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(Matr.-No, Signature)

Exam „Fluid Dynamics“

14 March 2023

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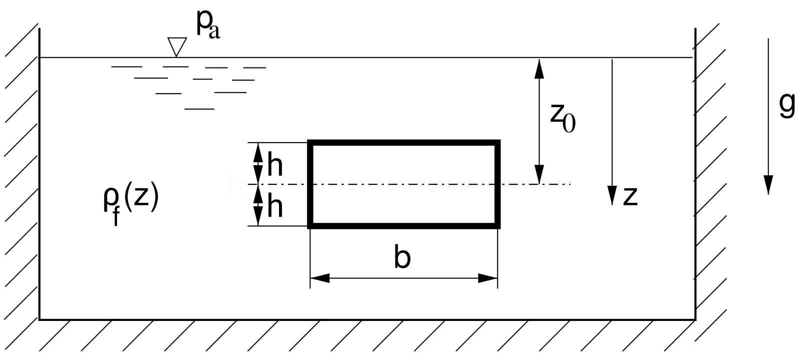
Problem 1 (7 Points)

a) Explain the theorem of Archimedes and give the equation to determine the buoyancy force. Specify the variables of the equation.

b) Which condition must be fulfilled for the equation to be valid?

In the following, a cuboid of length b, height $2h$ and width t is considered in a container filled with a fluid of density $\rho_f(z) = \rho_0 + \alpha\, z + \beta\, z^2$. The atmospheric pressure p_a is constant.



c) Determine the pressure integral over the surface of the cuboid and use it to determine the buoyancy of the cuboid.

d) Under which condition for the material coefficients $\{\alpha, \beta\}$ can the buoyancy of the body be determined via the Archimedes' principle using the average density $\overline{\rho_f} = \rho_f(z_0)$?

Given:

$\rho_0, \quad \alpha, \quad \beta, \quad b, \quad h, \quad t, \quad g, \quad z_0$

Hint:

- Check your results for unit and sign plausibility!

Problem 1

a) The buoyancy force of a body corresponds to the weight of the fluid displaced by it. $F_A = \rho_f g V_k$, where F_A denotes the buoyancy force, ρ_f the density of the fluid surrounding the body, g the acceleration due to gravity, and V_k the volume of the body.

b) constant density

c) Pressure Integral: $\vec{F} = \int\limits_{A_K} p(z)\vec{n}_A dA$ or $F_A = -\int\limits_{A_K} p(z) n_{z,A} dA$

The components of the surfaces parallel to the z-direction cancel each other out. It remains $F_A = -F_z = bt\, p(z_0 + h) - bt\, p(z_0 - h)$.

In general: $p(z) = p_a + \int\limits_0^z \rho(\tilde{z}) g d\tilde{z}$.

Inserted: $p(z) = p_a + g(\rho_0 z + \frac{1}{2}\alpha z^2 + \frac{1}{3}\beta z^3)$

Upper side: $p_o = p_a + g(\rho_0(z_0 - h) + \frac{1}{2}\alpha(z_0 - h)^2 + \frac{1}{3}\beta(z_0 - h)^3)$

Lower side: $p_u = p_a + g(\rho_0(z_0 + h) + \frac{1}{2}\alpha(z_0 + h)^2 + \frac{1}{3}\beta(z_0 + h)^3)$

Hence:

$$F_A = (p_u - p_o)bt = btg(2\rho_0 h + \tfrac{1}{2}\alpha((z_0 + h)^2 - (z_0 - h)^2) + \tfrac{1}{3}\beta((z_0 + h)^3 - (z_0 - h)^3))$$

d) Mean density: $\overline{\rho_f} = \rho_0 + \alpha z_0 + \beta z_0^2$

Volume of the body: $V = bt2h$

The following must apply: $F_A = \overline{\rho_f} g V$

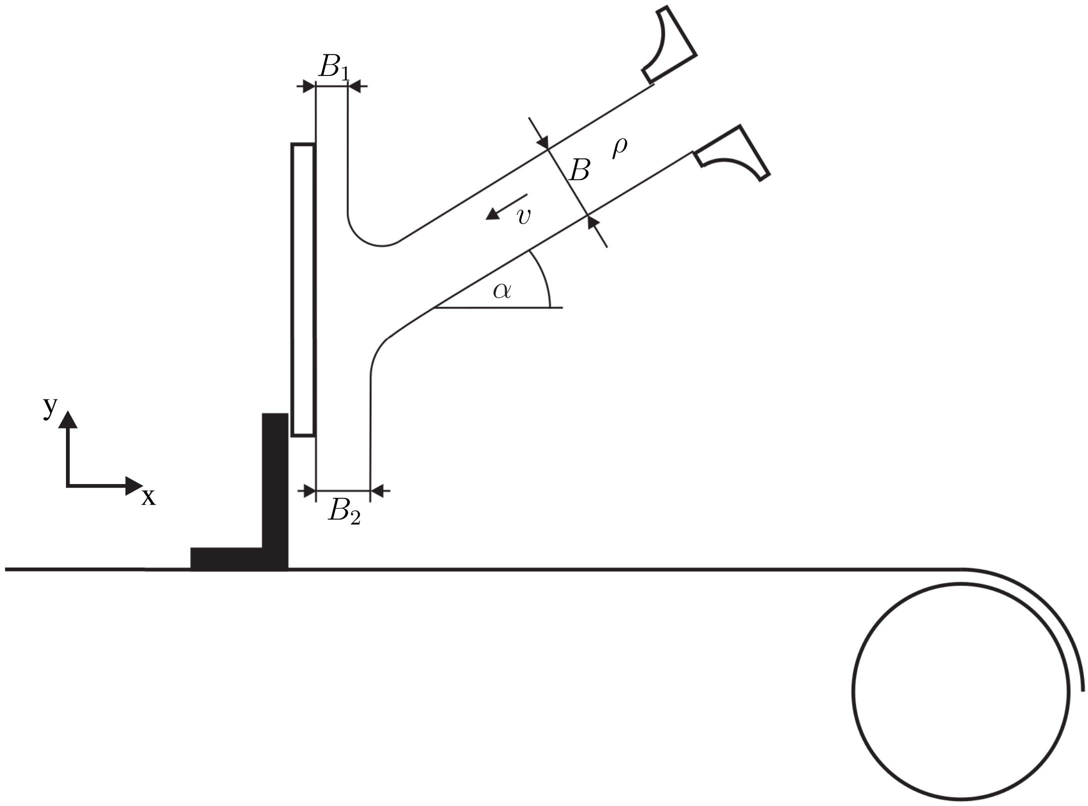
$F_A = btg(2\rho_0 h + \frac{1}{2}\alpha(4z_0 h) + \frac{1}{3}\beta(6z_0^2 h + 2h^3))$

$gbt2h(\rho_0 + \alpha z_0 + \beta z_0^2 + \beta\frac{h^2}{3}) = (\rho_0 + \alpha z_0 + \beta z_0^2)gbt2h$

The density distribution must be linear. Only for the linear distribution the mean value of the density lies in the center of the body $\Rightarrow \beta = 0$.

Problem 2 (8 Points)

A plane object is mounted vertically on a conveyor belt. This object is to be cleaned by a two-dimensional water jet. The water jet exits at the velocity v the nozzle of width B at the angle α against the horizontal. The jet is deflected by the object as shown in the sketch. The flow in the jet can be considered without any losses.



a) Calculate the components of the force per depth extent T exerted by the jet on the object when the conveyor belt is not moving.

b) Calculate the widths B_1 and B_2 of the deflected jet when the conveyor belt is not moving.

c) Determine the components of the force per depth extent T on the object when the conveyor belt moves with velocity v_F in the positive x-direction and the jet impinges upon the object at angle $\alpha = 0$.

d) What is the conveyor speed v_F^* when the force on the object is twice as large as in the non-moving state? In both cases, the angle α is considered $\alpha = 0$.

Given:

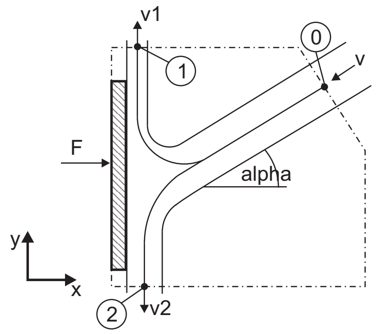
ρ, B, v_F, v, α

Hint:

- Check your results for unit and sign plausibility!

Problem 2

a) Momentum equation in the x-direction (conveyor belt at rest):



$$\frac{dI_x}{dt} = \rho v^2 \cos(\alpha) BT = F_x$$

$$\Rightarrow \frac{F_x}{T} = \rho v^2 \cos(\alpha) B$$

Force onto the object:

$$F_x' = -F_x = -\rho v^2 \cos(\alpha) B$$

$$\frac{F_y'}{T} = 0$$

b) Deflection without losses:

Bernoulli from 0 to 1 and from 0 to 2:

$$\Rightarrow p_a + \frac{\rho}{2} v^2 = p_a + \frac{\rho}{2} v_1^2 = p_a + \frac{\rho}{2} v_2^2$$

$$\Rightarrow v = v_1 = v_2$$

Continuity equation:

$$vB = v_1 B_1 + v_2 B_2 \Rightarrow B = B_1 + B_2$$

Momentum equation in the y-direction:

$$\frac{I_y}{dt} = \rho v^2 \sin(\alpha) BT + \rho v^2 B_1 T - \rho v^2 B_2 T$$

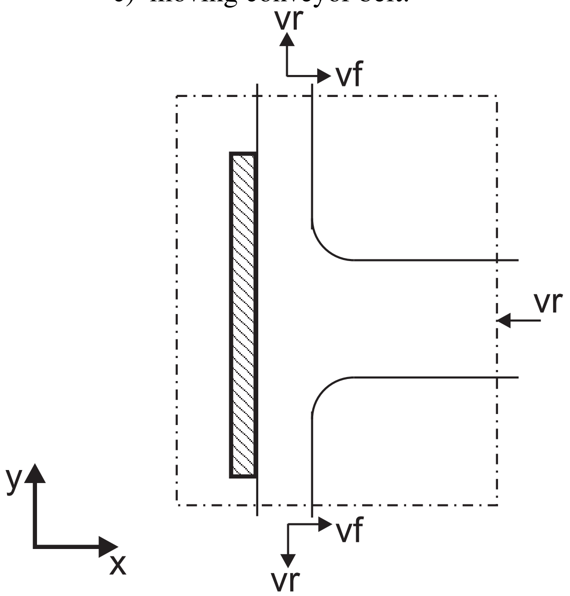
$$v^2 \sin(\alpha) B + v^2 B_1 - v^2 (B - B_1) = 0$$

$$(\sin(\alpha) - 1) B + 2B_1 = 0$$

$$B_1 = \frac{B}{2}(1 - \sin(\alpha))$$

$$B_2 = \frac{B}{2}(1 + \sin(\alpha))$$

c) moving conveyor belt:



Momentum equation in the x-direction for the moving control surface:

$$\frac{DI_x}{dt} = \int \rho v_{a,x}(\vec{v_r} \cdot \vec{n})dA = F_x$$

$$= \rho v v_r B + \rho v_F v_r \frac{B}{2} + \rho v_F v_r \frac{B}{2} = \frac{F_x}{T}$$

$$v_r = v_F - (-v)$$

$$\frac{F_x}{T} = \rho v(v + v_F)B + \rho v_F(v + v_F)B$$

$$\frac{F_x}{T} = \rho(v + v_F)^2 B$$

Force onto the object:

$$F_x' = -F_x = -\rho(v + v_F)^2 B$$

$$\frac{F_y'}{T} = 0$$

d) $F_{x,c} = 2F_{x,a}$

$$\rho(v + v_F^*)^2 B = 2\rho v^2 B \Rightarrow (v + v_F^*)^2 = 2v^2$$

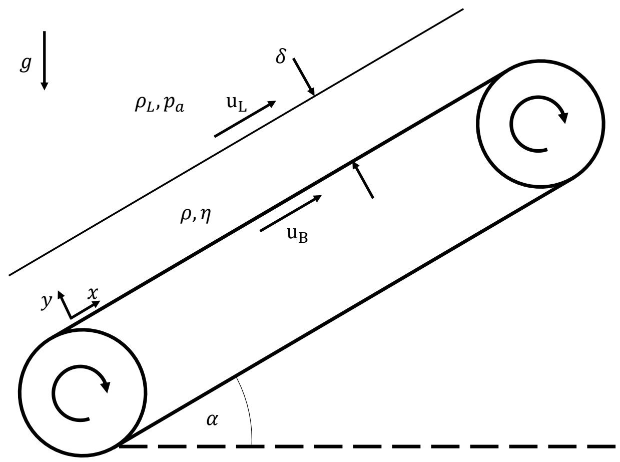
$${v_F^*}^2 + 2v v_F^* - v^2 = 0 \Rightarrow v_F^* = -v \pm \sqrt{2}v$$

Since $v_F > 0$:

$$v_F^* = v(\sqrt{2} - 1)$$

Problem 3 (14 Points)

A fluid is transported by an inclined conveyor belt. The belt moves with the velocity u_B. The ambient pressure p_a is constant. A fully developed laminar flow of thickness δ is established.



First, assume that the friction with the ambient air is negligible.

a) Formulate the equilibrium of forces in the flow direction for a fluid element and simplify it to formulate the differential equation for the shear stress.

b) Determine the velocity profile $u(y)$ and the shear stress profile $\tau(y)$.

c) Determine the transported volume flux per depth extent T.

The transported volume flux is now to be increased at constant belt speed. For this purpose, an additional air flow with the mean constant velocity u_L in the x-direction is used. The shear stress at the phase interface can be described approximately by $\tau = \frac{1}{2} c_f \rho_L u_L^2$. The velocity distribution of the air flow is to be neglected.

d) Determine the velocity of the air u_L such that the volume flux is doubled.

e) Carefully draw the two velocity profiles from b) and d) into one diagram.

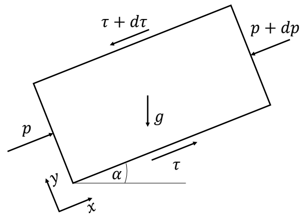
Given: $g, \quad \eta, \quad \rho, \quad u_B, \quad \delta, \quad \alpha, \quad \rho_L, \quad c_f$

Hints:

- The fluid exhibits Newtonian shear behaviour.
- Check your results for unit and sign plausibility!

Problem 3

a) Force balance in the x-direction:



$$(p-(p+\frac{dp}{dx}dx))dy+(\tau-(\tau+\frac{d\tau}{dy}))dx-\rho g\sin\alpha dxdy=0$$

$$-\frac{dp}{dx}dxdy-\frac{d\tau}{dy}dxdy-\rho g\sin\alpha dxdy=0$$

Free surface: $\frac{dp}{dx}=0$

$$\Rightarrow \frac{d\tau}{dy}=-\rho g\sin\alpha$$

b) 1st integration:

$$\int_{\tau}^{\tau_o}d\tau=-\int_{y}^{\delta}\rho g\sin\alpha dy$$

With $\tau_o=0$:

$$\Rightarrow \tau=-\eta\frac{du}{dy}=\rho g\sin\alpha(\delta-y)$$

2nd integration:

$$\int_{u_B}^{u}du=-\frac{\rho g\sin\alpha}{\eta}\int_{0}^{y}(\delta-y)dy$$

$$\Rightarrow u(y)=u_B+\frac{\rho g\sin\alpha}{\eta}(\frac{1}{2}y^2-\delta y)$$

c) Volume flux per depth extent: $\frac{\dot{V}}{T}=\int_0^{\delta}u(y)dy$

$$\frac{\dot{V}}{T}=u_B\delta+\frac{\rho g\sin\alpha}{\eta}(\frac{1}{6}\delta^3-\frac{1}{2}\delta^3)$$

$$\Rightarrow \frac{\dot{V}}{T}=u_B\delta-\frac{\rho g\sin\alpha}{3\eta}\delta^3$$

d) 1st integration as in b):

$$\tau(y) = \tau_o + \rho g \sin\alpha(\delta - y) = -\eta\frac{du}{dy}$$

2nd integration:

$$\int_{u_B}^{u} du = \int_0^y -\frac{\tau_o}{\eta} - \frac{\rho g \sin\alpha}{\eta}(\delta - y)dy$$

$$\Rightarrow u(y) = u_B - \frac{\tau_o}{\eta}y + \frac{\rho g \sin\alpha}{\eta}(\frac{1}{2}y^2 - \delta y)$$

Volume flux per depth extent: $\frac{\dot{V}}{T} = \int_0^\delta u(y)dy$

$$\frac{\dot{V}}{T} = u_B\delta - \frac{\tau_o}{2\eta}\delta^2 + \frac{\rho g \sin\alpha}{\eta}(\frac{1}{6}\delta^3 - \frac{1}{2}\delta^3)$$

$$\Rightarrow \frac{\dot{V}}{T} = u_B\delta - \frac{\tau_o}{2\eta}\delta^2 - \frac{\rho g \sin\alpha}{3\eta}\delta^3$$

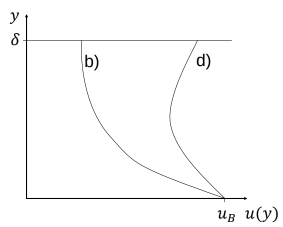
$$\frac{\dot{V}}{T} = \frac{\dot{V}_{alt}}{T} - \frac{\tau_o}{2\eta}\delta^2 = 2\frac{\dot{V}_{alt}}{T}$$

$$\frac{\tau_o}{2\eta}\delta^2 = -u_B\delta + \frac{\rho g \sin\alpha}{3\eta}\delta^3$$

$$\tau_o = -\frac{2\eta u_B}{\delta} + \frac{2}{3}\rho g \sin\alpha\delta = -\frac{1}{2}c_f\rho_L u_L^2$$

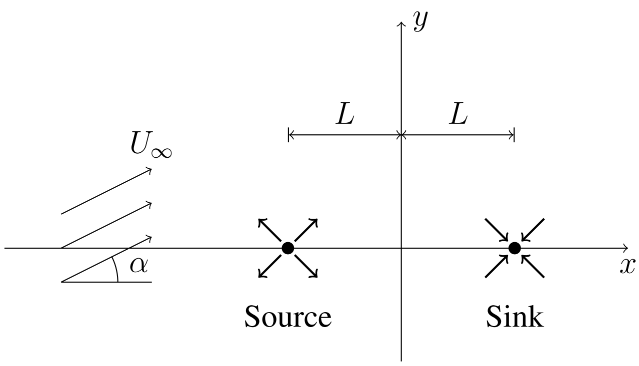
$$\Rightarrow u_L = \sqrt{\frac{2}{c_f\rho_L}(\frac{2\eta u_B}{\delta} - \frac{2}{3}\rho g \sin\alpha\delta)}$$

e) Sketch:



Problem 4 (13 Points)

The superposition of a source, a sink, and a parallel flow at the velocity U_∞ under an angle of attack α is considered.



a) Formulate the complex potential function $F(z)$ to represent the problem described. Indicate the signs of the constants in the elementary functions.

b) Determine the resulting velocity components $u(x,y)$ and $v(x,y)$ using the conjugate complex velocity $\overline{w}$.

It is known for the angle of attack $\alpha = 0$ that a stagnation point is located at $x_s = -2L$ and that the corresponding stagnation point streamline is symmetrical to the y-axis.

c) Determine the strength of the source and the sink.

d) Calculate the positions of the stagnation points for the angle of attack $\alpha = \pi$ and sketch qualitatively the corresponding flow field.

Given:

U_∞, L, α

Known complex potential functions:

Parallel flow:	$F(z) = (u_\infty - \mathrm{i}v_\infty)z$
Potential vortex:	$F(z) = -\frac{\mathrm{i}\Gamma}{2\pi}\ln z$
Source/Sink:	$F(z) = \frac{E}{2\pi}\ln z$
Stagnation point flow:	$F(z) = az^2$
Dipole:	$F(z) = \frac{M}{2\pi z}$

Hints:

- $z = x + \mathrm{i}y = r \cdot e^{\mathrm{i}\phi} = r(\cos\phi + \mathrm{i}\sin\phi)$
- $\dfrac{1}{z \pm a} = \dfrac{1}{(x \pm a) + \mathrm{i}y}$
- Check your results for unit and sign plausibility!

Problem 4 (13 Points)

a) using $u_\infty = U_\infty \cos(\alpha)$, $v_\infty = U_\infty \sin(\alpha)$ as well as the source strength $E_Q > 0$ and sink strength $E_S < 0$ follows 1_1

$$F(z) = U_\infty \left[\cos(\alpha) - \mathrm{i}\sin(\alpha)\right] z + \frac{E_Q}{2\pi}\ln(z+L) + \frac{E_S}{2\pi}\ln(z-L) \qquad 1_2$$

b) Using the definition of the conjugate complex velocity follows

$$\overline{w} = u - \mathrm{i}v = \frac{dF}{dz} = U_\infty \left[\cos(\alpha) - \mathrm{i}\sin(\alpha)\right] + \frac{E_Q}{2\pi}\frac{1}{z+L} + \frac{E_S}{2\pi}\frac{1}{z-L} \qquad 1_3$$

Expansion by the conjugate complex denominator

$$\frac{1}{z \pm L} = \frac{1}{(x \pm L) + \mathrm{i}y} = \frac{1}{(x \pm L) + \mathrm{i}y}\frac{(x \pm L) - \mathrm{i}y}{(x \pm L) - \mathrm{i}y} = \frac{(x \pm L) - \mathrm{i}y}{(x \pm L)^2 + y^2}$$

the expression can be simplified and decomposed into real and imaginary parts.

$$\frac{dF}{dz} = \underbrace{\left[U_\infty \cos(\alpha) + \frac{E_Q}{2\pi}\frac{x+L}{(x+L)^2+y^2} + \frac{E_S}{2\pi}\frac{x-L}{(x-L)^2+y^2}\right]}_{=u(x,y)} \qquad 1_4$$

$$-\mathrm{i}\underbrace{\left[U_\infty \sin(\alpha) + \frac{E_Q}{2\pi}\frac{y}{(x+L)^2+y^2} + \frac{E_S}{2\pi}\frac{y}{(x-L)^2+y^2}\right]}_{=v(x,y)} \qquad 1_5$$

c) For $\alpha = 0$, it follows from $v(x_s, y_s) = 0$ or due to symmetry reasons that the stagnation points lie on $y_s = 0$ and the magnitude of the strenght must be equal, i.e., $|E_Q| = |E_S| =: E$, where $E_Q = E$ and $E_S = -E$. Using $u(x_s = -2L, y_s = 0) = 0$, E is determined to be

$$u(x_s = -2L, y_s = 0, \alpha = 0) = U_\infty + \frac{E}{2\pi}\frac{-L}{(-L)^2} - \frac{E}{2\pi}\frac{-3L}{(-3L)^2} = 0 \qquad 1_6$$

$$\Rightarrow \quad 0 = U_\infty 2\pi L + E\left(-1 + \frac{1}{3}\right)$$

$$\Rightarrow \quad E = 3\pi U_\infty L \qquad 1_7$$

d) Using $\alpha = \pi : \sin(\alpha) = 0,\ \cos(\alpha) = -1$ and $E = E_Q = -E_S$ it follows

$$v(x,y) = \frac{E}{2\pi}y\left[\frac{1}{(x+L)^2+y^2} - \frac{1}{(x-L)^2+y^2}\right] \stackrel{!}{=} 0$$

$$y = 0 \quad \vee \quad \left[\frac{1}{(x+L)^2+y^2} - \frac{1}{(x-L)^2+y^2}\right] = 0$$

$$\Rightarrow [(x-L)^2 + y^2] - [(x+L)^2 + y^2] = 0 \quad \Rightarrow \quad x = 0 \qquad 1_8$$

Checking the zeros for the u component 1_{9}

$$u(x,y) = -U_\infty + \frac{E}{2\pi}\left[\frac{x+L}{(x+L)^2+y^2} - \frac{x-L}{(x-L)^2+y^2}\right]$$

$$u(x=0,y) = -U_\infty + \frac{E}{2\pi}\left[\frac{L}{(L)^2+y^2} - \frac{-L}{(-L)^2+y^2}\right] \overset{!}{=} 0$$

$$\Rightarrow 0 = -U_\infty + \frac{E}{\pi}\frac{L}{L^2+y^2} \Rightarrow y^2 = \frac{EL}{\pi U_\infty} - L^2 \overset{c)}{=} 2L^2$$

$$\Rightarrow (x_{sp}, y_{sp})_{1,2} = (0, \pm\sqrt{2}L) \qquad 1_{10}$$

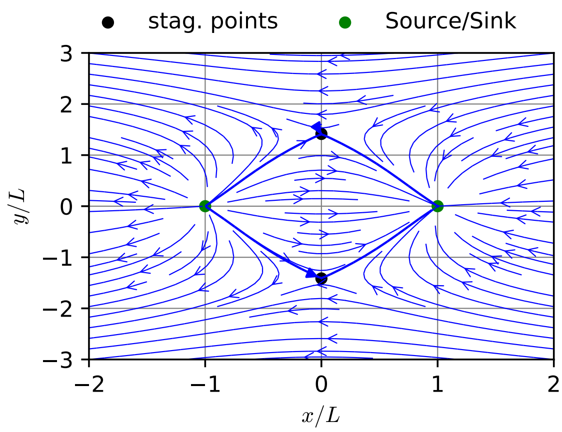
$$u(x,y=0) = -U_\infty + \frac{E}{2\pi}\left[\frac{x+L}{(x+L)^2+y^2} - \frac{x-L}{(x-L)^2+y^2}\right] \overset{!}{=} 0$$

$$\Rightarrow 0 = -U_\infty + \frac{E}{2\pi}\left[\frac{x+L}{(x+L)^2} - \frac{x-L}{(x-L)^2}\right] = -U_\infty + \frac{E}{2\pi}\left[\frac{(x-L)-(x+L)}{(x+L)(x-L)}\right]$$

$$= -U_\infty + \frac{E}{\pi}\left[\frac{-L}{x^2-L^2}\right] \rightarrow x^2 = L^2 - \frac{EL}{\pi U_\infty} \overset{c)}{=} -2L^2$$

$$\Rightarrow x = \pm\sqrt{-2L^2} \rightarrow \text{Physically meaningless, since imaginary} \qquad 1_{11}$$

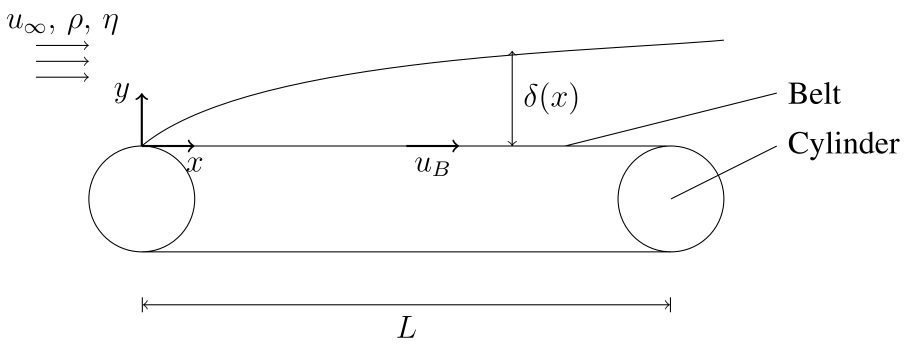
The two stagnation points are $(x_{sp}, y_{sp})_{1,2} = (0, \pm\sqrt{2}L)$.



1_{12} 1_{13}

Problem 5 (11 Points)

The flow at velocity u_∞ of an incompressible Newtonian fluid (density ρ, viscosity η) over a conveyor belt of length L is considered. The belt moves in the direction of the outer flow with the velocity $u_B = Ku_\infty$.



The following approach is assumed to approximate the velocity profile in the laminar boundary layer.

$$\frac{u(x,y)}{u_\infty} = a_0 + a_1\left(\frac{y}{\delta}\right) + a_2\left(\frac{y}{\delta}\right)^2 .$$

a) Determine the velocity profile $u(y/\delta)$ in the boundary layer.

b) Calculate the distribution of the boundary layer thickness $\delta(x)$.

c) What is the friction force per width F_R/B acting on the belt?

d) Give two technical methods that lead generate turbulent boundary layer. How does this affect the friction force compared to a laminar boundary layer? Justify your answer and sketch qualitatively the two velocity profile shapes.

Given:

L, u_∞, ρ, η, $u_B = Ku_\infty$, B

Hints:

- von Kármán integral equation

$$\frac{d\delta_2}{dx} + \frac{1}{u_a}\frac{du_a}{dx}(2\delta_2 + \delta_1) - \frac{\tau_W}{\rho u_a^2} = 0$$

- Check your results for unit and sign plausibility!

Problem 5 (11 Points)

a) Using three boundary conditions and the given ratio $\frac{u_B}{u_\infty} = K$ the unknowns a_i can be determined

no-slip: $\frac{y}{\delta} = 0 : u = u_B \quad \rightarrow a_0 = \frac{u_B}{u_\infty} = K$

edge of boundary layer: $\frac{y}{\delta} = 1 : u = u_\infty \quad \rightarrow a_1 + a_2 = 1 - \frac{u_B}{u_\infty} = 1 - K$

wall compatibility condition: $\frac{y}{\delta} = 0 : \eta \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \frac{\partial p}{\partial x} \overset{here}{=} 0 \quad \rightarrow a_2 = 0$

$$\Rightarrow \frac{u}{u_\infty} = K + (1-K)\left(\frac{y}{\delta}\right) \qquad 1_1$$

b) By $u_a \equiv u_\infty \rightarrow \frac{du_a}{dx} = 0$ the integral relation of Kármán simplifies. Via the wall shear stress and the momentum thickness, δ can be determined.

$$\begin{aligned}
\delta_2 &= \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy = \delta \int_0^1 \left[\frac{u}{u_\infty} - \left(\frac{u}{u_\infty}\right)^2\right] d\left(\frac{y}{\delta}\right) \qquad 1_2 \\
&= \delta \int_0^1 \left[\left(K + (1-K)\left(\frac{y}{\delta}\right)\right) - \left(K + (1-K)\left(\frac{y}{\delta}\right)\right)^2\right] d\left(\frac{y}{\delta}\right) \\
&= \delta \int_0^1 \left[\left(K + (1-K)\left(\frac{y}{\delta}\right)\right) - \left(K^2 + 2K(1-K)\left(\frac{y}{\delta}\right) + (1-K)^2\left(\frac{y}{\delta}\right)^2\right)\right] d\left(\frac{y}{\delta}\right) \\
&= \delta \int_0^1 \left[(K - K^2) + (1 - 3K + 2K^2)\left(\frac{y}{\delta}\right) - (1-K)^2\left(\frac{y}{\delta}\right)^2\right] d\left(\frac{y}{\delta}\right) \\
&= \delta \left.\left[(K-K^2)\left(\frac{y}{\delta}\right) + \frac{1}{2}(1-3K+2K^2)\left(\frac{y}{\delta}\right)^2 - \frac{1}{3}(1-K)^2\left(\frac{y}{\delta}\right)^3\right]\right|_0^1 \\
&= \delta\left[\frac{1}{6} + \frac{1}{6}K - \frac{1}{3}K^2\right] = \frac{\delta}{6}\underbrace{\left[1 + K - 2K^2\right]}_{:=\phi} \qquad 1_3
\end{aligned}$$

$$\tau_W = -\tau(y=0) = \eta \left.\frac{\partial u}{\partial y}\right|_{y=0} = \eta u_\infty (1-K)\frac{1}{\delta} \qquad 1_4$$

$$\frac{d\delta_2}{dx} - \frac{\tau_W}{\rho u_\infty^2} = 0 \qquad 1_5$$

$$\Rightarrow \frac{\phi}{6}\frac{d\delta}{dx} = \frac{1}{\rho u_\infty^2}\frac{\eta u_\infty (1-K)}{\delta}$$

$$\Rightarrow \int \delta d\delta = \frac{6\eta(1-K)}{\phi \rho u_\infty}\int dx \quad \text{with } \delta(x=0) = 0 \text{ follows}$$

$$\Rightarrow \delta(x) = \sqrt{\frac{12\eta(1-K)}{\phi\rho u_\infty}x} \qquad 1_6$$

c)

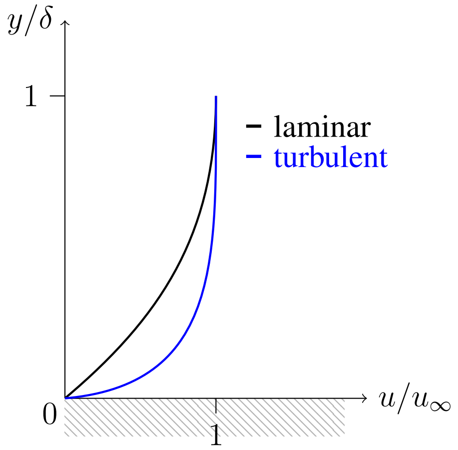
$$\frac{F_R}{B} = \int_0^L \tau_W(x)dx \qquad (1)_7$$

$$= \int_0^L \frac{\eta u_\infty (1-K)}{\delta(x)} dx = \int_0^L \frac{\eta u_\infty (1-K)}{1} \sqrt{\frac{\phi \rho u_\infty}{12\eta(1-K)x}} dx$$

$$= \eta u_\infty (1-K) \sqrt{\frac{\phi \rho u_\infty}{12\eta(1-K)}} 2\sqrt{L} \qquad (1)_8$$

d) The change from a laminar to a turbulent boundary layer can be achieved by imposing disturbances to the flow. Technical methods are e.g. trip wires, roughness changes of the surface or vortex generators. $(1)_9$

In the case of a turbulent boundary layer, higher momentum is transported to the near wall region due to additional mixing movements, which makes the mean velocity profile fuller and increases the frictional force. $(1)_{10}$



$(1)_{11}$

Problem 6 (7 Points)

a) Using Reynolds averaging, show that $\overline{fg} = \overline{f}\,\overline{g} + \overline{f'g'}$ holds.

b) In a laminar flow, two fluids are layered normal to the flow direction. The fluids have the viscosities η_1 and η_2. What conditions must be fulfilled at the interface for the velocity component in the flow direction?

c) Explain the concept of the viscous sublayer.

d) Why is it possible to achieve a different number of parameters for the same physical problem using Buckingham's Π theorem instead of the method of differential equations?

Problem 6

a) Decomposition of the function in $f = \overline{f} + f'$

$$\overline{f} = \frac{1}{T}\int_T f dt \quad \frac{1}{T}\int_T f' dt = 0$$

$$\overline{fg} = \overline{(\overline{f}+f')(\overline{g}+g')} = \overline{\overline{f}\,\overline{g}} + \overline{f'\overline{g}} + \overline{\overline{f}g'} + \overline{f'g'}$$

$$\Rightarrow \overline{fg} = \overline{f}\,\overline{g} + \overline{f'g'}$$

b) Kinematics: $u_1 = u_2$

Force balance: $\tau_1 = \tau_2 \Rightarrow \eta_1 \frac{du_1}{dy} = \eta_2 \frac{du_2}{dy}$

c) The viscous sublayer is a very thin layer in the vicinity of the wall, in which the laminar shear stress dominates the turbulent shear stress.

d) Via Buckingham's Π-theorem one obtains the maximum number of parameters of a physical problem. Since there is more information in the differential equation, the number of parameters obtained by the method of differential equations is less than or equal to the number of parameters obtained by Buckingham's Π-theorem. 1