frac{1}{Pr} \cdot \frac{1}{M^2} \cdot \frac{1}{\gamma - 1}$$

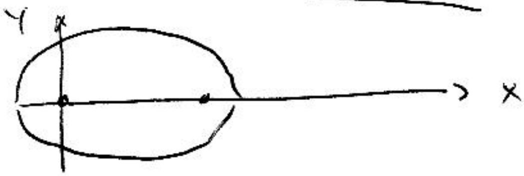
6. Aufgabe

a)

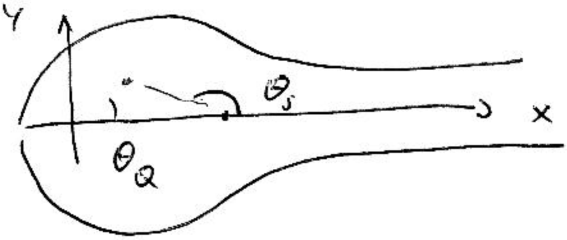
1. $E_S \ll E_Q$



2. $E_S = E_Q$



3. $E_S = E_Q/2$



b) $$\phi = u_\infty x + \frac{E_Q}{2\pi} \ln r - \frac{E_S}{2\pi} \ln |\vec{r} - \vec{x}_S|$$

$$= u_\infty x + \frac{E_Q}{2\pi} \ln \sqrt{x^2 + y^2} - \frac{E_S}{2\pi} \ln \sqrt{(x - x_S)^2 + y^2}$$

c) $$\psi = u_\infty y + \frac{E_Q}{2\pi} \underbrace{\mathrm{alen}\, \frac{y}{x}}_{\theta_Q} - \frac{E_S}{2\pi} \ln \underbrace{\frac{y}{x - x_S}}_{\theta_S}$$

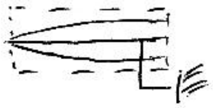
d) kont.: $E_Q - E_S = 2 u_\infty \cdot y_\infty$

$$\longrightarrow y_\infty = \frac{E_Q - E_S}{2 u_\infty}$$

7. Aufgabe | a) Impuls

$$\int_0^\delta \rho u^2 B\,dy + \Delta\dot{m}\, u_\infty - \rho u_\infty^2 \delta \cdot B = \frac{1}{2} F_W$$

Masse: $\int_0^\delta \rho u B\,dy + \Delta\dot{m} = \rho u_\infty \delta B$



$$\Delta\dot{m} = \rho B \int_0^\delta (u_\infty - u)\,dy = \rho B u_\infty \delta \int_0^1 \left(1 - \frac{u}{u_\infty}\right) d\frac{y}{\delta}$$

$$= \rho B u_\infty \delta \int_0^1 1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^3 d\frac{y}{\delta}$$

$$= \rho B u_\infty \delta \left(1 - \frac{3}{4} + \frac{1}{8}\right) = \frac{3}{8}\rho B u_\infty \delta$$

$$\int_0^\delta \rho u^2 B\,dy = \rho u_\infty^2 B \cdot \delta \int_0^1 \left(\frac{u}{u_\infty}\right)^2 d\frac{y}{\delta}$$

$$= \rho u_\infty^2 B \delta \int_0^1 \frac{9}{4}\left(\frac{y}{\delta}\right)^2 - \frac{3}{2}\left(\frac{y}{\delta}\right)^4 + \frac{1}{4}\left(\frac{y}{\delta}\right)^6 d\frac{y}{\delta}$$

$$= \rho u_\infty^2 B \delta \frac{17}{35}$$

$$F_W = 2\left(\rho u_\infty^2 B \delta \left(17/35 + \frac{3}{8} - 1\right)\right) =$$

$$= -\frac{39}{140}\rho u_\infty^2 B \delta = -5.79 \cdot 10^{-2}\,N$$

b) $$\tau = -\eta \frac{\partial u}{\partial y} = -\nu \rho \frac{u_\infty}{\delta} \frac{\partial(u/u_\infty)}{\partial(y/\delta)}$$

$$= -\nu\rho \frac{u_\infty}{\delta} \frac{\pi}{2} \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$$Re_x = \frac{\rho u_\infty x}{\eta} = \frac{u_\infty x}{\nu} = 300.000 \rightarrow \delta = 4.382\,mm$$

$$\rightarrow \tau = -0.0188\ \frac{N}{m^2}$$

8. Aufgabe 1

a) Energiesatz: $c_p T_0 = c_p T + \frac{1}{2} u^2$

$$\frac{T_0}{T} = 1 + \frac{1}{2 c_p T} u^2$$

$$c_p = \frac{\gamma}{\gamma-1} R \rightarrow \frac{T_0}{T} = 1 + \frac{\gamma-1}{2\gamma R T} u^2 \qquad a^2 = \gamma R T \quad M = u/a$$

$$\rightarrow \frac{T_0}{T} = 1 + \frac{1}{2}(\gamma-1) M^2$$

$$\rightarrow \frac{p}{p_0} = \left(\frac{T_0}{T}\right)^{-\gamma/\gamma-1} = \left(1 + \frac{1}{2}(\gamma-1) M^2\right)^{-\gamma/\gamma-1}$$

b) $p_k = \frac{p_k}{p_1} \cdot \frac{p_1}{p_2} \cdot p_2 \qquad p_2 = p_a$

$$= \frac{\left(1 + \frac{1}{2}(\varkappa-1) M^2\right)^{\gamma/\varkappa-1}}{1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)} p_a = \frac{36.732}{10.333} p_a = 3{,}555 \text{ b}$$

$$M_1^* \cdot M_2^* = 1 \rightarrow u_1 \cdot u_2 = a^{*2}$$

$$u_2 = \frac{\varkappa R \frac{T^*}{T_0} T_0}{M_1 \sqrt{\varkappa R T_1}} = \frac{\sqrt{\varkappa R T_0}}{M_1} \frac{T^*}{T_0} \sqrt{\frac{T_0}{T_1}}$$

$$T^*/T_0 = 0.8333 \qquad T_0/T_1 = 2{,}8$$

$$\rightarrow u_2 = 159{,}48 \text{ m/s}$$