Hydrostatics

$\longrightarrow$ mechanics of fluids in static equilibrium

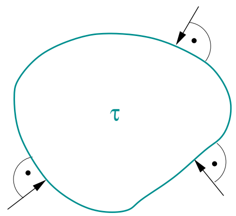
Fluids: materials, that can be deformed by the affection of tangential stresses

$\longrightarrow$ no tangential stresses in static fluids

Hydrostatics

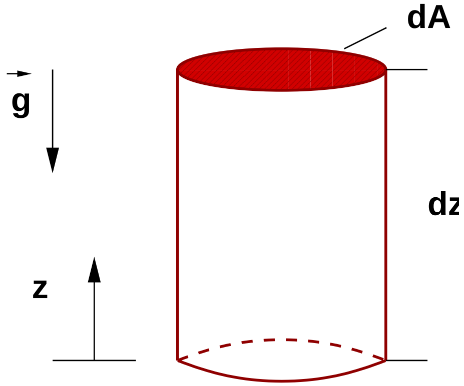
Hydrostatics:

- total amount of all outer forces vanishes
- the fluid elemnts are not moving or are moving with constant velocity
- only normal stresses, no shear stresse



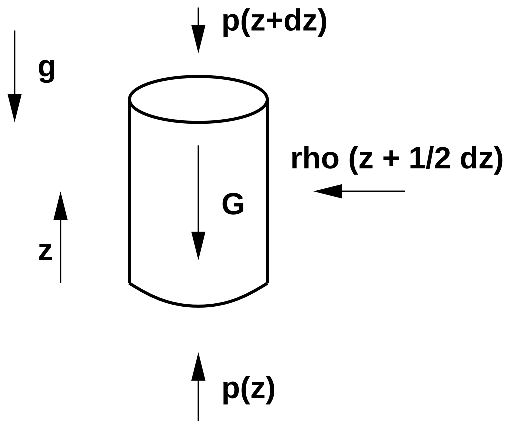
Normal stresses are pressures
(no internal molecular forces)

essential hydrostatic equation



all quantities (pressure p, density $\rho, \ldots$)
are functions coordinate z
$p(z), \rho(z), \ldots$

equilibrium of forces



essential hydrostatic equation

Taylor expansion of p and ρ:

$$p(z+dz) = p(z) + \frac{dp}{dz}dz + \frac{d^2p}{dz^2}\frac{dz^2}{2} + \cdots$$

$$\rho(z+\frac{dz}{2}) = \rho(z) + \frac{d\rho}{dz}\frac{dz}{2} + \frac{d^2\rho}{dz^2}\frac{dz^2}{4} + \cdots$$

$$p\,dA - (p + \frac{dp}{dz}dz - (\rho + \frac{d\rho}{dz}\frac{dz}{2}))g\,dz\,dA = 0$$

$$-\frac{dp}{dz}dz - \rho\,g\,dz\,dA - \underbrace{\frac{d\rho}{dz}\frac{dz^2}{2}g\,dA}_{\approx 0} = 0$$

$$\longrightarrow \boxed{\frac{\mathbf{dp}}{\mathbf{dz}} = -\rho\,\mathbf{g}}$$ E. H. E.

pressure distribution

Integration for **incompressible** fluids
($\rho = const$ and $\vec{g} = const$)

$$\frac{dp}{dz} = -\rho\, g \longrightarrow dp = \; -\rho\, g\, dz$$

$$\longrightarrow \boxed{\mathbf{p} + \rho\mathbf{gz} = \mathbf{const}} \quad \text{H. G. G.}$$

Integration for **compressible** fluids

Assumption: **perfect gas**: $\rho = \dfrac{p}{RT}$

isothermal atmosphere: $T = T_0 = konst$

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pressure distribution

$$\frac{dp}{dz} = -\rho\, g \longrightarrow dp = -\rho(z)\, g = -\frac{p(z)}{RT} g$$

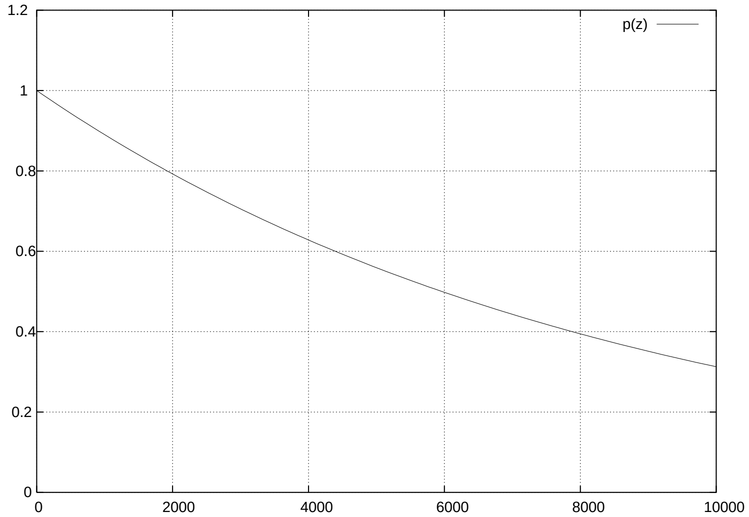
$$\int_{p_0}^{p_1} \frac{dp}{p} = -\int_{z_0}^{z_1} \frac{g}{RT} dz$$

$$\ln p_1 - \ln p_0 = \ln \frac{p_1}{p_0} = -\frac{g(z_1 - z_0)}{RT_0}$$

$$\longrightarrow \boxed{\mathbf{p_1 = p_0\, e^{-\frac{g \Delta z}{RT_0}}}}$$

scale height relation!

pressure distribution



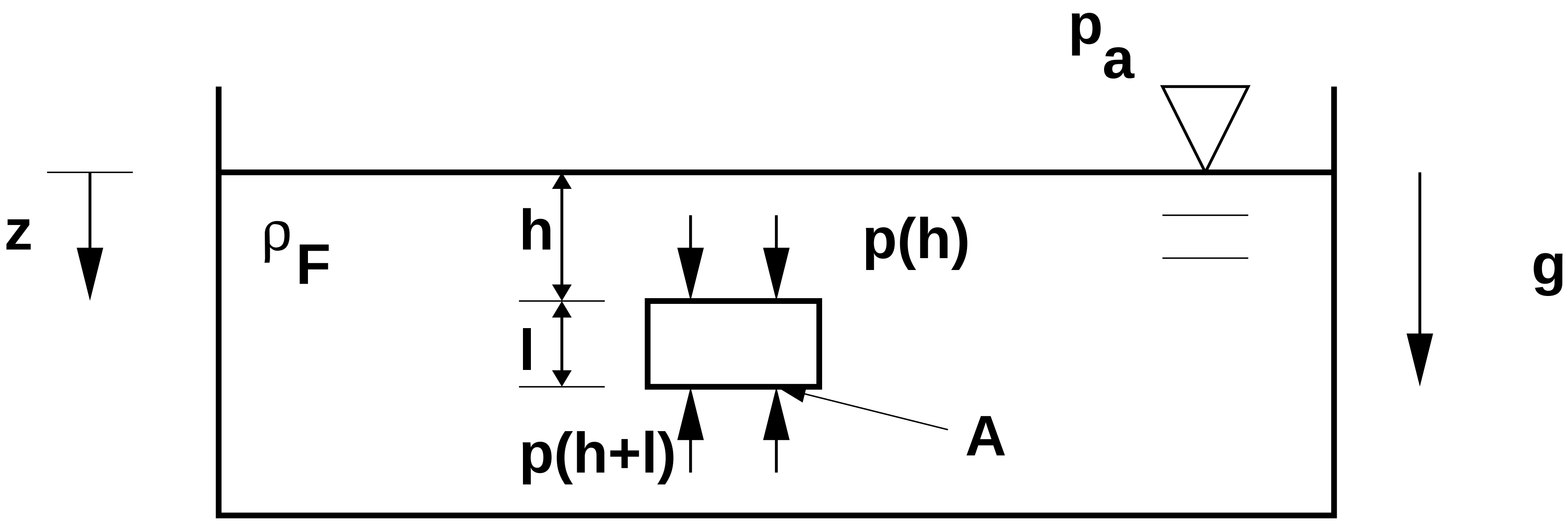
scale height relation

Hydrostatic lift

A body, that is fully or partly submerged in a fluid undergoes an apparent loss of weight.

$\longrightarrow$ **Lift**

parallel epiped in fluid with density ρ_F



Hydrostatic lift

Resulting force F_p in z-direction:

$$F_p = \left(p(h) - p(h+l) \right) A$$

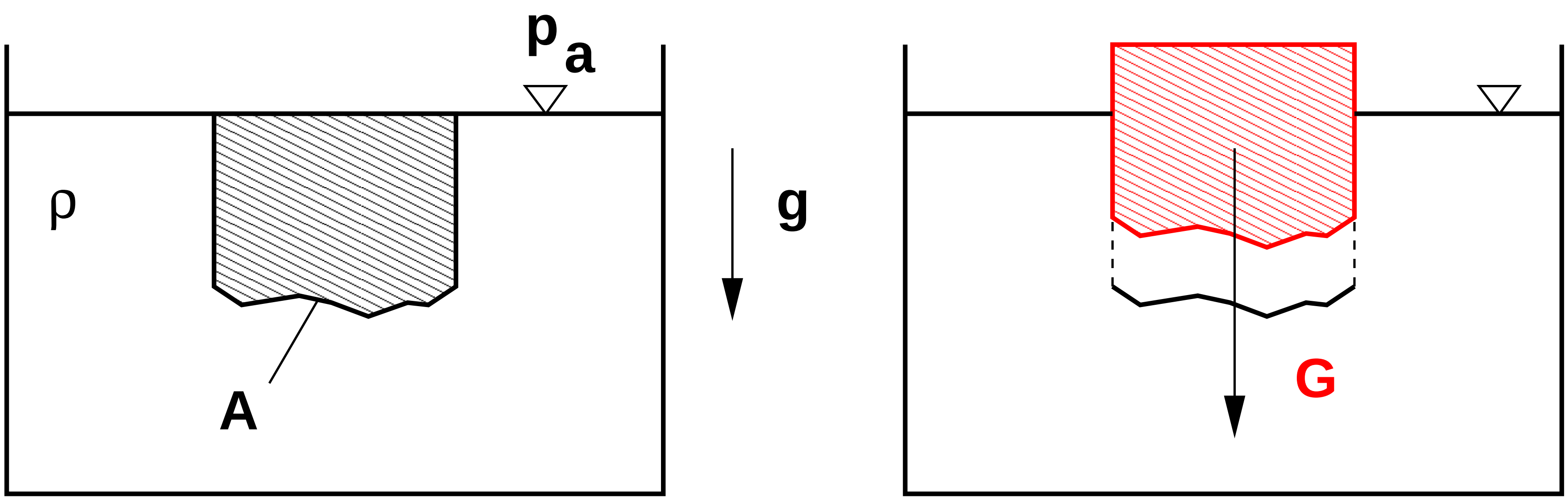
Hydrostatic pressure: $p(z) = p_a + \rho_F\, g z$

$$\longrightarrow F_p = \left(\cancel{p_a} + \cancel{\rho_F g h} - \cancel{p_a} - \rho_F g (\cancel{h} + l) \right) A$$

$$\mathbf{F_p} = -\rho_\mathbf{F}\, \mathbf{g} \underbrace{lA}_{\text{volum}} = -\rho_\mathbf{F}\, \mathbf{g}\, \tau = \mathbf{F_L} \quad (\text{ARCHIMEDES})$$

Lift force $\Longleftrightarrow$ resulting pressure force

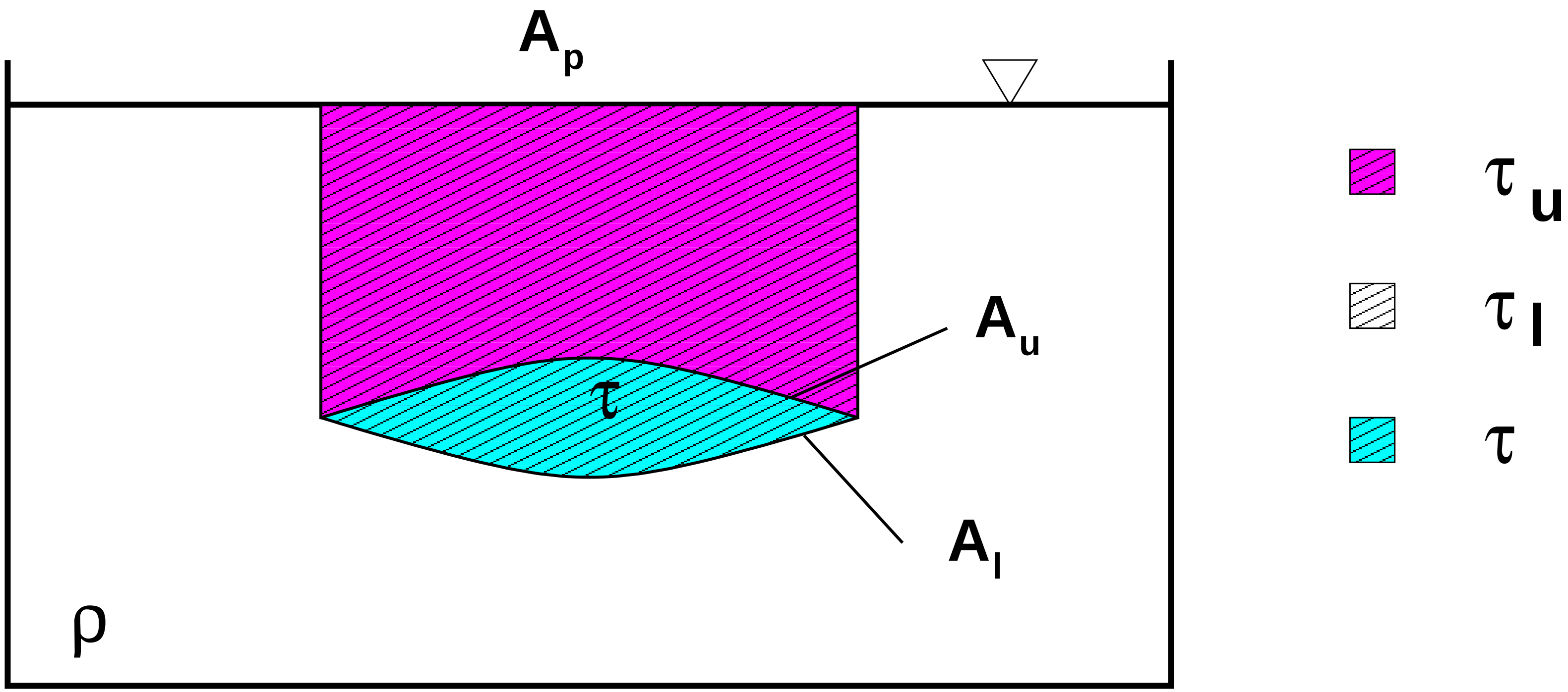
STEVIN principle of solidification



The force on an arbitrary area A in the fluid corresponds to weight of the fluid column above + the outer pressure multiplied with the projected area.

$$\mathbf{F} = \mathbf{G} + \mathbf{p_a A}$$

STEVIN principle of solidification

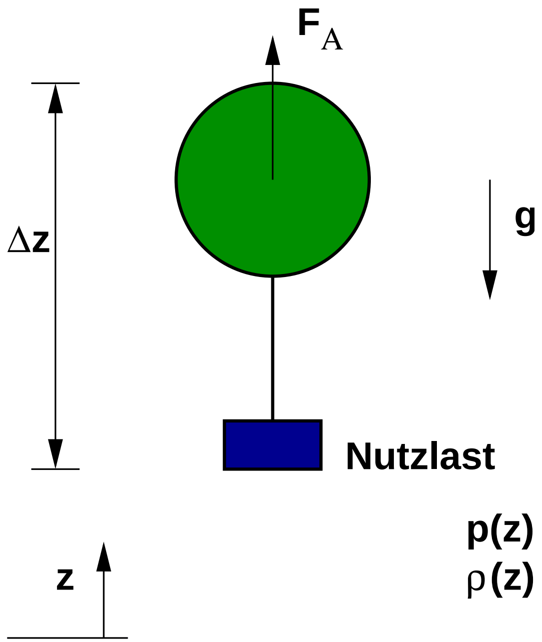


Total force on a body with the volume τ

$$F_L = p_a\,A_p + \rho g \tau_u - p_a\,A_p - \rho g \tau_l =$$
$$= -\rho g(\tau_l - \tau_u) = -\rho g \tau$$

$$\longrightarrow \boxed{F_L = -\rho\, g\, \tau}$$

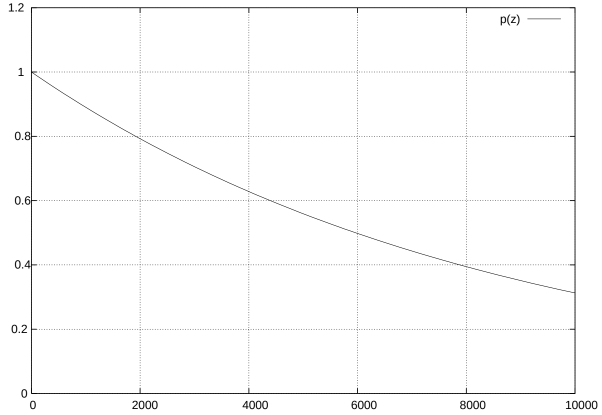
Example: Balloon in atmosphere



Atmosphere $\longrightarrow$ Gas $\rho = \rho(z)$

scale height relation

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} = e^{-\frac{gz}{R_L T_0}}$$



pressure distribution

Example: Balloon in atmosphere

typical values

$\Delta z = 10\ m$ change of density in Δz

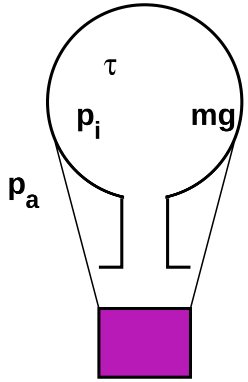
$T_0 = 290\ K$ $\Longleftrightarrow$ $\dfrac{\rho(z+dz)-\rho(z)}{\rho(z)} = e^{-\frac{g\Delta z}{R_L T_0}} - 1$

$R_L = 288\ \frac{Nm}{kg\,K}$ $\approx 1.2\ 0/00$

$\longrightarrow$ change of density across the height of the balloon is neglectible.

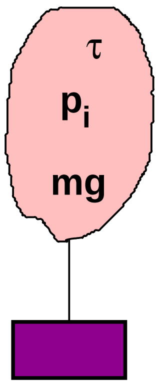
different envelopes of balloons

1.) rigid, open
(hot air balloon)



open $\longrightarrow$ $\boxed{p_i = p_a}$
rigid $\longrightarrow$ $\boxed{\tau = konst}$
open $\longrightarrow$ $\boxed{m \neq konst}$

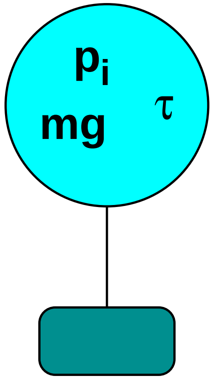
2.) perfectly loose
closed
(weather balloon)



no forces $\longrightarrow$ $\boxed{p_i = p_a}$
closed $\longrightarrow$ $\boxed{mg = konst}$
loose $\longrightarrow$ $\boxed{\tau \neq konst}$

different envelopes of balloons

3.) rigid
closed
(zeppelin)



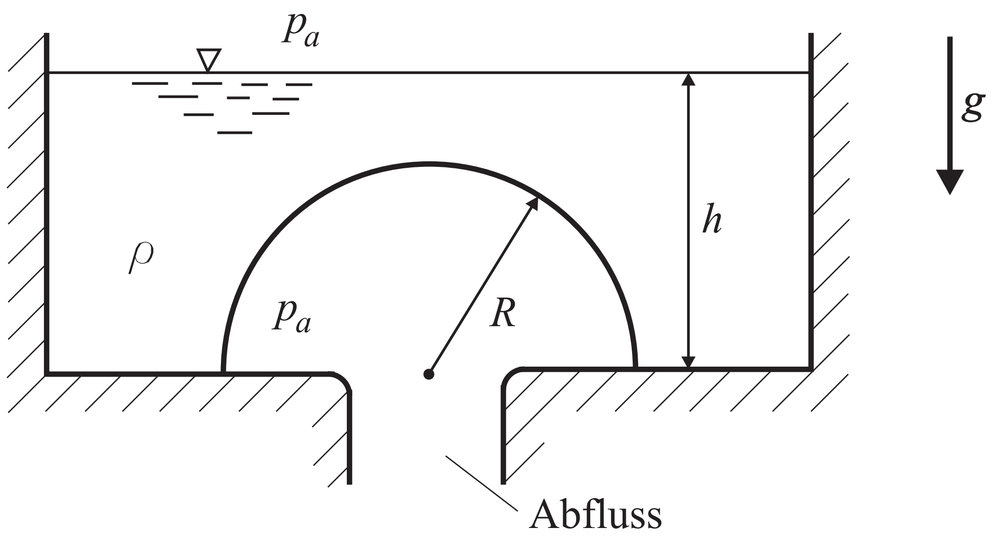
no pressure compensation $\longrightarrow$ $\boxed{p_i \neq p_a}$

closed $\longrightarrow$ $\boxed{mg = konst}$

no deformation $\longrightarrow$ $\boxed{\tau = konst}$

A container is filled with a fluid of the density ρ. The drain of the container, filled up to a height h, is closed with a hollow hemisphere (radius R, weight G).

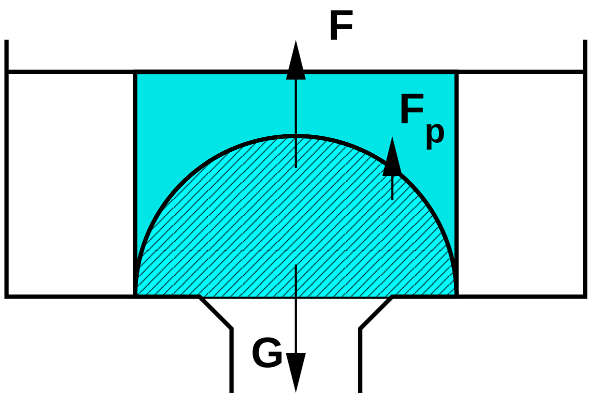
Given: $h,\ \rho,\ R,\ G,\ g$



Determine the necessary force F to open the drain.

Hint: Volume of a sphere: $V_k = \frac{4}{3}\pi R^3$

5.2



$$\Sigma F = 0$$
$$F - G + F_p = 0$$
$$F = G - F_p$$

$$F_p = V_{HK}\,\rho_w\,g - \rho_w\,g\,h A_{HK}$$

The hemisphere is not fully covered.

$$F_p = \frac{1}{2}\frac{4}{3}\pi R^3 \rho_w\,g - \rho_w\,g\,h\,\pi R^2$$

$$\longrightarrow F = G - \rho_w\,g\,\pi R^2 \left(\frac{2}{3}R - h\right)$$

Beispiel

A balloon with a rigid envelope has an opening for the pressure compensation with the surrounding The weight of the balloon without gas is G. Before launch the balloon is fixed with the force F_s.

$G = 1000N \quad ; \quad F_s = 1720N \quad ; \quad R = 287Nm/kgK \quad ; \quad T = 273K$
$g = 10m/s^2$

Compute the ceiling in isothermal atmosphere.

Beispiel

$$\left.\begin{aligned} F_A(z) &= G + G_{Gas}(z) \\ F_A(0) &= G + G_{Gas}(0) + F_s \end{aligned}\right\} \text{with} \quad G_{Gas} = \rho_G(z) g\tau = \frac{p_G(z)}{R_G T_G} g\tau$$

$$\begin{aligned} p_L(z) &= \rho_L(z) R_L T_L \\ p_L(0) &= \rho_L(0) R_L T_L \end{aligned} \rightarrow \frac{\rho_L(z)}{\rho_L(0)} = \frac{p_L(z)}{p_L(0)} = e^{-\dfrac{gz}{R_L T_L}}$$

$$\begin{aligned} p_G(z) &= \rho_G(z) R_G T_G \\ p_G(0) &= \rho_G(0) R_G T_G \end{aligned} \rightarrow \frac{\rho_G(z)}{\rho_G(0)} = \frac{p_G(z)}{p_G(0)} = \frac{p_L(z)}{p_L(0)} = e^{-\dfrac{gz}{R_L T_L}}$$

Beispiel

$$\rho_G(z) = \rho_G(0)e^{-\frac{gz}{R_L T_L}} \qquad \rho_L(z) = \rho_L(0)e^{-\frac{gz}{R_L T_L}}$$

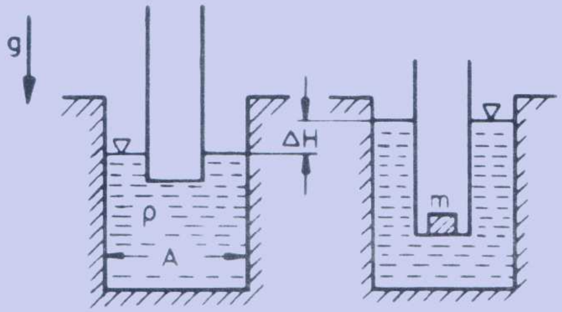
$$\rho_G(0) = \frac{\rho_L(0)\tau g - G - F_s}{\tau g}$$

$$\rho_L(0)\tau g e^{-\frac{gz}{R_L T_L}} = G + \frac{\rho_L(0)\tau g - G - F_s}{\tau g}\tau g e^{-\frac{gz}{R_L T_L}}$$

$$0 = G - (G + F_s)e^{-\frac{gz}{R_L T_L}} \quad ; \quad z = \frac{R_L T_L}{g}\ln\left(1 + \frac{F_s}{G}\right) = 7.84\,\text{km}$$

Beispiel

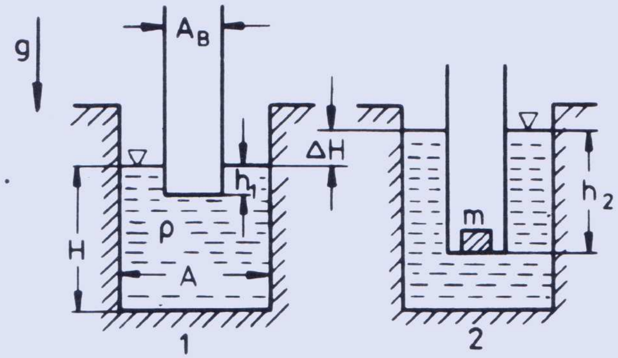
2.4 Ein zylindrisches Gefäß schwimmt in einem mit Wasser gefüllten zylindrischen Behälter. Durch Zuladen einer Masse m steigt der Wasserspiegel um ΔH.



Gegeben: ρ, A, m

Berechnen Sie die Höhendifferenz ΔH !

2.4



$$G_1 = F_{A1} = \rho A_B h_1 g$$

$$G_2 = F_{A2} = \rho A_B h_2 g$$

$$G_2 = G_1 + mg$$

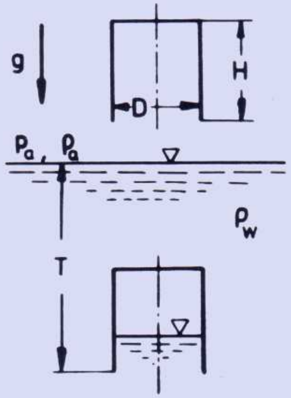
Das vom Wasser eingenommene Volumen bleibt konstant.

$$AH - A_B h_1 = A(H + \Delta H) - A_B h_2$$

$$\Delta H = \frac{m}{\rho A}$$

Beispiel

2.6 Eine Taucherglocke mit Gewicht G wird abgesenkt.



$D = 3$ m $\quad H = 3$ m $\quad T = 22$ m $\quad G = 8 \cdot 10^4$ N

$\rho_W = 10^3$ kg/m^3 $\quad \rho_a = 1{,}25$ kg/m^3 $\quad p_a = 10^5$ N/m^2

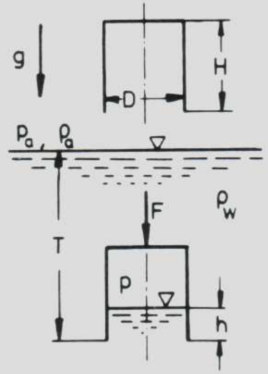
$g = 10$ m/s^2

a) Wie hoch steigt das Wasser in der Glocke bei gleichbleibender Temperatur?

b) Mit welcher Kraft (Größe und Richtung) muß die Glocke gehalten werden?

c) Bei welcher Eintauchtiefe wird die Haltekraft Null?

<u>2.6</u>



a) $p_a \frac{\pi}{4} D^2 H = p \frac{\pi}{4} D^2 (H-h)$

$p = p_a + \rho_W g \; (T-h)$

$$h = \frac{\rho_W g (T+H) + p_a}{2 \rho_W g} - \sqrt{\left(\frac{\rho_W g (T+H) + p_a}{2 \rho_W g}\right)^2 - T H} =$$

$= 2$ m

b) $F = F_A - G_{Luft} - G$

$= \frac{\pi}{4} D^2 g [(H-h) \rho_W - H \rho_a] - G = -9{,}58 \cdot 10^3$ N

c) $0 = F_A - G_{Luft} - G$

$$h_o = H\left(1 - \frac{\rho_a}{\rho_W}\right) - \frac{4G}{\pi D^2 \rho_W g}$$

$$T_o = h_o \left(1 + \frac{p_a}{\rho_W g (H - h_o)}\right) = 18{,}3 \text{ m}$$