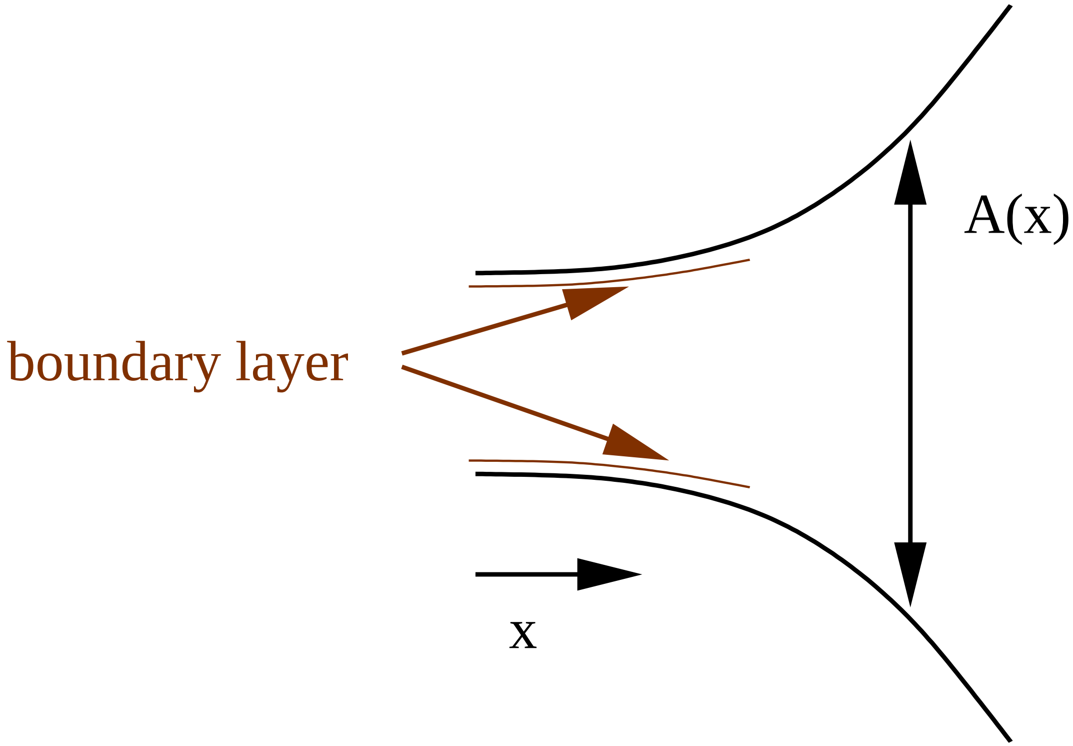
separated flow

A flow impressed by a positive pressure gradient can separate

i.e.: flow in a diffuser



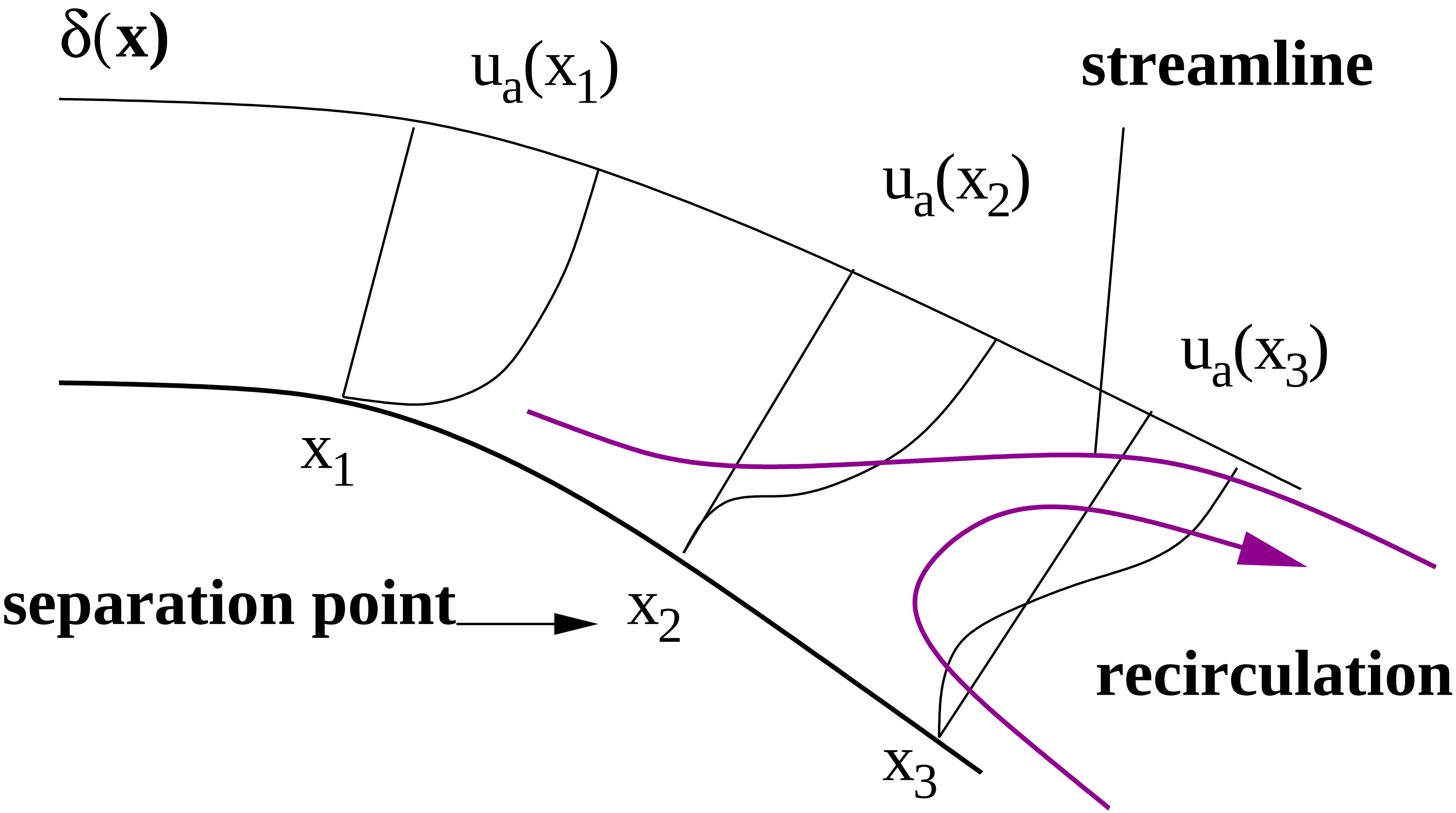
frictionless flow:

$$\rho u_a \frac{\partial u_a}{\partial x} = -\frac{\partial p}{\partial x}$$

$$\frac{\partial A(x)}{\partial x} > 0; \quad \frac{\partial u_a}{\partial x} < 0 \text{ conti}$$

$$\Rightarrow \frac{\partial p}{\partial x} > 0$$

separated flow



separated flow

the recirculation zone usually is larger than the boundary layer

$\rightarrow$ the friction force are no longer limited to a thin layer

$\rightarrow$ boundary layer approximation is no longer valid

The separation point still can be computed with the Kàrmàn-Pohlhausen method

Boundary conditions for the separation point

1. no-slip (Stokes) for $\frac{y}{\delta} = 0 \rightarrow u = v = 0 \qquad (u_B = u_w)$

2. boundary leayer edge $\frac{y}{\delta} = 1 \rightarrow u = U$

separated flow

3. pressure distribution is unknown $\rightarrow$ wall equation is not possible
   but the separation condition holds $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$

4. $\frac{y}{\delta} > 1 \rightarrow \frac{\partial u}{\partial y} = 0$
   continous transition from the boundary layer to the outer flow

5. $\frac{y}{\delta} = 1 \rightarrow \frac{\partial^2 u}{\partial y^2} = 0$
   frictionless flow

sequence of the separation

The pressure in the separated region cannot reach the value of the frictionless flow (circular cylinder)

⇒ Increase of drag (pressure drag)

Strong decrease of the lift for aerofoils (Stall)

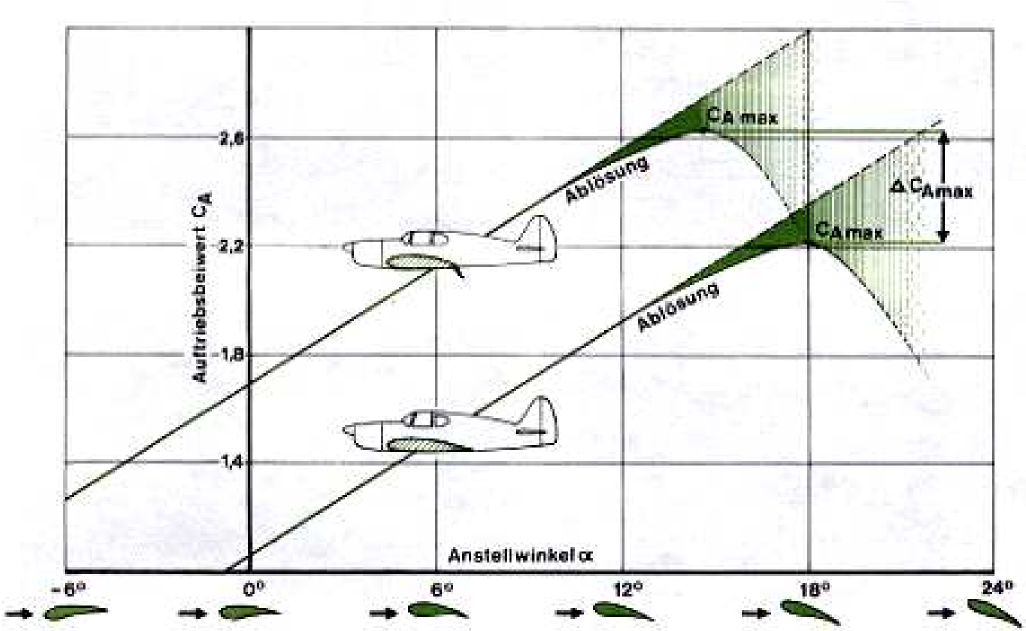


Bild 3: Bei Anderung des Anstellwinkels α verandert sich auch der Auftriebsbeiwert C_A

http://www.ultraleichtflugschule.de/auftrieb.html

prevention of separation

1. Enforcing of laminar-turbulent transition

- trip wire
- Roughness on the surface
- for airfoils: “Vortex Generator”

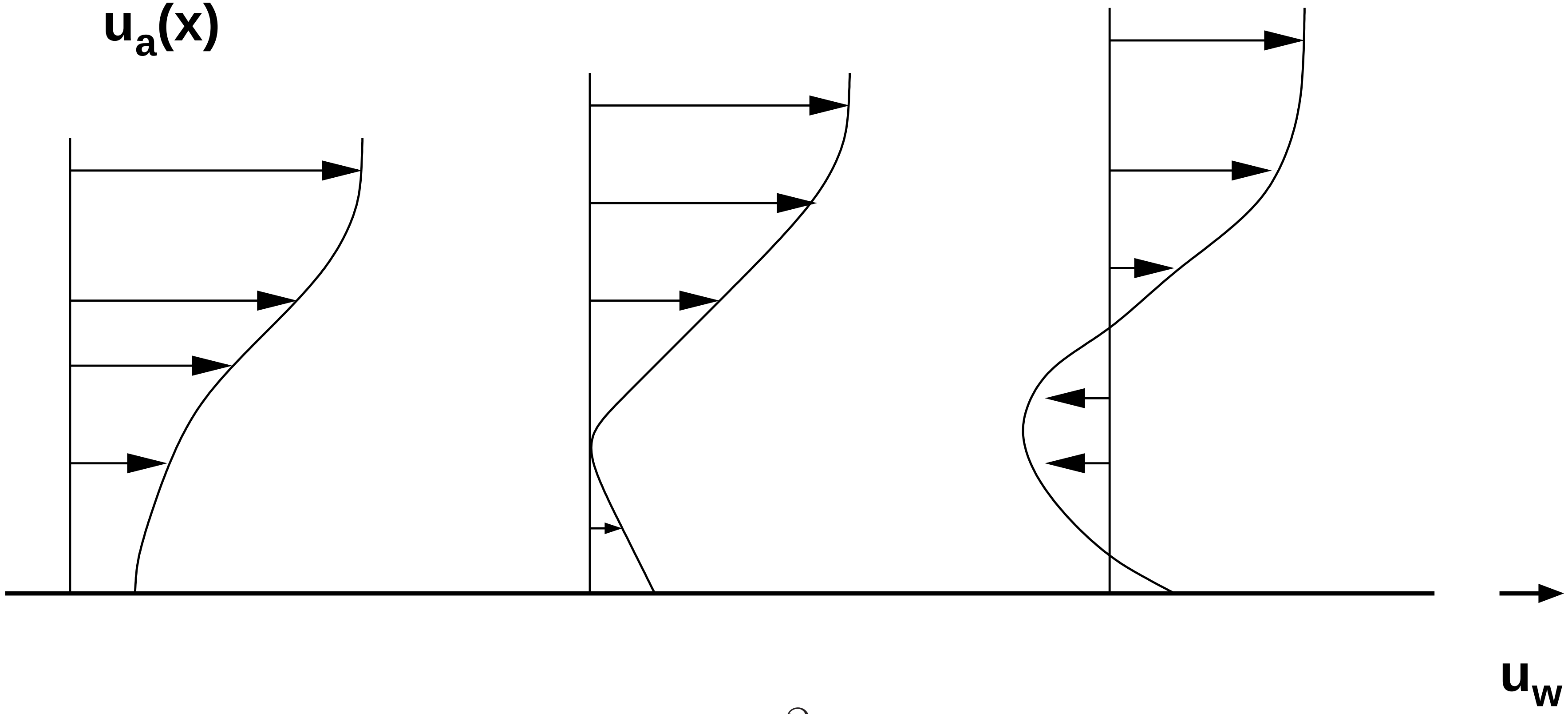
turbulent flow:
due to the mixing, more energy is the vicinity of the wall
⇒ a turbulent boundary layer can overcome a lerger pressure gradient than a laminar boundary layer

2. Moving wall

- avoids the generation of the boundary layer
- no velocity difference between wall and frictionless outer flow (no-slip)
- diffult to handle for curved bodies
- Experimental investigation of an aerofoil with an endless conveyor on the upper side:
  Angle of attack until ca. $\alpha = 55°$
  maximum lift coefficient: $c_a \approx 3.5$

2. moving wall

- different form of separation $u_w \neq u_a; \; \frac{\partial p}{\partial x} > 0$



criterion for separation $u = 0; \; \frac{\partial u}{\partial y} = 0$ not at the wall

suction of the boundary layer

A grid of flat plates is flown against with the velocity u_∞. A part of the boundary layer is sucked up. The sucking velocity v_A is such that the velocity at the boundary layer equals the incoming velocity. The flow is laminar.

Determine

a) the relation between v_A and δ_1

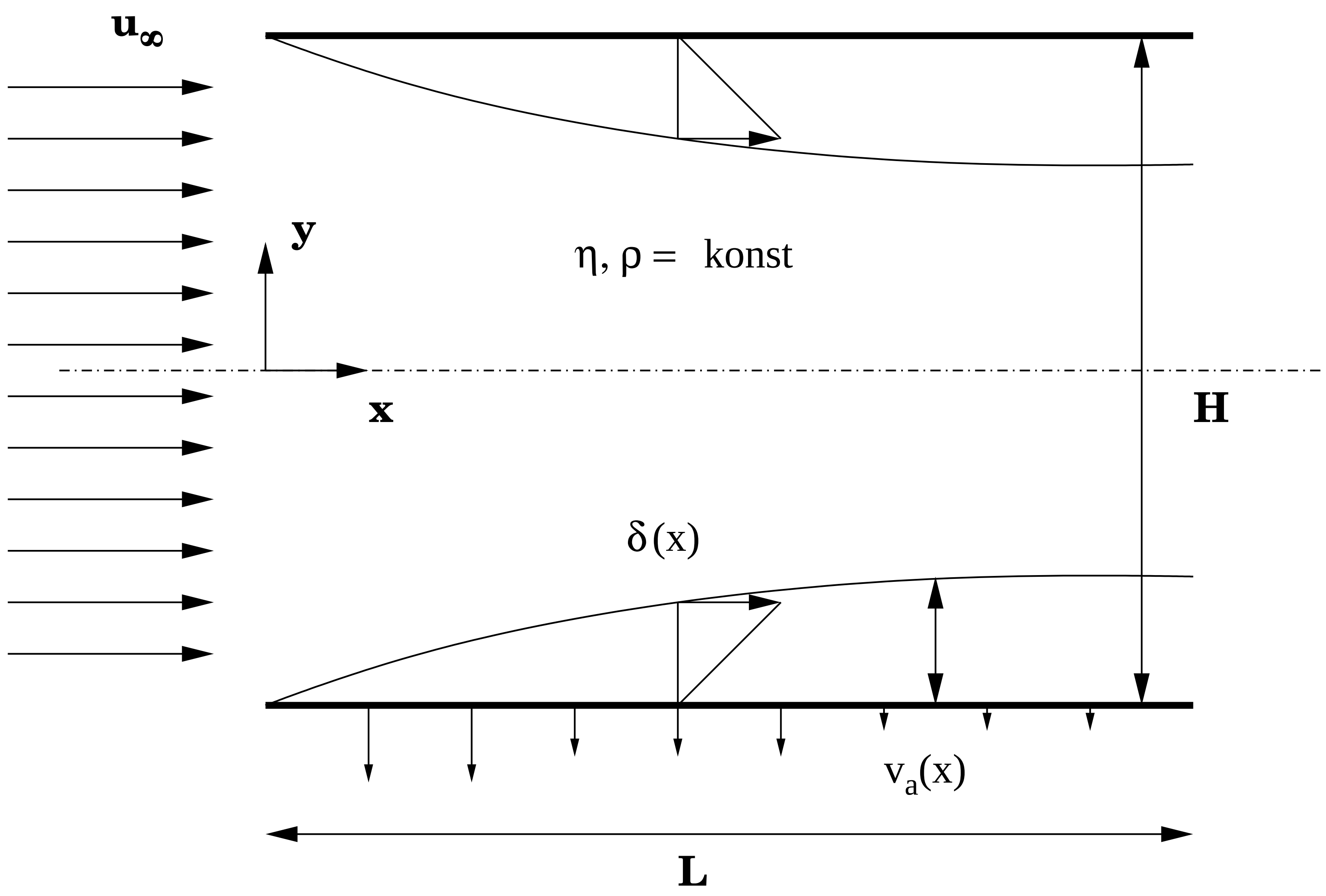
b) the distribution of the boundary layer thickness $\delta(x)$

c) the sucking velocity $v_A(x)$

d) the drag coefficient of one plate

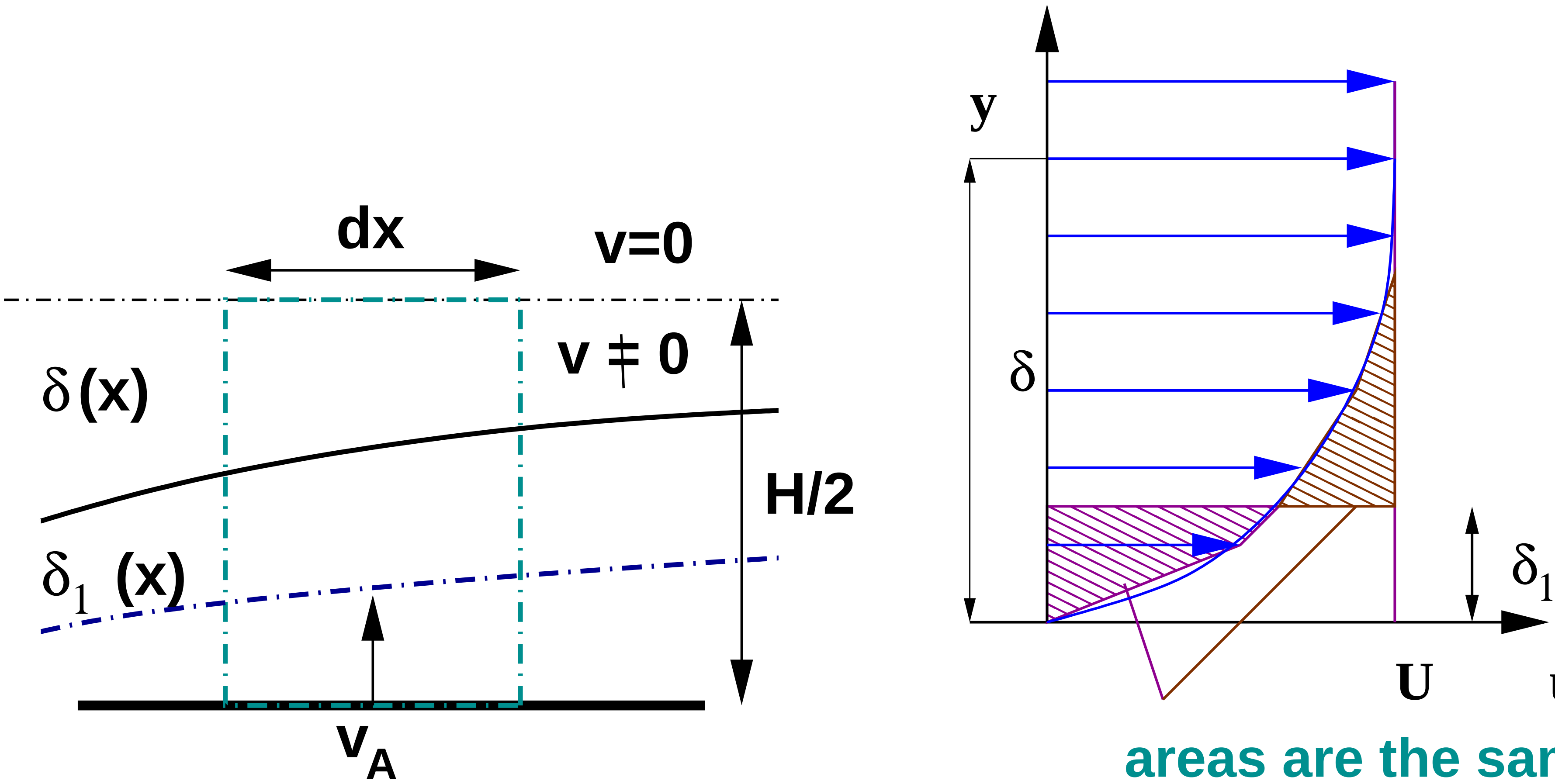
given: $\dfrac{u}{u_\infty} = \dfrac{y}{\delta};\; u_\infty;\; L;\; H;\; \eta;\; \rho$

suction of the boundary layer



suction of the boundary layer

a) balance for an infinitesimal element



areas are the same

Definition of δ_1

suction of the boundary layer

$$\dot{Q} = \int\limits_0^{H/2} u\,dy = u_\infty \int\limits_0^{H/2} \frac{u}{u_\infty}\,dy =$$

$$u_\infty \left[\frac{H}{2} - \left(\frac{H}{2} - \int\limits_0^{H/2} \frac{u}{u_\infty}\,dy\right)\right] =$$

$$= u_\infty \left(\frac{H}{2} - \int\limits_0^{H/2} 1 - \frac{u}{u_\infty} dy\right) =$$

$$= u_\infty \left(\frac{H}{2} - \delta_1\right)$$

suction of the boundary layer

volume balance for the element

$$u_\infty \left(\frac{H}{2} - \delta_1(x) \right) + v_A(x + \frac{dx}{2}) = u_\infty \left(\frac{H}{2} - \delta_1(x + dx) \right)$$

Remark: due to $v(x, H/2) = 0$: no volume flux across the symmetry plane $y = H/2$

$v_A(x), \delta_1(x)$ with Taylor expansion:

$$v_A(x + \frac{dx}{2}) = v_A(x) + \frac{dv_A}{dx} \frac{dx}{2} + \cdots$$

$$\delta_1(x + dx) = \delta_1(x) + \frac{d\delta_1}{dx} dx + \cdots$$

suction of the boundary layer

$\Longrightarrow$ introduce in volume balance

$$-u_\infty \delta_1 + v_A\, dx + \frac{dv_A}{dx}\frac{dx}{2}\, dx = -u_\infty \delta_1 - u_\infty \frac{d\delta_1}{dx}\, dx$$

$$\mathcal{O}(dx^2)$$

$$\Longrightarrow \qquad v_A = -u_\infty \frac{d\,\delta_1}{d\,x}$$

b) Distribution of the boundary layer $\delta(x)$
von Kàrmàn-Pohlhausen for boundary layer with suction
Integration von $v\frac{\partial u}{\partial y}$ from $y = 0$ to $y = \delta$

$$\frac{d\delta_2}{dx} + \frac{1}{u_\infty}\frac{du_\infty}{dx}(2\delta_2 + \delta_1) = -\frac{\tau(y=0)}{\rho u_\infty^2} + \frac{v_A(x)}{u_\infty}$$

$$\Rightarrow \frac{d\delta_2}{dx} - \frac{v_A}{u_\infty} = -\frac{\tau(y=0)}{\rho u_\infty^2} \Longrightarrow \frac{d\delta_2}{dx} + \frac{d\delta_1}{dx} = -\frac{\tau(y=0)}{\rho u_\infty^2}$$

suction of the boundary layer

Determination of $\delta_2(\delta)$ and $\delta_1(\delta)$

$$\delta_2 = \int\limits_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

Linear function for the velocity profile $\dfrac{u}{u_\infty} = \dfrac{y}{\delta}$

Transformation of the independent variable

$$\eta^\star = \frac{y}{\delta} \Rightarrow \frac{dy}{d\eta^\star} = \delta \Rightarrow dy \; = \; \delta \, d\eta^\star$$

$$\delta_2 = \int\limits_0^1 \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) \delta d\eta^\star$$

suction of the boundary layer

$$\Longrightarrow \frac{\delta_2}{\delta} = \int_0^1 \eta^\star - {\eta^\star}^2 \, d\eta^\star = \frac{1}{2}{\eta^\star}^2 - \frac{1}{3}{\eta^\star}^3 \Big|_0^1 = \frac{1}{6}$$

the same for δ_1

$$\delta_1 = \int_0^1 1 - \frac{u}{u_\infty} \, dy$$

$$\frac{\delta_1}{\delta} = \int_0^1 1 - \eta^\star \, d\eta^\star = \eta^\star - \frac{1}{2}{\eta^\star}^2 \Big|_0^1 = \frac{1}{2}$$

suction of the boundary layer

Determination of the $\tau(y=0)$:

in general
$$\frac{\tau(y=0)}{\rho u_\infty^2} = -\eta \frac{\partial u}{\partial y}\frac{1}{\rho u_\infty^2} = -\frac{\eta}{\rho u_\infty^2}\frac{\partial(u/u_\infty)}{\partial(y/\delta)}\frac{u_\infty}{\delta}$$

from Ansatz for the profile
$$\frac{\partial(u/u_\infty)}{\partial(y/\delta)} = 1 \Longrightarrow \frac{\tau(y=0)}{\rho u_\infty^2} = -\frac{\eta}{\rho u_\infty \delta}$$

Plug into Kàrmàn-Pohlhausen

$$\frac{d\delta_2}{dx} = \frac{d\delta_2}{d\delta}\frac{d\delta}{dx} = \frac{1}{6}\frac{d\delta}{dx}$$

$$\frac{d\delta_1}{dx} = \frac{d\delta_1}{d\delta}\frac{d\delta}{dx} = \frac{1}{2}\frac{d\delta}{dx}$$

suction of the boundary layer

$$\Longrightarrow \frac{1}{6}\frac{d\delta}{dx} + \frac{1}{2}\frac{d\delta}{dx} = \frac{\eta}{\rho\, u_\infty\, \delta}$$

$$\Rightarrow \frac{2}{3}\delta\, d\delta = \frac{\eta}{\rho\, u_\infty}\, dx \Longrightarrow \frac{1}{3}\delta^2 = \frac{\eta}{\rho\, u_\infty}\, x + C$$

initial condition for δ

$$x = 0 \Rightarrow \delta(x) = 0 \Longrightarrow C = 0$$

$$\Longrightarrow \delta(x) = \sqrt{3}\,\frac{\eta x}{\rho u_\infty} \Longrightarrow \frac{\delta}{x} = \sqrt{3}\,\frac{1}{\mathrm{Re}_x}$$

c) sucking velocity v_A

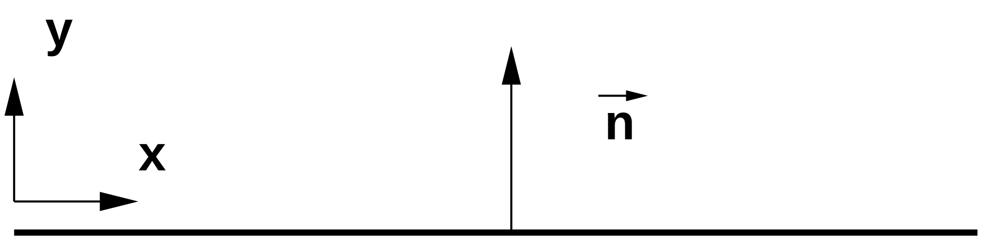
$$\text{from b)} \quad \frac{\delta_1}{\delta} = \frac{1}{2}$$

$$\Rightarrow \text{ from a) } v_A(x) = -u_\infty \frac{d\delta_1}{dx} = -u_\infty \frac{1}{2} \frac{d}{dx} \left(\sqrt{3} \frac{1}{\mathrm{Re}_x} \right)$$

$$= -u_\infty \frac{1}{2} \sqrt{\frac{3\eta}{\rho u_\infty}} \frac{1}{2\sqrt{x}} = -\frac{\sqrt{3}}{4} u_\infty \frac{1}{\sqrt{\mathrm{Re}_x}}$$

d) Definition of c_w

$$c_w = \frac{F_w}{\frac{1}{2}\rho u_\infty^2 \, L \, B}$$



$$F_p = \int -p\vec{n} \, dA \text{ und } \vec{n}_x = 0 \Longrightarrow F_{w_p} = 0$$

Pressure forces dont influence the drag.
The drag results from the friction forces on the surface.

suction of the boundary layer

$$F_W = \int_0^L (\tau_{w_o} + \tau_{w_u})\ B\ dx$$ B: Width of the plate

The flow is symmetric to the plate $\Rightarrow \tau_{w_o} = \tau_{w_u} = \tau_w$

$$\Rightarrow c_w = \frac{4}{L}\int_0^L \frac{\tau_w}{\rho u_\infty^2}\,dx = \frac{4}{L}\int_0^L \frac{\eta}{\rho u_\infty \delta}\,dx$$

δ from b) $\rightarrow$ $$\frac{4}{L}\sqrt{\frac{\eta}{\rho u_\infty 3}}\int_0^L \frac{1}{\sqrt{x}}\,dx = \frac{4}{L}\sqrt{\frac{\eta}{3\rho u_\infty}}\,2\sqrt{x}\Big|_0^L = \frac{8}{\sqrt{3}}\,\frac{1}{\sqrt{\mathrm{Re}_x}}$$