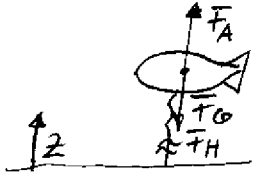


Aufgabe 1

a) Haltekraft: Kräftegleichgewicht am Boden ($z=0$)



$$\sum_i F_{i,z} = 0 = F_A - F_G - F_H, \text{ mit } F_G = (m_z + m_p + m_{\text{Gas}})g$$

$$\begin{aligned} \Rightarrow F_H &= F_A(z=0) - (m_z + m_p + m_{\text{Gas}})g \\ &= \rho_L(z=0) V_z g - (m_z + m_p + \rho_{\text{Gas}}(z=0) V_z)g \\ &= (\rho_L(z=0) - \rho_{\text{Gas}}(z=0)) V_z g - (m_z + m_p)g \\ &\stackrel{\text{id Gas}}{=} \frac{p_a}{T_0} \left(\frac{1}{\rho_L} - \frac{1}{\rho_{\text{Gas}}} \right) V_z g - (m_z + m_p)g \end{aligned}$$

$$\Rightarrow F_H(H_2) = 166,61 \text{ RN}, F_H(H_0) = 146,22 \text{ RN}$$

b) $z=0$: $m_{\text{Gas}} a_0 = F_A - F_G$, $m_{\text{Gas}} = m_z + m_p + \rho_{\text{Gas}}^* V_z$

$$\Rightarrow m_{\text{Gas}} (a_0 + g) = \rho_L V_z g$$

$$\Rightarrow \rho_{\text{Gas}}^* = \rho_L \frac{g}{a_0 + g} - \frac{m_p + m_z}{V_z} = 0,1546 \frac{\text{kg}}{\text{m}^3}$$

c) Anfangsbeschleunigung:

$$z=0: m_{\text{Gas}} \cdot a = F_A - F_G$$

$$\Rightarrow (m_z + m_p + \rho_{\text{Gas}} V_z) (a + g) = \rho_L g V_z$$

$$\Rightarrow a = \frac{\rho_L g V_z}{m_z + m_p + \rho_{\text{Gas}} V_z} - g$$

$$\Rightarrow \begin{cases} a_{\text{He}} = 9,679 \frac{\text{m}}{\text{s}^2} < a_0 \\ a_{\text{H}_2} = 12,7467 \frac{\text{m}}{\text{s}^2} > a_0 \end{cases} ; \text{ mit } a_0 = 10 \frac{\text{m}}{\text{s}^2}$$

d) max. Steighöhe:

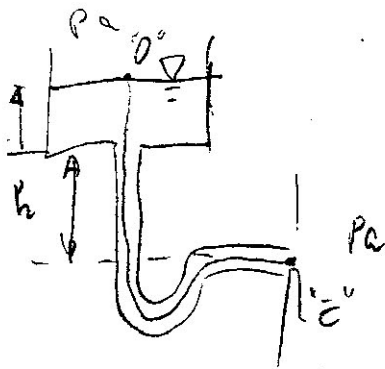
$$z = z_{\text{max}}: \sum_i F_{i,z} = 0 = F_A(z_{\text{max}}) - F_G$$

$$\Rightarrow \rho_L(z_{\text{max}}) V_z g = (m_z + m_p + \rho_{\text{Gas}} V_z) g$$

$$\Rightarrow \frac{p(z_{\text{max}})}{\rho_L T_0} = \frac{(m_z + m_p)}{V_z} + \frac{p(z=0)}{\rho_{\text{Gas}} T_0} ; \text{ barometr. Höhenformel. } p(z) = p_0 e^{-\frac{\rho_L g z}{p_0}}$$

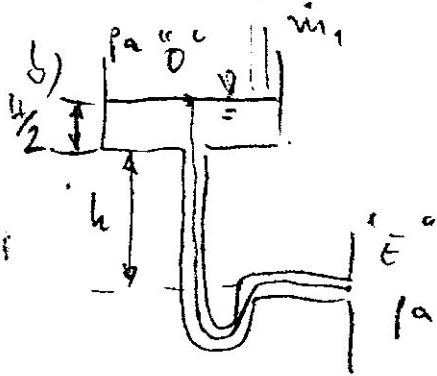
$$\Rightarrow z_{\text{max}} = -\frac{\rho_L T_0}{g} \ln \left\{ (m_z + m_p) \frac{\rho_L T_0}{V_z} + \frac{\rho_L}{\rho_{\text{Gas}}} \right\} \Rightarrow \begin{cases} z_{\text{max}}(\text{He}) = 569,23 \text{ m} \\ z_{\text{max}}(\text{H}_2) = 691,5 \text{ m} \end{cases}$$

Aufg. 2) a) Bernoulli $0 \rightarrow E$



$$A \gg d^2 \Rightarrow p_a + \rho g (h + z) = p_a + \frac{\rho}{2} v_E^2$$

$$\Rightarrow v_E = \sqrt{2g(h+z)}$$

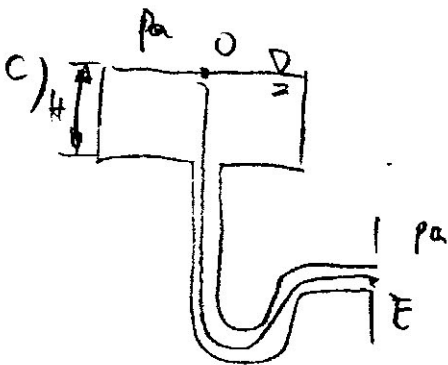


Verlustbeh. Bernoulli $0 \rightarrow E$

$$p_a + \rho g (h + \frac{h}{2}) = p_a + \frac{\rho}{2} v_{E1}^2 (1 + \xi_{Ges})$$

$$\Rightarrow \xi_{Ges} \leq \frac{2g(h + \frac{h}{2})}{v_{E1}^2} - 1$$

mit $v_{E1} = \frac{\dot{m}_1}{\rho \frac{\pi}{4} d^2}$



Verlustbeh. Bernoulli $0 \rightarrow E$

$$p_a + \rho g (h + H) = p_a + \frac{\rho}{2} v_{E2}^2 (1 + \xi_{Ges} + \xi_v)$$

mit $v_{E2} = \frac{4 \dot{m}_{max}}{\rho \pi d^2}$

$$2g(h+H) = \left(\frac{4 \dot{m}_{max}}{\rho \pi d^2} \right)^2 \cdot (1 + \xi_{Ges} + \xi_v)$$

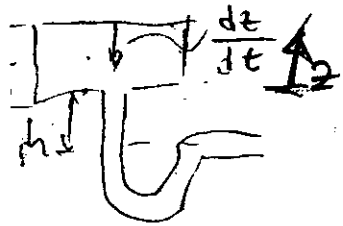
$$\Rightarrow \dot{m}_{max} = \sqrt{\frac{2g(h+H)}{1 + \xi_{Ges} + \xi_v}} \cdot \frac{\rho \pi d^2}{4}$$

$$= \sqrt{\frac{2g(h+H)}{1 + \frac{2g(h + \frac{h}{2})}{v_{E1}^2} - 1 + \xi_v}} \cdot \frac{\rho \pi d^2}{4}$$

$$v_{\max} = \sqrt{\frac{(h+H)}{\left(h + \frac{H}{2}\right) + \xi_v \frac{v_{E1}^2}{2g}}} \cdot \frac{3\pi d^2}{4} \cdot v_{E1}$$

$$= \sqrt{\frac{(h+H)}{\left(h + \frac{H}{2}\right) + \frac{\xi_v}{2g} \left(\frac{4\dot{m}_1}{3\pi d^2}\right)^2}} \cdot \dot{m}_1$$

d) Quasistationär : $\int \frac{\partial v}{\partial t} ds = 0$



$$\frac{\rho}{2} \left(\frac{dz}{dt} \right)^2 + \rho g (h+z) = \frac{\rho}{2} v_E^2(t) (1 + \xi_{Ges})$$

mit Konti : $v_E \frac{\pi d^2}{4} = - \frac{dz}{dt} \cdot A \Rightarrow v_E(t) = - \frac{dz}{dt} \frac{4A}{\pi d^2}$

$$\Rightarrow \left(\frac{dz}{dt} \right)^2 \cdot \left[\left(\frac{4A}{\pi d^2} \right)^2 (1 + \xi_{Ges}) - 1 \right] = 2g (h+z)$$

Trennung der Variablen:

$$\sqrt{\frac{\left(\frac{4A}{\pi d^2} \right)^2 (1 + \xi_{Ges}) - 1}{2g}} \cdot \frac{dz}{\sqrt{h+z}} = dt$$

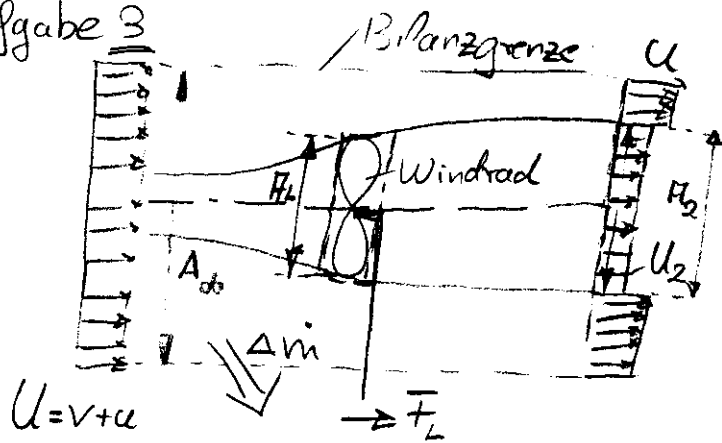
$$\Rightarrow T = \int_h^0 \sqrt{\frac{\left(\frac{4A}{\pi d^2} \right)^2 (1 + \xi_{Ges}) - 1}{2g}} \cdot \frac{dz}{\sqrt{h+z}}$$

$$= k \cdot 2 \sqrt{h+z} \Big|_h^0 = 2k \cdot (\sqrt{h} - \sqrt{h+H})$$

$$= \sqrt{\frac{2}{g} \left[\left(\frac{4A}{\pi d^2} \right)^2 (1 + \xi_{Ges}) - 1 \right]} \cdot (\sqrt{h} - \sqrt{h+H})$$

Aufgabe 3

a)

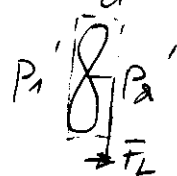


Massenstrom:

$$\begin{aligned} -\rho U A_0 + \rho U (A_0 - A_2) + \rho U_2 A_2 + \Delta m &= 0 \\ \Rightarrow \Delta m &= \rho A_2 (U - U_2) \end{aligned}$$

Impulssatz: $-\rho U^2 A_0 + \rho U^2 (A_0 - A_2) + \rho U_2^2 A_2 + \Delta m U = F_L$
mit $\Delta m = \dots$
 $\Rightarrow \rho A_2 U_2 (U_2 - U) = F_L$ *

Impulssatz um Windrad:



$$-\rho U'^2 A_L + \rho U_2^2 A_L = (p_1' - p_2') A_L + F_L$$

Bernoulli: $p_1' - p_2' = \left(p_\infty + \frac{\rho}{2} (U^2 - U'^2) \right) - \left(p_\infty + \frac{\rho}{2} (U_2^2 - U'^2) \right)$

$$\Rightarrow F_L = -\frac{\rho}{2} (U^2 - U_2^2) A_L$$
 **

* und ** gleichsetzen:

$$-\frac{\rho}{2} (U^2 - U_2^2) A_L = \rho A_2 U_2 (U_2 - U); \text{ Konti: } \rho A_L U' = \rho A_2 U_2$$

$$\Rightarrow \frac{1}{2} (U + U_2) = A_2 U_2 = A_L U'$$

$$\Rightarrow U' = \frac{U + U_2}{2}$$

$$\begin{aligned} P_L = -F_L \cdot U' &= \frac{\rho}{4} U^3 \left(1 - \left(\frac{U_2}{U} \right)^2 \right) \left(1 + \frac{U_2}{U} \right) A_L; \quad \frac{U_2}{U} = \xi \\ &= \frac{\rho}{4} U^3 A_L (1 - \xi^2) (1 + \xi) \end{aligned}$$

$$\frac{\partial P_L}{\partial \xi} = 0 = \frac{\rho}{4} U^3 A_L \underbrace{(1 + \xi)(-2\xi) + 1 - \xi^2}_{=0}$$

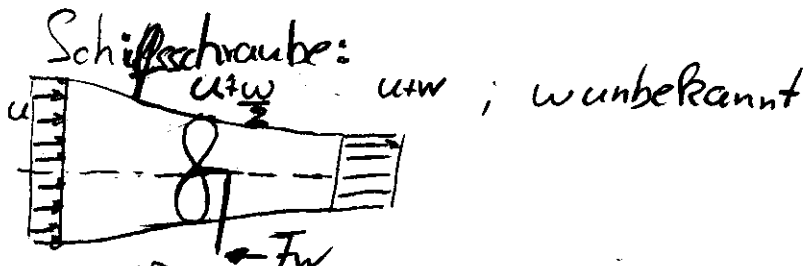
$$\Rightarrow \xi_{1,2} = -\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{3}{9}} = -\frac{1}{3} \pm \frac{2}{3}$$

$$\Rightarrow \xi = \frac{1}{3}$$

$$\Rightarrow P_{L,opt} = -\bar{F}_{L,opt} \cdot u'_{opt} = -\bar{F}_{L,opt} \cdot \frac{2}{3} U = -\frac{8}{27} \rho U^3 A_L$$

$$\bar{F}_{opt} - \bar{F}_{L,opt} = \frac{4}{9} \rho U^2 A_L$$

- b) keine mechanischen Verluste $\Rightarrow P_L + P_w = 0$
 kein Widerstand vom Bootskörper $\Rightarrow \bar{F}_{opt} + \bar{F}_w = 0$



$$P_{L,opt} = -P_w = -(u + \frac{w}{2}) \bar{F}_w = -\bar{F}_L \frac{2}{3} (u+v) \quad = U$$

mit $\bar{F}_L = \bar{F}_w$ $\Rightarrow w = \frac{1}{3} (4v - 2u)$

$$\bar{F}_{L,opt} = \frac{4}{9} \rho U^2 A_L = \rho_w A_w w \frac{(u+w+u)}{2} = -\bar{F}_w$$

$$\Leftrightarrow \frac{4}{9} \left(\frac{\rho_L A_L}{\rho_w A_w} \right) U^2 = w \left(2u + \frac{4}{3}v - \frac{2}{3}u \right)$$

$$\stackrel{\leq 1}{=} w \frac{2}{3} (u+v)$$

$$= \frac{2}{9} (4v - 2u) (u+v)$$

mit $U = u+v$ folgt:

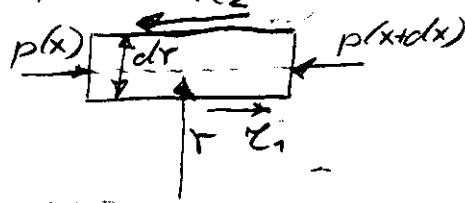
$$\frac{4}{9} \rho (u+v)^2 = \frac{2}{9} (4v - 2u) (u+v)$$

$$\Leftrightarrow 2\rho (u+v) = 2(2v - u)$$

$$\Leftrightarrow u = \frac{2-\rho}{1+\rho} v$$

Aufgabe 4

- a) Geschwindigkeitsverteilung:
IES am diff. Volumenelement:



$$0 = \tau_1 2\pi(r - \frac{dr}{2}) dx - \tau_2 2\pi(r + \frac{dr}{2}) dx + p(x) 2\pi r dr - p(x+dx) 2\pi r dr$$

$$\Rightarrow 0 = -(\tau_1 + r \frac{d\tau}{dr}) - r \frac{dp}{dx}$$

$$\Rightarrow \frac{d(r \cdot \tau)}{dr} = -r \frac{dp}{dx}$$

1. Integration:

$$r \cdot \tau = - \frac{dp}{dx} \frac{r^2}{2} + C_1$$

$$\Leftrightarrow \tau = - \frac{dp}{dx} \frac{r}{2} + \frac{C_1}{r} ; \text{ mit R.B. } \tau(r=0) = 0 \text{ folgt } C_1 = 0$$

$$\Rightarrow \tau(r) = - \frac{dp}{dx} \frac{r}{2} = - \eta \frac{du}{dr} \quad (\text{Newton'sche Fließgesetz})$$

$$\Leftrightarrow \frac{du}{dr} = \frac{r}{2\eta} \frac{dp}{dx}$$

2. Integration:

$$u_2(r) = \frac{1}{4\eta} \frac{dp}{dx} \cdot r^2 + C_2 ; \text{ mit R.B. } u(r=R) = 0 \text{ folgt } C_2 = - \frac{R^2}{4\eta} \frac{dp}{dx}$$

$$\Rightarrow u_2(r) = - \frac{R^2}{4\eta} \frac{dp}{dx} \left(1 - \left(\frac{r}{R}\right)^2\right)$$

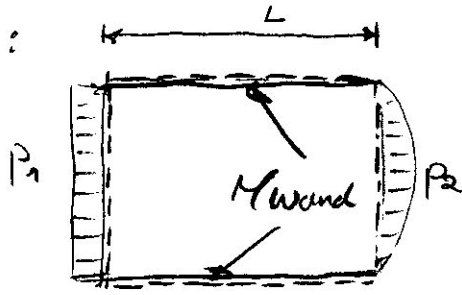
$$\frac{dp}{dx} = ?$$

$$Q = \text{konst} = u_1 \pi R^2 = 2\pi R \int_0^R u_2(r) \frac{r}{R} d\left(\frac{r}{R}\right)$$

$$= - \frac{R^4}{8\eta} \frac{dp}{dx} \Rightarrow \frac{dp}{dx} = - \frac{8 u_1 \eta}{R^2}$$

$$\Rightarrow u_2(r) = 2 u_1 \left(1 - \left(\frac{r}{R}\right)^2\right)$$

b) IES :



$$-\dot{I}_{\text{Ein}} + \dot{I}_{\text{aus}} = \Delta p \pi R^2 - F_R$$

$$\text{mit } \Delta p = p_1 - p_2$$

$$F_R = \frac{1}{2} (\tau_{\text{wand}}(x=0) + \tau_{\text{wand}}(x=L)) 2\pi RL$$

$$\tau_{\text{wand}}(x=L) = -\eta \left. \frac{du_2}{dr} \right|_{r=R} = 4\eta \frac{u_1}{L R}$$

$$\tau_{\text{wand}}(x=0) = C \cdot \tau_{\text{wand}}(x=L)$$

$$\Rightarrow F_R = \tau_{\text{wand}}(x=L) (C+1) \pi RL$$

$$\dot{I}_{\text{Ein}} = -\int u_1^2 \pi R^2$$

$$\begin{aligned} \dot{I}_{\text{aus}} &= 2\pi R^2 \int_0^R u_2 \left(\frac{r}{R}\right)^2 \frac{r}{R} d\left(\frac{r}{R}\right) \\ &= \frac{4}{3} \pi R^2 \int_0^R u_1^2 \end{aligned}$$

$$\Rightarrow \frac{\int u_1^2 \pi R^2}{3} = \Delta p \pi R^2 - \pi RL (C+1) 4\eta \frac{u_1}{LR}$$

$$\Leftrightarrow \Delta p = \frac{4\eta u_1 L (C+1)}{R^2} + \frac{\int u_1^2}{3}$$

A5/ a) Einflußgrößen: $g, u_\infty, S, \gamma, h, f$
Dimensionen: m, s, kg

$\Rightarrow k=6, r=3 \Rightarrow k-r=3$ Kenngrößen

b) Wiederkehrende Variable

u_∞, h, S

$$\Pi_1 = g \cdot u_\infty^{\alpha_1} \cdot h^{\beta_1} \cdot S^{\gamma_1}$$

$$[m]: 1 + \alpha_1 + \beta_1 - 3\gamma_1 = 0$$

$$[kg]: 0 + 0 + 0 + \gamma_1 = 0 \Rightarrow \gamma_1 = 0$$

$$[s]: -2 - \alpha_1 + 0 + 0 = 0 \Rightarrow \alpha_1 = -2$$

$$\Rightarrow \beta_1 = 1$$

$$\Pi_1 = \frac{g \cdot h}{u_\infty^2} = \frac{1}{Fr^2}$$

$$\Pi_2 = \gamma \cdot u_\infty^{\alpha_2} \cdot h^{\beta_2} \cdot S^{\gamma_2}$$

$$[m]: -1 + \alpha_2 + \beta_2 - 3\gamma_2 = 0$$

$$[kg]: 1 + 0 + 0 + \gamma_2 = 0 \Rightarrow \gamma_2 = -1$$

$$[s]: -1 - \alpha_2 + 0 + 0 = 0 \Rightarrow \alpha_2 = -1$$

$$\Rightarrow \beta_2 = -1$$

$$\Pi_2 = \frac{\eta}{u_\infty h S} = \frac{1}{Re}$$

$$\bar{u}_3 = f \cdot u_{\infty}^{\alpha_3} h^{\beta_3} g^{\gamma_3}$$

$$[m]: \quad 0 + \alpha_3 + \beta_3 - 3\gamma_3 = 0$$

$$[kg]: \quad 0 + 0 + 0 + \gamma_3 = 0 \Rightarrow \gamma_3 = 0$$

$$[s]: \quad -1 - \alpha_3 + 0 + 0 = 0 \Rightarrow \alpha_3 = -1$$

$$\Rightarrow \beta_3 = 1$$

$$\bar{u}_3 = \frac{f \cdot h}{u_{\infty}} = \text{Sr}$$

Aufgabe 6:

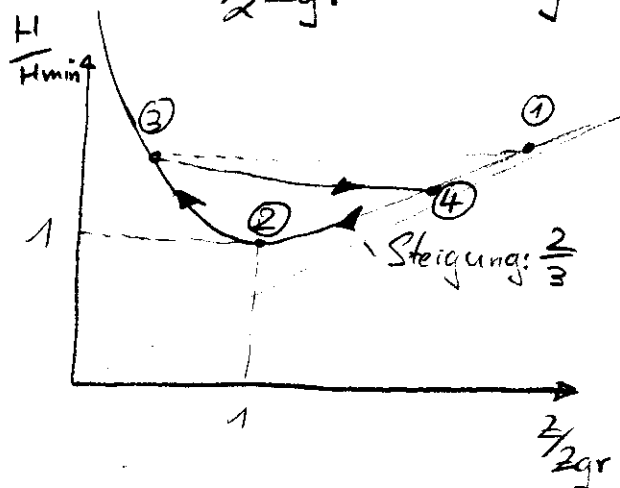
a) $z + \frac{v^2}{2g} = H$, Neigung wird vernachlässigt?

mit $Q = z B v$ / dgt: $H = z + \frac{Q^2}{2g B^2 z^2}$;

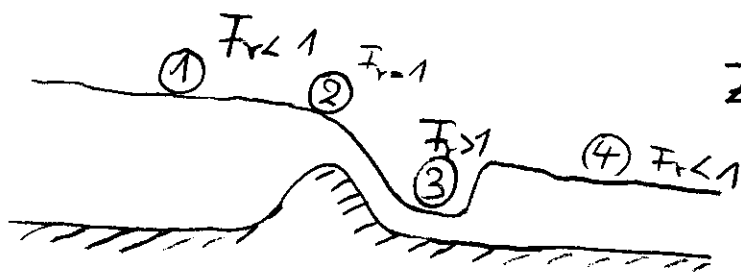
$$H_{min}, z_{gr}: \frac{dH}{dz} = 0 = 1 - \frac{Q^2}{g B^2 z^3} \Leftrightarrow z_{gr} = \sqrt[3]{\frac{Q^2}{g B^2}}$$

$$\Rightarrow H_{min} = H(z_{gr}) = z_{gr} + \frac{Q^2}{2g z_{gr}^2 B^2} = z_{gr} + \frac{1}{2} \frac{z_{gr}^3}{z_{gr}^2} = \frac{3}{2} z_{gr}$$

$$\Rightarrow \frac{H}{H_{min}} = \frac{z + \frac{Q^2}{2g z^2 B^2}}{\frac{3}{2} z_{gr}} = \frac{2}{3} \frac{z}{z_{gr}} + \frac{1}{3} \left(\frac{z}{z_{gr}} \right)^{-2}$$



b)



Zustandsänderungen
siehe Skizze 2.1a)

c)

$$z_3: H_1 = H_3 = z_1 + \frac{Q^2}{2g B^2 z_1^2} = z_3 + \frac{Q^2}{2g B^2 z_3^2}$$

$$\Leftrightarrow z_1 - z_3 = \frac{Q^2}{2g B^2} \left(\frac{1}{z_3^2} - \frac{1}{z_1^2} \right) = \frac{Q^2}{2g B^2} \frac{(z_1 + z_3)(z_1 - z_3)}{z_1^2 z_3^2}$$

$$\Rightarrow z_3 = \frac{Q^2}{4g B^2 z_1^2} \pm \sqrt{\left(\frac{Q^2}{4g B^2 z_1^2} \right)^2 + \frac{Q^2}{2g B^2 z_1}}$$

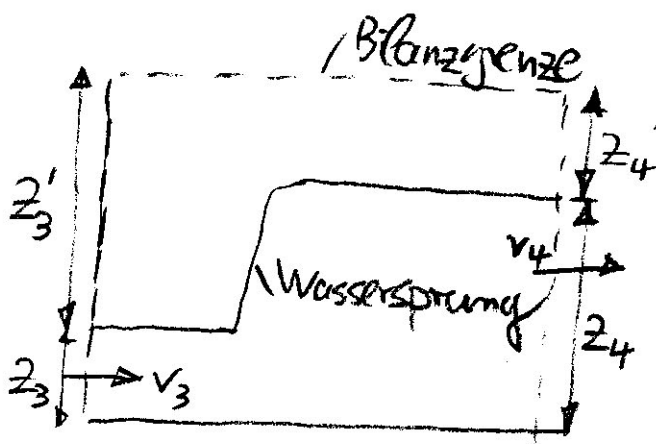
$$\Rightarrow Fr_3 = \frac{v_3}{\sqrt{g z_3}} = \frac{Q}{B \sqrt{g z_3^3}}$$

$$H_3 = H_1 = z_3 + \frac{Q^2}{2gB^2 z_3^2}$$

$$y_w + H_{\min} = H_1; \text{ mit } H_{\min} = \frac{3}{2} z_{gr}; \quad z_{gr} = \sqrt[3]{\frac{Q^2}{gB^2}}$$

$$\Rightarrow H_1 - \frac{3}{2} z_{gr} = z_1 + \frac{Q^2}{2gB^2 z_1^2} - \frac{3}{2} z_{gr}$$

d)



$$\text{konti: } \rho z_3 v_3 = \rho z_4 v_4 \\ \Rightarrow v_4 = v_3 \frac{z_3}{z_4}$$

Impulssatz in x-Richtung über Wassersprung

$$-\rho z_3 v_3^2 B + \rho z_4 v_4^2 B = p_a B z_3 + B \int_0^{z_3} (p_a + \rho g z) dz \\ - p_a B z_4 - B \int_0^{z_4} (p_a + \rho g z) dz$$

$$\Rightarrow -z_3 v_3^2 + z_4 v_4^2 \left(\frac{z_3}{z_4} \right)^2 = \frac{g}{2} (z_3^2 - z_4^2)$$

$$\Leftrightarrow z_{4/2} = -\frac{1}{2} z_3 \pm \sqrt{\frac{1}{4} z_3^2 + \frac{2v_3^2 z_3}{g}}$$

$$\Rightarrow \Delta H = \overset{\text{mit } H_1 = H_3}{z_1} - z_4 + \frac{Q^2}{2gB^2} \left(\frac{1}{z_1^2} - \frac{1}{z_4^2} \right)$$

Aufgabe 7

a) Koeffizienten a_0, a_1, a_2, a_3

① $y=0 : u=0 \Rightarrow a_0=0$

② $y=\delta : u=U \Rightarrow 1 = a_0 + a_1 + a_2 + a_3$

③ $y=0 : -\nu \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}$

mit $\nu = \frac{\eta}{\rho} = C \eta U ; \frac{\partial p}{\partial x} = 0$

$\Rightarrow -\frac{a_1}{\delta} C \eta U = \frac{2}{\delta} \frac{2a_2}{\delta^2}$

$\Leftrightarrow a_1 = -\frac{2}{\delta} \frac{2a_2}{C \eta U}$

④ $y=\delta : \frac{\partial u}{\partial y} = 0 : 0 = a_1 + 2a_2 + 3a_3$

3. ②-④ ergibt: $3 = 2a_1 + a_2$

mit ③ folgt: $3 = -\frac{2}{\delta} \frac{4a_2}{C \eta U} + a_2 = a_2 \left(1 - \frac{4\eta}{\delta C \eta U}\right)$

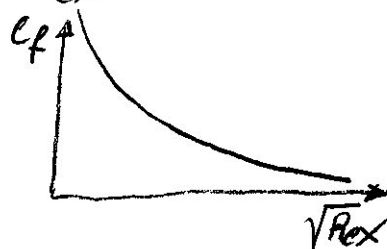
$\Leftrightarrow a_2 = \frac{3 \delta C \eta U}{\delta C \eta U - 4\eta}$

$a_1 = -\frac{6\eta}{\delta C \eta U - 4\eta}$

$a_3 = 1 - a_1 - a_2 = -\frac{2(\delta C \eta U - \eta)}{\delta C \eta U - 4\eta}$

b) $\frac{\delta(x)}{x} = C \cdot \frac{1}{\sqrt{Re_x}} ; \text{ mit } C=4,9 \text{ siehe Lösungsskript}$

c) $c_f = \frac{0,664}{\sqrt{Re_x}}$



d) $Re_{krit} \approx 500000$

beeinflussende Größen u.a. - Oberflächenrauigkeit,
- Störungen in der freien Einströmung

Aufgabe 8

a) Konti: $\dot{m} = \rho u A = \text{konst}$; $d\dot{m} = 0 = d\rho u A + \rho du A + \rho u dA$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

Impuls: $u du = - \frac{dp}{\rho} = - \frac{dp}{\rho} \frac{d\rho}{\rho}$

mit $c^2 = \frac{dp}{d\rho}$ folgt: $-\frac{u^2}{c^2} \frac{du}{u} + \frac{du}{u} + \frac{dA}{A} = 0$

$$\Rightarrow \frac{du}{u} (1 - M^2) = - \frac{dA}{A}$$

$$\Leftrightarrow \frac{du}{u} = - \frac{dA}{A} \frac{1}{1 - M^2}$$

$$\underline{M < 1}$$

$$dA > 0$$

$$\Rightarrow du < 0$$

$$\underline{M > 1}$$

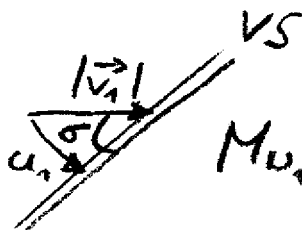
$$dA > 0$$

$$\Rightarrow du > 0$$

und jeweils
umgekehrt für
 $dA < 0$

b) Konti über Verdichtungsstoß:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{u_1^2}{u_1 u_2} \stackrel{\text{Prandtl-Boz}}{=} \frac{u_1^2}{c^{*2}} = M_{N1}^{*2}$$



$$M_{N1} = \frac{u_1}{c_1} = \frac{|\vec{u}_1| \sin \sigma}{c_1} = M_1 \sin \sigma$$

mit Hinweis

$$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1 + \frac{2}{(M_1 \sin \sigma)^2}} ; \sigma = 90^\circ \text{ für } \perp VS$$