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(Name, Matr.-Nr, Unterschrift)

Klausur Strömungsmechanik II

15. 03. 2017

1. Aufgabe

a) Grenzschichtannahmen $u \gg v$ und $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$
Größenordnung konvektiver Term = diffusiver Term:

$$\frac{u_\infty^2}{L} \sim \nu \frac{u_\infty}{\delta^2} \quad \Rightarrow \quad \delta \sim \sqrt{\frac{\nu L}{u_\infty}} = \frac{L}{\sqrt{Re}} \quad \text{mit} \quad Re = \frac{u_\infty L}{\nu}$$

Auf Referenzgrößen bezogene dimensionslose Größen:

$$\bar{x} = \frac{x}{L}\,, \quad \bar{u} = \frac{u}{u_\infty}\,, \quad \bar{\rho} = \frac{\rho}{\rho_\infty}\,, \quad \bar{T} = \frac{T}{T_\infty}\,, \quad \bar{p} = \frac{p}{p_{ref}}$$
$$\bar{y} = \frac{y}{\delta} = \frac{y}{L/\sqrt{Re}}\,, \quad \bar{v} = \frac{v}{u_\infty/\sqrt{Re}}$$

Energiegleichung in dimensionsloser Form:

$$\rho_\infty c_p \frac{u_\infty T_\infty}{L} \bar{\rho} \left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} \right) = \frac{u_\infty p_{ref}}{L} \left(\bar{u}\frac{\partial \bar{p}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{p}}{\partial \bar{y}} \right) + \lambda \frac{T_\infty}{L^2} \left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + Re \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right)$$
$$+ \eta \frac{u_\infty^2}{L^2} \left[2\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 + 2\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2 + \left(\frac{1}{\sqrt{Re}} \frac{\partial \bar{v}}{\partial \bar{x}} + \sqrt{Re}\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 - \frac{2}{3}\left(\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} \right) \right]$$

Mit $\frac{\partial p}{\partial y} = 0$ und $\frac{\partial p}{\partial x} = 0$ für die ebene Platte und Multiplikation mit $L/(\rho_\infty c_p u_\infty T_\infty)$:

$$\bar{\rho} \left(\underbrace{\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}}_{\mathcal{O}(1)} + \underbrace{\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}}_{\mathcal{O}(1)} \right) = \lambda \frac{1}{c_p \eta} \underbrace{\frac{1}{Re}}_{\mathcal{O}(\delta^2)} \left(\underbrace{\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}}_{\mathcal{O}(1)} + \underbrace{Re \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}}_{\mathcal{O}(\frac{1}{\delta^2})} \right)$$
$$+ \frac{u_\infty^2}{c_p T_\infty} \underbrace{\frac{1}{Re}}_{\mathcal{O}(\delta^2)} \left[2\underbrace{\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2}_{\mathcal{O}(1)} + 2\underbrace{\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2}_{\mathcal{O}(1)} + \left(\underbrace{\frac{1}{\sqrt{Re}} \frac{\partial \bar{v}}{\partial \bar{x}}}_{\mathcal{O}(\delta)} + \underbrace{\sqrt{Re}\frac{\partial \bar{u}}{\partial \bar{y}}}_{\mathcal{O}(\frac{1}{\delta})} \right)^2 - \frac{2}{3}\left(\underbrace{\frac{\partial \bar{u}}{\partial \bar{x}}}_{\mathcal{O}(1)} + \underbrace{\frac{\partial \bar{v}}{\partial \bar{y}}}_{\mathcal{O}(1)} \right) \right]$$

Grenzschicht: Größenordnungsabschätzung mit $Re \gg 1 \quad (\delta \ll 1)$, alle Terme mit $\mathcal{O} < 1$ fallen weg:

$$\bar{\rho} \left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} \right) = \lambda \frac{1}{c_p \eta} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{u_\infty^2}{c_p T_\infty} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2$$

b) Aus a) ergeben sich folgende Kennzahlen:

$$K_1 = \frac{\lambda}{c_p \eta} \quad K_2 = \frac{u_\infty^2}{c_p T_\infty}$$

c)

$$K_1 = \frac{\lambda}{c_p \eta} = \frac{1}{Pr}$$
$$K_2 = \frac{u_\infty^2}{c_p T_\infty} = \frac{u_\infty^2 (\gamma - 1)}{\gamma R T_\infty} = M_\infty^2 (\gamma - 1)$$

2. Aufgabe

a)

$$\frac{\partial^2 u_\varphi}{\partial h^2} = \frac{1}{\eta}\frac{dp}{d\varphi}$$

$$\Leftrightarrow \frac{\partial u_\varphi}{\partial h} = \frac{1}{\eta}\frac{dp}{d\varphi}h + c_1(\varphi)$$

$$\Leftrightarrow u_\varphi(h,\varphi) = \frac{1}{2\eta}\frac{dp}{d\varphi}h^2 + c_1 h + c_2$$

Randbedingungen: $h = 0: \quad u_\varphi = \omega R_w, \quad h = h(\varphi): \quad u_\varphi = 0$

$$\Rightarrow c_2 = \omega R_w$$

$$\Rightarrow c_1 = -\frac{1}{2\eta}\frac{dp}{d\varphi}H(\varphi) - \frac{\omega R_w}{H(\varphi)}$$

$$\Rightarrow u_\varphi(h,\varphi) = \omega R_w\left(1 - \frac{h}{H(\varphi)}\right) - \frac{1}{2\eta}\frac{dp}{d\varphi}H^2(\varphi)\frac{h}{H(\varphi)}\left(1 - \frac{h}{H(\varphi)}\right)$$

b)

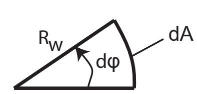
$$\tau(\varphi) = \eta\left.\frac{\partial u_\varphi(h,\varphi)}{\partial h}\right|_{h=0}$$

$$\tau(\varphi) = \eta\left(-\frac{\omega R_w}{H(\varphi)}\right) + \frac{1}{4}\frac{dp}{d\varphi}h - \left.\frac{1}{2}\frac{dp}{d\varphi}H(\varphi)\right|_{h=0}$$

$$\tau(\varphi) = -\frac{\eta\omega R_w}{H(\varphi)} - \frac{1}{2}\frac{dp}{d\varphi}H(\varphi)$$

c)

$$-M = F_R \cdot R_w = \int_0^{2\pi} \tau(\varphi)dA \cdot R_w = \int_0^{2\pi} \tau(\varphi)LR_w d\varphi \cdot R_w = LR_w^2 \int_0^{2\pi} \tau(\varphi)d\varphi$$



$$M = LR_w^2 \int_0^{2\pi}\left(\frac{\eta\omega R_w}{H(\varphi)} + \frac{1}{2}\frac{dp}{d\varphi}H(\varphi)\right)d\varphi$$

3. Aufgabe

a) $F(z) = az^2 + \frac{E}{2\pi} \ln z = \Phi + i\Psi$
mit $z = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$
$Re(F(z)) = \Phi(r,\varphi) = ar^2 \cos 2\varphi + \frac{E}{2\pi} \ln r$
$Im(F(z)) = \Psi(r,\varphi) = ar^2 \sin 2\varphi + \frac{E}{2\pi}\varphi$

b) $v_r = \frac{1}{r}\frac{\partial\Psi}{\partial\varphi} = 2ar\cos 2\varphi + \frac{E}{2\pi r}$
$v_\varphi = -\frac{\partial\Psi}{\partial r} = -2ar\sin 2\varphi$

c) Staupunkt $v_r = v_\varphi = 0$
$v_\varphi = 0 \Rightarrow \varphi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$
$v_r = 0 \Rightarrow \cos 2\varphi = -\frac{E}{2\pi}\frac{1}{2ar^2} < 0$
allgemein gilt: $\cos x \leq 0 \Rightarrow$ für $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$
$\Rightarrow \frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4} \Rightarrow \varphi_{st} = \frac{\pi}{2}$.
Es ist
$\cos\pi = -1 \quad \Rightarrow \quad \frac{E}{2\pi} = 2ar^2$
$\Rightarrow \quad r = \sqrt{\frac{E}{4a\pi}} = 1$ (Einheitskreis)
$\Rightarrow \quad E = 4a\pi$

d) Konturstromlinie:
$ar_k^2 \sin 2\varphi_k + \frac{E}{2\pi}\varphi_k = \Psi_{st}$
$\varphi_{st} = \frac{\pi}{2}$ und $r_{st} = 1$
$\Rightarrow \Psi_{st} = \frac{E}{4}$
$\Rightarrow r_k^2 a \sin 2\varphi_k = \frac{E}{4} - \frac{E}{2\pi}\varphi_k$
$r_k = \sqrt{\frac{\pi - 2\varphi}{\sin 2\varphi}}$

e) $\frac{\partial\Phi}{\partial r} = v_r$
$\frac{1}{r}\frac{\partial\Phi}{\partial\varphi} = v_\varphi$
$\frac{\partial\Psi}{\partial r} = -v_\varphi$
$\frac{1}{r}\frac{\partial\Psi}{\partial\varphi} = v_r$
$\nabla\Phi \cdot \nabla\Psi = v_r(-v_\varphi) + v_\varphi v_r = 0$

4. Aufgabe

a) $\frac{u(x,y)}{U(x)} = a_0 + a_1 \left(\frac{y}{\delta}\right) + a_2 \left(\frac{y}{\delta}\right)^2 + a_3 \left(\frac{y}{\delta}\right)^3$ mit $U(x) = u_\infty = konst.$

Randbedingungen:

a) $y = 0: \; u(x,y) = 0 \Rightarrow a_0 = 0$ (Haftbedingung)

b) $y = \delta: \; u(x,y) = u_\infty \Rightarrow a_1 + a_2 + a_3 = 1$ (Grenzschichtrand)

c) $y = 0: \; 0 = \eta \frac{\partial^2 u}{\partial y^2} \Rightarrow 2 \cdot a_2 = 0$ (Wandbindungsgleichung)

d) $y = \delta: \; \frac{\partial u}{\partial y} = 0 \Rightarrow a_1 + 3 \cdot a_3 = 0$ (glatter Übergang)

aus (c) $\Rightarrow a_2 = 0$

aus (b)+(d) $\Rightarrow a_1 = \frac{3}{2}, \; a_3 = -\frac{1}{2}$

$\Rightarrow \frac{u(x,y)}{u_\infty} = \frac{3}{2}(\frac{y}{\delta}) - \frac{1}{2}(\frac{y}{\delta})^3$

b) $\tau_W = \tau_W(\delta, u_\infty)$

Newton: $\tau = -\eta \frac{du}{dy} = -\eta u_\infty \left(\frac{3}{2} \cdot \frac{1}{\delta} - \frac{3}{2} \cdot \frac{y^2}{\delta^3}\right)$

$\tau_W = -\tau(y=0) \Rightarrow \tau_W(x) = \frac{3}{2} \frac{\eta}{\delta(x)} u_\infty$

c) aus Hinweis: von Kármánsche Integralbeziehung: $\frac{d\delta_2}{dx} + \frac{1}{U}\frac{dU}{dx}(2\delta_2 + \delta_1) = \frac{\tau_W}{\rho U^2}$

$U(x) = u_\infty = konst. \Rightarrow \frac{dU}{dx} = 0$, also: $\frac{d\delta_2}{dx} = \frac{\tau_W}{\rho u_\infty^2}$

$\delta_2 = \delta \int\limits_0^1 \left(1 - \frac{u}{u_\infty}\right) \frac{u}{u_\infty} d(\frac{y}{\delta}) = \delta \int\limits_0^1 \frac{3}{2}(\frac{y}{\delta}) - \frac{1}{2}(\frac{y}{\delta})^3 - \left[\frac{3}{2}(\frac{y}{\delta}) - \frac{1}{2}(\frac{y}{\delta})^3\right]^2 d(\frac{y}{\delta}) = \frac{39}{280}\,\delta(x)$

$\Rightarrow \frac{d\delta_2}{dx} = \frac{39}{280}\,\frac{d\delta(x)}{dx} = \frac{\tau_W}{\rho u_\infty^2} = \frac{3}{2}\frac{\eta}{\delta(x)\rho u_\infty}$

$\delta \, d\delta = \frac{140}{13}\,\frac{\eta}{\rho u_\infty}\,dx$

Integration: $\frac{1}{2}\,\delta^2 = \frac{140}{13}\,\frac{\eta}{\rho u_\infty} \cdot x$

$\Leftrightarrow \delta(x) = \sqrt{\frac{280}{13}}\,\sqrt{\frac{\eta}{\rho u_\infty}}\sqrt{x}$

d) Haftreibung bis zum Einsetzen der Bewegung:

$F_{max} = \mu \cdot m \cdot g = F_{Strömung} = \int\limits_0^L \tau_W(x) \cdot T \, dx$

$\tau_W(x) = \sqrt{\frac{117}{1120}}\frac{\sqrt{\eta\rho u_\infty^3}}{\sqrt{x}} \;\Rightarrow\; \mu \cdot m \cdot g = \left[\sqrt{\frac{117}{1120}}\sqrt{\eta\rho u_\infty^3}2 \cdot T\sqrt{x}\right]_0^L$

$\Rightarrow u_\infty = \sqrt[3]{\frac{280}{117}\,\frac{(\mu\;m\;g)^2}{\eta\;\rho\;T^2\;L}}$

5. Aufgabe

a) Konti: $\rho_1^* c_1^* A_1 = \rho_2^* c_2^* A_2$

$$T_{0,1} = T_{0,2} = konst. \quad \Rightarrow \quad \frac{c_1^*}{c_2^*} = \sqrt{\frac{T_1^*}{T_2^*}} = \sqrt{\frac{T_{0,1}}{T_{0,2}}} = 1$$

$$\frac{\rho_1^*}{\rho_2^*} = \frac{\frac{\rho_1^*}{\rho_{0,1}}\rho_{0,1}}{\frac{\rho_2^*}{\rho_{0,2}}\rho_{0,2}} = \frac{\rho_{0,1}}{\rho_{0,2}} = \frac{p_{0,1}}{p_{0,2}} \cdot \frac{T_{0,2}}{T_{0,1}} = \frac{p_{0,1}}{p_{0,2}}$$

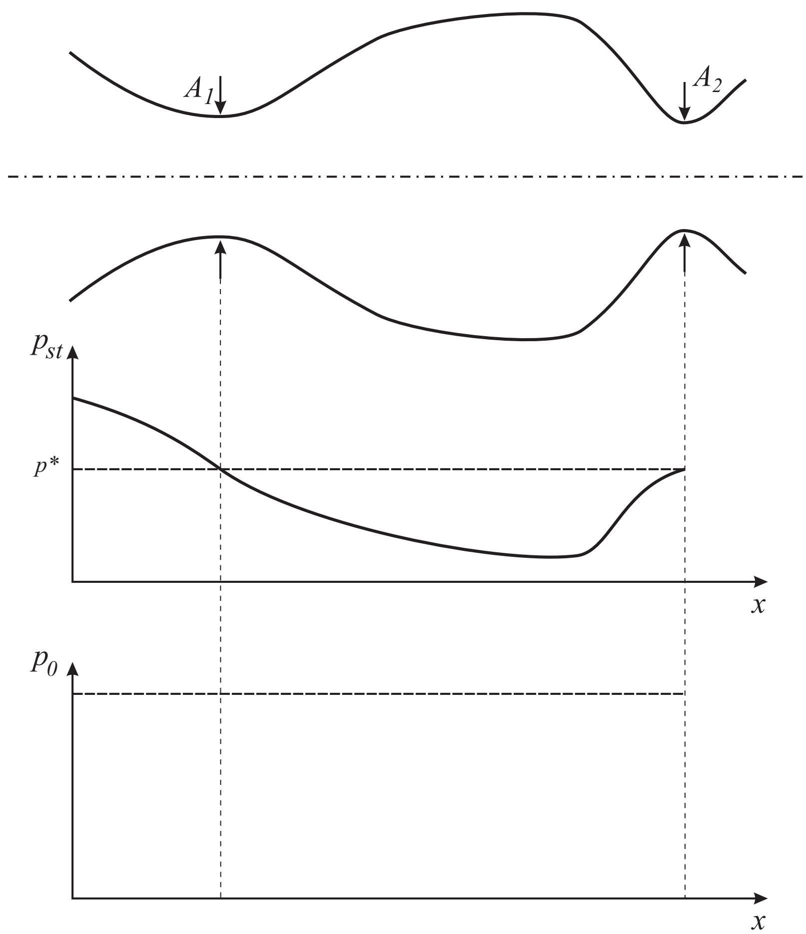
(I) isentrop: $p_{0,1} = p_{0,2}$:

$$A_2 = A_1$$

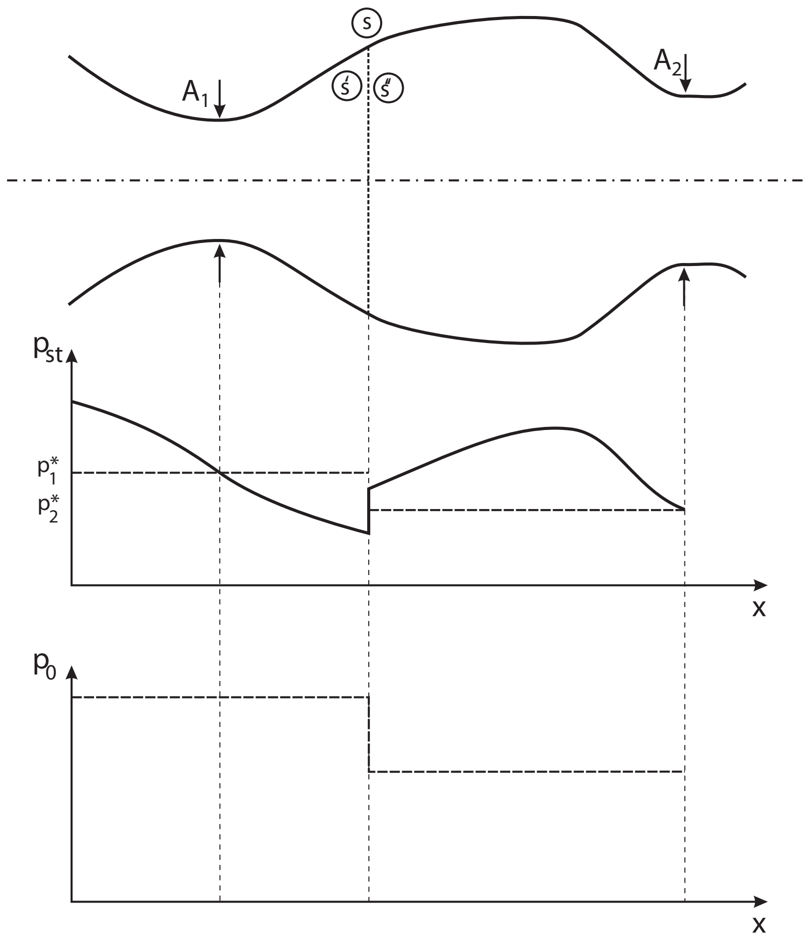
(II) Verdichtungsstoß: $p_{0,1}/p_{0,2} = p_{0,s'}/p_{0,s''}$:

$$A_2 = A_1 \frac{p_{0,s'}}{p_{0,s''}}$$

b) (I) Skizze:



(II) Skizze:



s
s'
s''
A1
A2
pst
p*1
p*2
x
p0
x

6. Aufgabe

a) $Sr = \frac{l}{\tau u_\infty}$ für quasistationäre Strömungen gilt $Sr \to 0$.

b) $\frac{\delta}{L} \sim \frac{1}{\sqrt{Re_L}} \quad \to \quad \frac{\delta_1}{\delta_2} = \sqrt{\frac{L_1}{L_2}}$

c) Betrachte Strömung an bewegter Oberfläche
$v(y=0) = 0, \quad \frac{\partial u}{\partial x}(y=0) = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2}|_{y=0} = \frac{1}{\eta}\frac{\partial p}{\partial x}|_{y=0}$

d) Der Ablösepunkt ist als der Ort definiert, an dem die Wandschubspannung verschwindet bzw. an dem $\frac{\partial u}{\partial y}|_{Wand} = 0$.

e) $u^* = c^* = \sqrt{u_1 u_2}$