laminar boundary layer

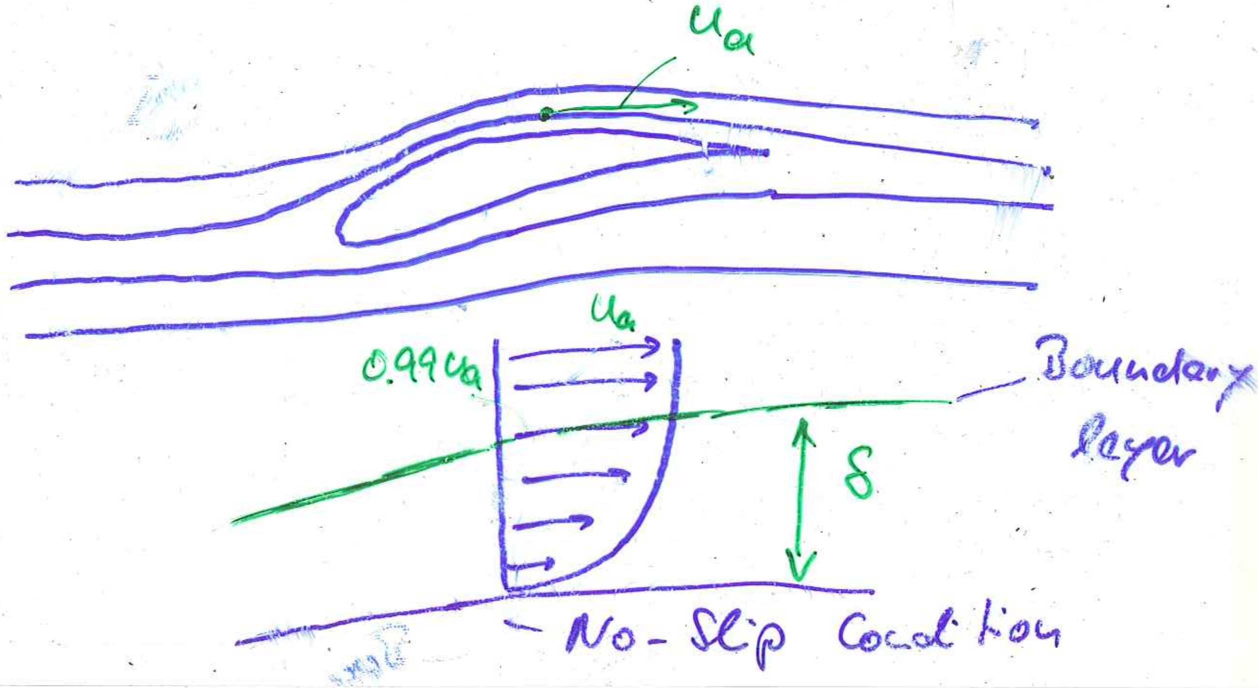
Why B.L. theory?

- With the potential theory only pressure distributions can be determined
  $\Rightarrow$ Lift force

- The drag force cannot be described with the potential theory. Viscous forces have to be considered.

The pressure distribution at slender bodies fits well with the theoretical distribution from the potential theory, if $Re \gg 1$. The influence of the viscous forces is limited to a thin layer near the wall $\rightarrow$ boundary layer

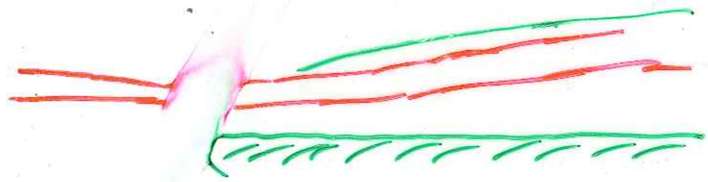
example:



• The segmentation of the flow into two parts (the frictionless outer part + the viscous boundary layer) allows a complete description of the flow field.

• The boundary layer theory is not valid in nose regions!

• Due to the deceleration in the boundary layer the stream lines are pushed away from the wall. The stream lines are no longer parallel to the wall.



• The line $\delta(x)$, describing the edge of the boundary layer (b.l. thickness) is not a stream line. It denotes the line where the velocity reaches the value of the outer flow up to a certain amount.
Usually 99%

$$\frac{u(y)}{u_a} = 0.99$$ arbitrary

Approximation of the boundary layer thickness

In the boundary layer:

$$O(\text{Inertia}) \approx O(\text{viscous forces})$$

$$\rho u \frac{\partial u}{\partial x} \approx \eta \frac{\partial^2 u}{\partial y^2}$$

dimensionless values $\rightarrow O(1)$

$$\bar{u} = \frac{u}{u_\infty} \qquad \bar{x} = \frac{x}{L} \qquad \bar{y} = \frac{y}{\delta}$$

$$\rightarrow \rho u_\infty \frac{u_\infty}{L} \approx \eta \frac{u_\infty}{\delta^2}$$

$$\rightarrow \delta \sim \frac{L}{\sqrt{\frac{u_\infty \rho L}{\eta}}} = \frac{L}{\sqrt{Re_L}} \qquad \boxed{\delta \sim \sqrt{L}}$$

Introduction of dimensionless variables in the Navier-Stokes equations for 2-d, steady, incompressible flows. Dimensionless variables are $O(1)$.

Neglect all terms with the factor $\underline{\underline{\frac{1}{Re}}}$ or smaller

$\rightarrow$ Boundary layer equations are valid for $Re \gg 1$

Continuity : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

x-mom : $\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\underbrace{\frac{\partial p}{\partial x}}_{= \frac{dp}{dx}} + \eta \frac{\partial^2 u}{\partial y^2}$

y-mom : $\frac{\partial p}{\partial y} = 0$

y-mom : The pressure is constant normal to the main stream direction. It is impressed from the frictionless outer flow.

Boundary layer edge for a flat plate ($y = \delta$)

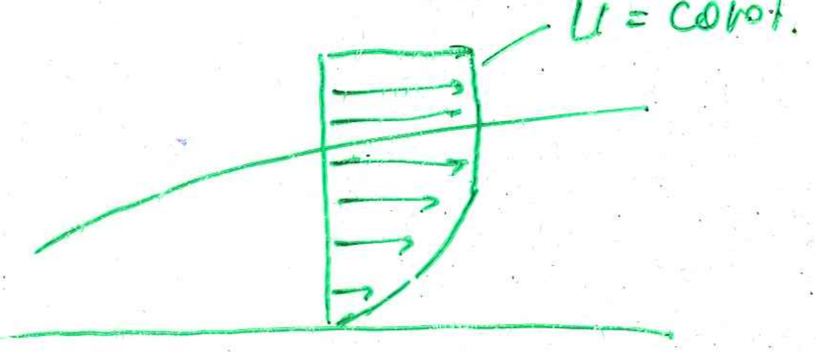
$u = U \Rightarrow \frac{\partial u}{\partial y} = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0$ frictionless outer flow field

x-mom : $\boxed{\rho U \frac{\partial U}{\partial x} = -\frac{\partial p}{\partial x}}$ Euler-equation for $y = \delta$

3 cases depending on the pressure gradient

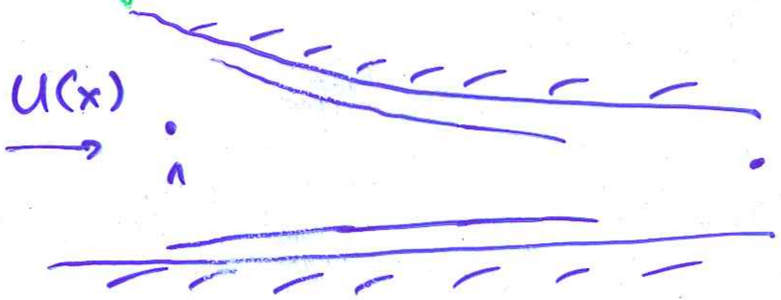
1. $\frac{\partial p}{\partial x} = 0 = \frac{\partial U}{\partial x} = 0 \Rightarrow U = \text{const.}$

flat plate boundary layer (Blasius)



2. $\frac{\partial p}{\partial x} < 0 \Rightarrow \frac{\partial u}{\partial x} > 0$ ≙ accelerated flow

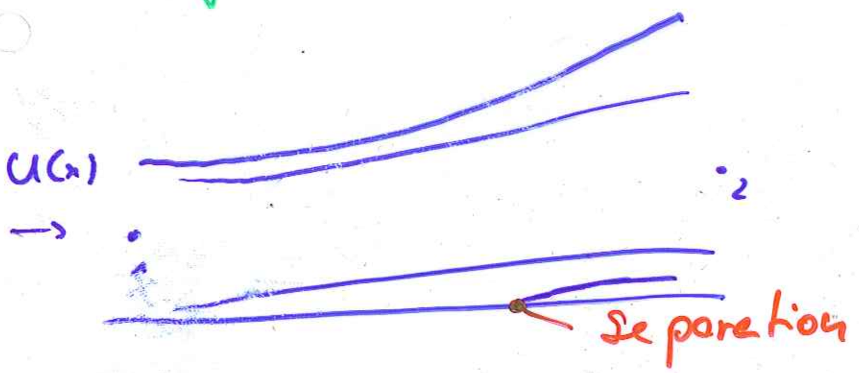
convergent channel (nozzle)



$u_1 < u_2$

3. $\frac{\partial p}{\partial x} > 0 \Rightarrow \frac{\partial u}{\partial x} < 0$ ≙ decelerated flow

divergent channel (diffusor)



$u_1 > u_2$

At the wall ($y=0$)

No-slip condition: $u = v = 0$

$\Rightarrow$ x-mom: $\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2}\Big|_{y=0}$ $\qquad \tau_w = \eta \frac{\partial u}{\partial y}$

$$\rightarrow \boxed{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}\Big|_{y=0}}$$

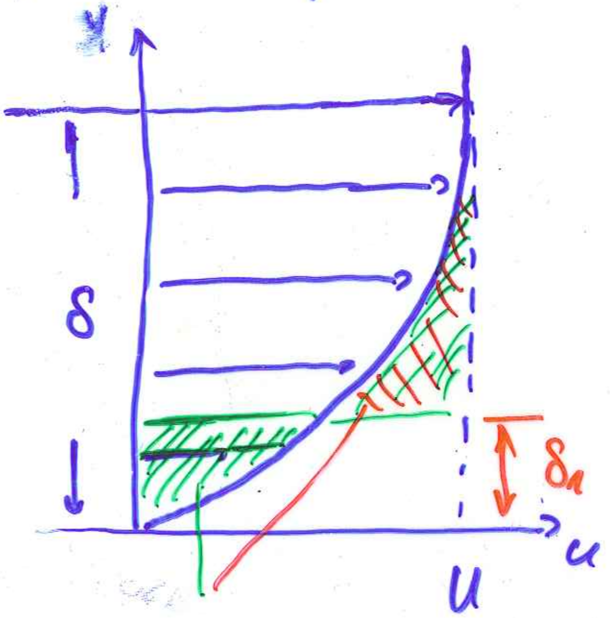
flat plate: $\frac{\partial p}{\partial x} = 0$ (no pressure gradient, U = const.)

$\rightarrow \frac{\partial^2 u}{\partial y^2}\Big|_{y=0} = 0$ $\qquad \left(\frac{\partial u}{\partial y}\Big|_{y=0} = \text{const}\right)$

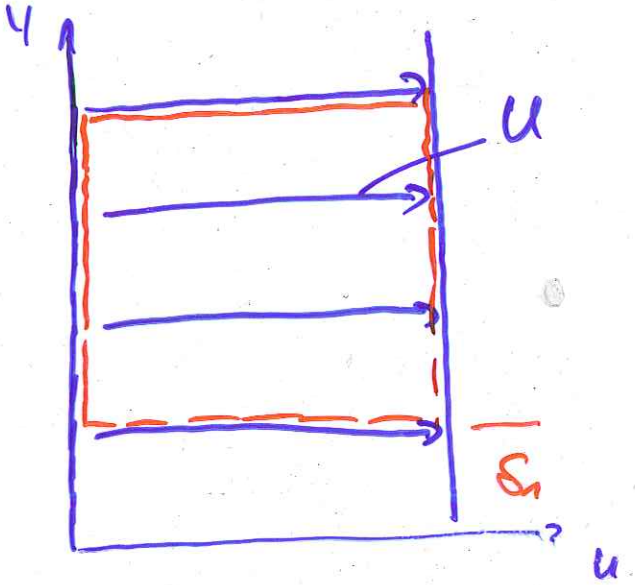
$\rightarrow$ no curvature of the velocity profile at the wall

Displacement- and momentum thickness

δ_1 (displacement-thickness): characteristic measure for the displacement of an undisturbed streamline



Areas are the same



$\frac{\text{massflux}}{\text{width}} = \text{const}$

δ_1 from $\dot{m} = \text{const.}$

$$U(\delta - \delta_1) = \int_0^\delta u\,dy \rightarrow \int_0^\delta U - u\,dy = U\delta_1$$

$$\rightarrow \boxed{\delta_1 = \int_0^\delta \left(1 - \frac{u(y)}{U}\right) dy}$$

in dimensionless form $\frac{\delta_1}{\delta} = \int_0^1 \left(1 - \frac{u(y)}{U}\right) d\frac{y}{\delta}$ $(\delta \neq f(y))$

Due to friction some momentum losses occur against the undisturbed flow.

From a momentum balance:

$$\boxed{\delta_2 = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$

$$\boxed{\frac{\delta_2}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right)}$$

To compute the drag, these two measures + the von Kármán-Integral-equation is used

Integration of the x-momentum equation

$$\frac{d}{dx}\left(U^2 \delta_2\right) + \delta_1 U \frac{dU}{dx} = \frac{\tau_w}{\rho}$$

or

$$\frac{d\delta_2}{dx} + \frac{1}{U}\frac{dU}{dx}\left(2\delta_2 + \delta_1\right) = \frac{\tau_w}{\rho U^2}$$

- Assume a polynomial for the velocity.
- Use the boundary conditions to compute the coefficients
- Compute δ_1 and δ_2
- Use the von Kármán-equation to compute $\tau_w(x)$ or $\delta(x)$

Assumption: polynomial profile

$$\frac{u(x,y)}{U(x)} = \sum_{i=0}^{n} a_i \left(\frac{y}{\delta}\right)^i = f\left(x, \frac{y}{\delta}\right)$$

selfsimilar profile $a_i(x)$, $\delta(x)$

boundary conditions

1.) no-slip condition (Stokes) for $\frac{y}{\delta} = 0 \Rightarrow u = v = 0$
($u_B = u_{\infty}$)

2.) B.L. edge $\frac{y}{\delta} = 1 - u = U$

3.) from x-momentum

$$\eta \frac{\partial^2 u}{\partial y^2}\Big|_{y=0} = \frac{\partial p}{\partial x} \quad \left(= 0 \text{ for a flat plate} \right)$$

$\frac{\partial p}{\partial x}$ from Euler-equation
Bernoulli

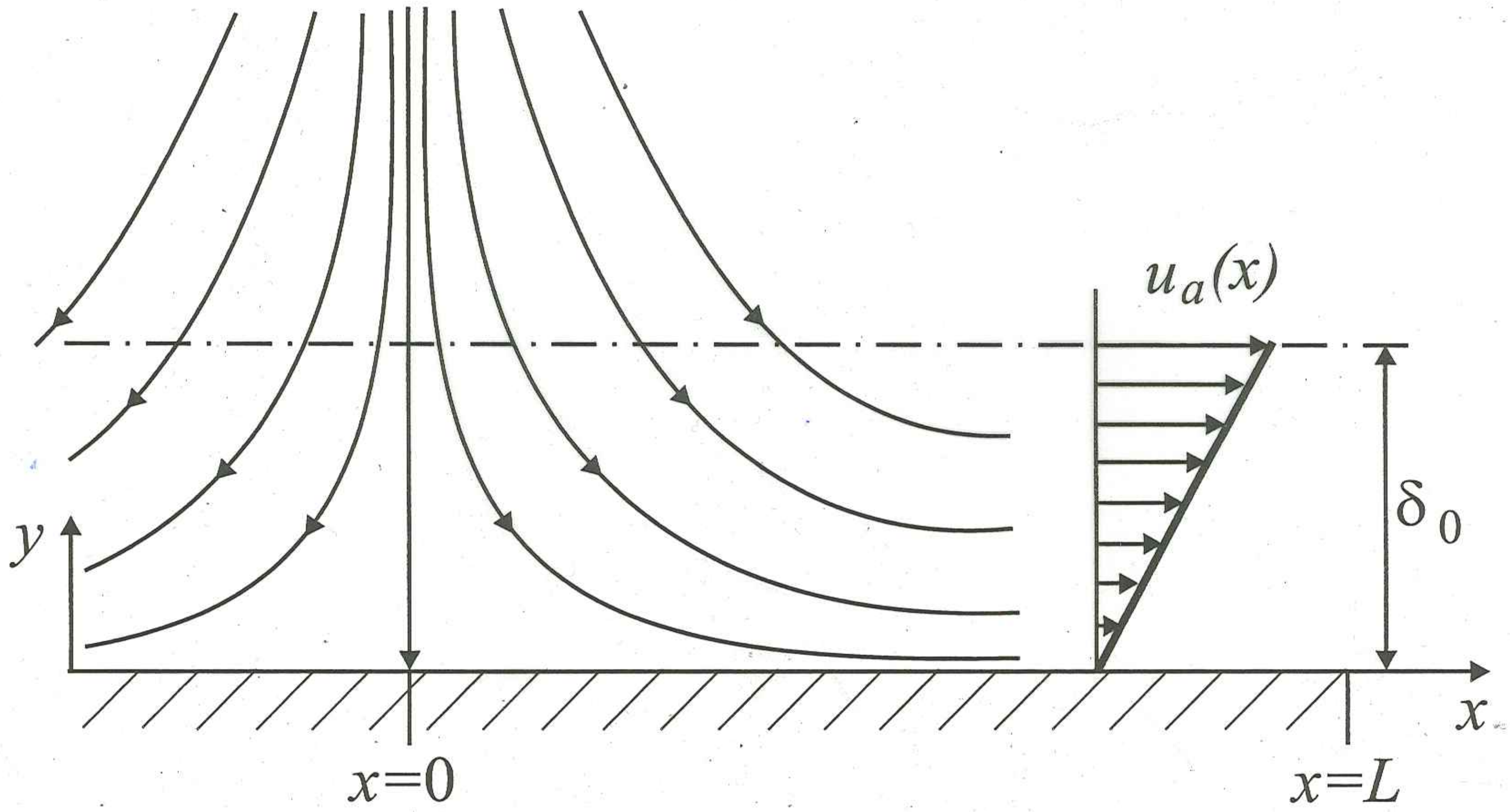
Only, if the degree of the polynomial is > 2, other
boundary conditions are necessary
from the continuity at the b.l. edge

4.) $\frac{y}{\delta} = 1 \rightarrow \frac{\partial u}{\partial y} = 0$

continuous transition from b.l to outer flow

5) $\frac{y}{\delta} = 1 \rightarrow \frac{\partial^2 u}{\partial y^2} = 0$

frictionless flow



$u_a(x)$
δ_0
y
x
$x=0$
$x=L$

15.8 In the stagnation point of a flat plate that is flown against normally to the outer flow $u_a(x)$ is accelerated in such a way that a constant boundary layer thickness δ_0 is generated. The velocity profile is assumed to be linear as a first approximation.

$$\frac{u(x,y)}{u_a(x)} = a_0 + a_1 \frac{y}{\delta_0}$$

Determine:

a) the constants $a_0,\ a_1$
b) the distribution of the outer velocity $u_a(x)$ using the von Kármán integral equation.
c) the tangential force that is applied between $x = 0$ and $x = L$ on the plate with the width B.

Given: $\delta_0,\ L,\ \eta,\ \rho,\ B$

von Kármán integral equation

$$\frac{d\delta_2}{dx} + \frac{1}{u_a}\frac{du_a}{dx}(2\delta_2 + \delta_1) = \frac{\tau_w}{\rho u_a^2}$$

[15.8] Remark: $u_a = U_a = U = u_e = \dots$ different notations

a) $a_0, a_1 = ?$

no-slip condition $u(y=0)=0 \rightarrow a_0 = 0$

at the b.l. edge $\left.\frac{u}{U}\right|_{y/\delta_0 = 1} = 1 \rightarrow a_1 = 1$

$$\Rightarrow \frac{u(x,y)}{U(x)} = \frac{y}{\delta_0}$$

b) displacement thickness: $\frac{\delta_1}{\delta_0} = \int_0^1 \left(1 - \frac{u}{U}\right) d\frac{y}{\delta}$

$$= \left[\frac{y}{\delta_0} - \frac{1}{2}\left(\frac{y}{\delta_0}\right)^2\right]_0^1 = \frac{1}{2} \rightarrow \delta_1 = \frac{1}{2}\delta_0$$

momentum thickness $\frac{\delta_2}{\delta_0} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\frac{y}{\delta}$

$$= \frac{1}{2}\left(\frac{y}{\delta_0}\right)^2 - \frac{1}{3}\left(\frac{y}{\delta}\right)^3 \Big|_0^1 = \frac{1}{6} \rightarrow \delta_2 = \frac{1}{6}\cdot\delta_0$$

$$\frac{\partial \delta_2}{\partial \delta_0} = \frac{1}{6} = \text{const} \rightarrow \frac{\partial \delta_2}{\partial x} = 0$$

wall shear stress: $\tau_w = \eta \left.\frac{\partial u}{\partial y}\right|_{y/\delta = 0}$

$$\tau_w = \eta \frac{U}{\delta_0} \frac{\partial(u/U)}{\partial(y/\delta_0)} = \eta \frac{U}{\delta_0}$$

von Kármán - equation

$$\frac{dU}{dx} = U \cdot \frac{1}{\rho U^2} \eta \frac{U}{\delta_0^2} \left(\frac{1}{2 \cdot \frac{1}{6} + \frac{1}{2}} \right) = \frac{\eta}{\rho \delta_0^2} \frac{6}{5}$$

$$U(x) = \frac{\eta}{\rho \delta_0^2} \frac{6}{5} x$$ (usually $\delta = \delta(x)$)

3) $$F = \int_0^L \tau(x) \, B \cdot dx$$

$$= \frac{6}{5} \frac{\eta}{\rho \delta_0^2} \frac{\eta}{\delta_0} B \int_0^L x \, dx$$

$$\boxed{F = \frac{3}{5} \frac{\eta^2}{\rho} \frac{B L^2}{\delta_0^3}}$$

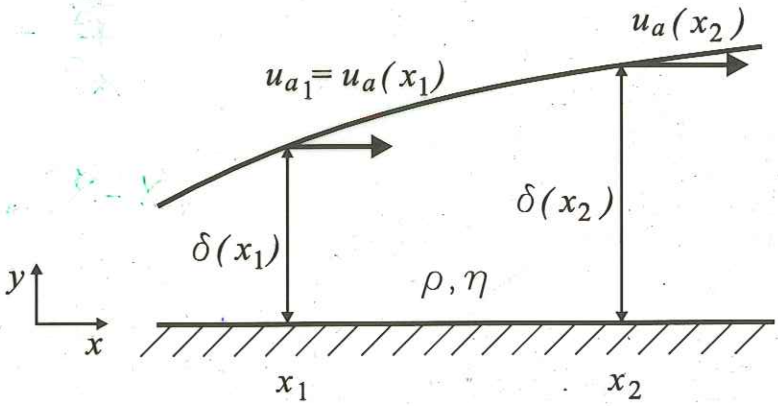
15.4 The velocity profile of a laminar incompressible boundary layer with constant viscosity η can be described with a polynomial:

$$\frac{u(x,y)}{u_a(x)} = a_0 + a_1(x)\left(\frac{y}{\delta}\right) + a_2(x)\left(\frac{y}{\delta}\right)^2 + a_3(x)\left(\frac{y}{\delta}\right)^3$$

The outer velocity $u_a(x)$ is given with the following approach:

$$u_a(x) = u_{a1} - C \cdot (x - x_1)^2$$

u_{a1} is the outer velocity at x_1 and C is a positive constant. The boundary layer thickness at x_2 is $\delta(x_2)$.



Given: $\rho,\ \eta,\ x_1,\ u_{a1},\ \delta(x_2),\ C,$ with: $C > 0$

Determine:

a) the pressure gradient $\partial p/\partial x$ in the flow as a function of x.
b) the coefficient a_0 and the coefficients $a_1(x),\ a_2(x),\ a_3(x)$.

[15.4]

a) $\frac{\partial p}{\partial x} = ?$

frictionless outer flow:

x-mom $\quad \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \eta\frac{\partial^2 u}{\partial y^2}$

at the b.l. edge: $\frac{\partial u}{\partial y} = 0$; $\frac{\partial^2 u}{\partial y} = 0$

$\Rightarrow \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} \qquad u = u_1 - c(x-x_1)^2$

$\Rightarrow \frac{\partial u}{\partial x} = -2c(x-x_1)$

$$\boxed{\Rightarrow \frac{\partial p}{\partial x} = 2c\rho\left[u_1(x-x_1) - c(x-x_1)^3\right]}$$

$\frac{\partial p}{\partial x}\Big|_{x=x_1} = 0$

b) 4 coefficients $\rightarrow$ 4 boundary conditions:

1.) no-slip condition : $\frac{y}{\delta} = 0 \rightarrow u = 0 \rightarrow a_0 = 0$

2.) b.l. edge : $\frac{y}{\delta} = 1$: $u = U$

$\Rightarrow \overset{\nearrow 0}{\cancel{a_0}} + a_1 + a_2 + a_3 = 1$

3.) at the wall: $\frac{y}{\delta} = 0 = \eta\frac{\partial^2 u}{\partial y}\Big|_{y=0} = \frac{\partial p}{\partial x}$

$$u = \left[a_1 \left(\frac{y}{\delta}\right) + a_2 \left(\frac{y}{\delta}\right)^2 + a_3 \left(\frac{y}{\delta}\right)^3\right]$$

$$\frac{\partial u}{\partial y} = \left(\frac{a_1}{\delta} + 2\frac{a_2}{\delta^2} y + 3 a_3 \left(\delta^{-3} y^2\right)\right) U$$

$$\frac{\partial^2 u}{\partial y^2} = \left(0 + 2\frac{a_2}{\delta^2} + 6 a_3 \frac{y}{\delta^3}\right) U$$

$$\frac{y}{\delta} = 0 \Rightarrow \frac{\partial p}{\partial x} = \eta\, 2 \frac{a_2}{\delta^2} U \Rightarrow a_2(x) = \frac{1}{2}\frac{\delta^2}{\eta U}\frac{\partial p}{\partial x}$$

$$a_2(x) = \frac{\delta^2}{2\eta}\,\frac{2 \rho C (x - x_1)(U - C(x-x_1)^2)}{U - C(x-x_1)^2}$$

$$\Rightarrow a_2(x) = \frac{\delta^2 \rho C}{\eta}(x - x_1)$$

4.) continous distribution at b.l. edge

$$\frac{y}{\delta} = 1 \Rightarrow \frac{\partial u}{\partial y} = 0 \Rightarrow \frac{a_1}{\delta} + 2\frac{a_2}{\delta} + 3\frac{a_3}{\delta} = 0$$

$$\rightarrow \begin{array}{l} a_1 + a_2 + a_3 = 1 \\ a_1 + 2a_2 + 3a_3 = 0 \end{array} \rightarrow \begin{array}{l} a_3 = -\frac{1}{2}(1 + a_2) \\ a_1 = \frac{1}{2}(3 - a_2) \end{array}$$

$$a_1(x) = \frac{3}{2} - \frac{1}{2}\frac{\delta^2 \rho C}{\eta}(x - x_1)$$

$$a_3(x) = -\frac{1}{2} - \frac{1}{2}\frac{\delta^2 \rho C}{\eta}(x - x_1)$$

16.1 A flat plate is flown against parallel to the surface with air.

$u_\infty = 45\ m/s \qquad \upsilon = 1,5 \cdot 10^{-5}\ m^2/s$

Determine

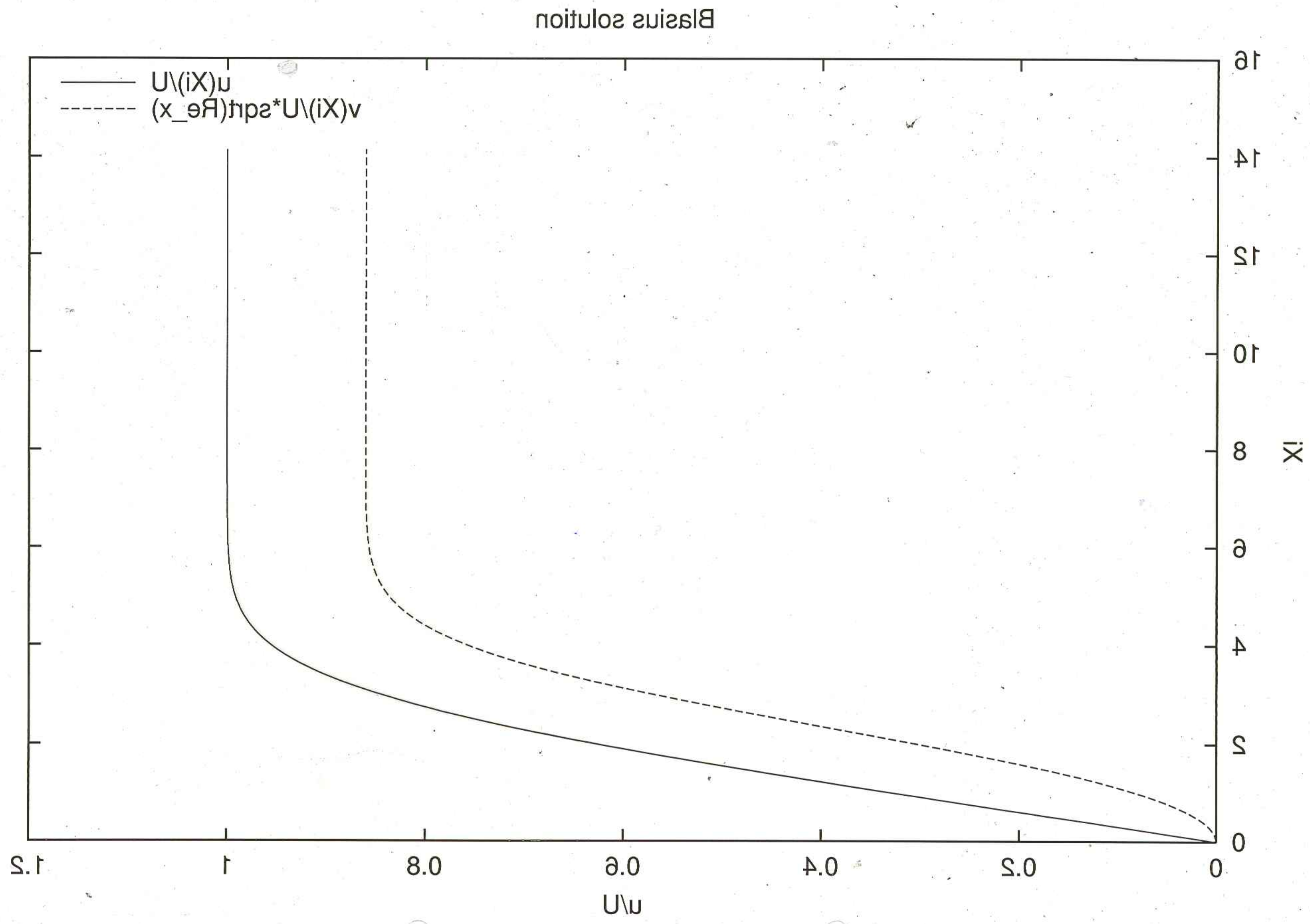
a) the transition point for $Re_{krit} = 5 \cdot 10^5$,

b) the velocity in point $x = 0,1\ m,\ y = 2 \cdot 10^{-4}\ m$ using the Blasius solution. What is the coordinte y with the same velocity at $x = 0,15\ m$

Sketch

c) the distribution of the boundary layer thickness $\delta(x)$ and a velocity profile for $x < x_{krit}$ and $x > x_{krit}$ respectively.

d) the wall shear stress as a function of x for $dp/dx < 0$, $dp/dx = 0$, and $dp/dx > 0$.



Blasius solution
u(Xi)/U
v(Xi)/U*sqrt(Re_x)
Xi
u/U
0
0.2
0.4
0.6
0.8
1
1.2
2
4
6
8
10
12
14
16

[16.1] flat plate $\Rightarrow$ Blasius

$u_\infty = 45\ m/s \qquad \nu = \frac{\eta}{\rho} = 1.5 \cdot 10^{-5}\ \frac{m}{s}$

a) $Re_{crit} = 5 \cdot 10^5 = \frac{\rho\, u_\infty\, x_{crit}}{\eta} = \frac{u_\infty\, x_{crit}}{\nu}$

$x_{crit} = Re_{crit} \cdot \frac{\nu}{u_\infty} = 0{,}167\ m$

b) $u(x = 0.1\,m\,;\ y = 2 \cdot 10^{-4}\,m) = ?$

$x = 0.1\,m < x_{crit} \Rightarrow$ laminar boundary layer

$\Rightarrow$ Blasius $\frac{u}{u_\infty} = g(\xi)$

$\xi = \frac{y}{\delta(x)}$ with $\delta(x) = \sqrt{\frac{\nu \cdot x}{u_\infty}}$

$\Rightarrow \frac{u}{u_\infty} = f\left(y \cdot \sqrt{\frac{u_\infty}{\nu x}}\right)$

$\Rightarrow$ similar profiles

$\xi = y \cdot \sqrt{\frac{u_\infty}{\nu x}} = \frac{y}{x} \sqrt{\frac{u_\infty x}{\nu}} = \frac{y}{x} \sqrt{Re_x}$

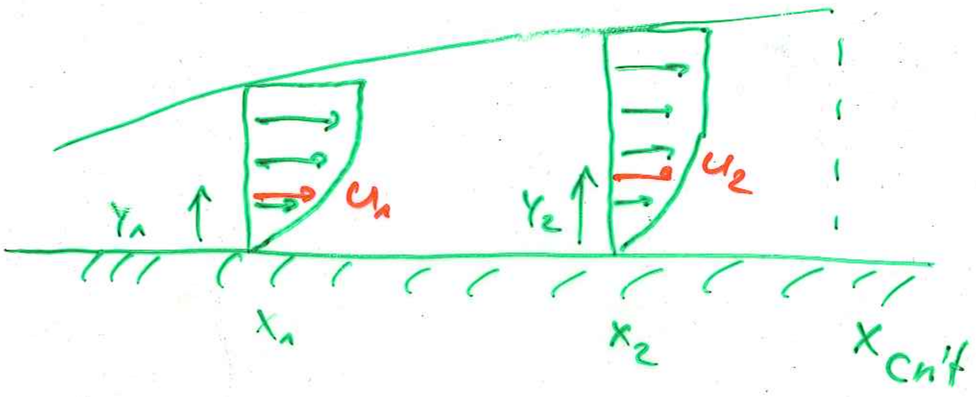
$x = 0.1\,m \rightarrow Re_x = 0.3 \cdot 10^6$

$y = 2 \cdot 10^{-4}\,m \rightarrow \xi = 1.095$

$\Rightarrow \frac{u}{u_\infty} = 0.36 \rightarrow u(x,y) = 16.2\ \frac{m}{s}$

$u(x = 0.15\,m;\ y = ?\ ) = u(0.1\,m;\ 2\cdot 10^{-4}\,m)$

$x = 0.15$ the flow is still laminar



$u_1 = u_2$

$$Re_2 = \frac{u_\infty x_2}{\nu} = 4.5\cdot 10^5$$

$$\frac{u_1}{u_\infty} = \frac{u_2}{u_\infty} \rightarrow \xi_1 = \xi_2 \overset{!}{=} 1.095 \text{ (selfsimilar solution)}$$

$$\xi = \frac{y}{x}\sqrt{Re_x} \rightarrow \boxed{y = \frac{\xi\cdot x}{\sqrt{Re_x}} = 2.45\cdot 10^{-4}\,m > y_1}$$

c) Sketch of $\delta(x)$

laminar : $o(\text{inertia}) = o(\text{friction})$

$$\Rightarrow \frac{\delta}{x} = o\left(\frac{1}{\sqrt{Re_x}}\right) \Rightarrow \boxed{\delta(x) \sim \sqrt{x}}$$

d) $\tau_w = f(x)$ for $\frac{dp}{dx} < 0$ $\quad \frac{dp}{dx} = 0$

$$\frac{\partial p}{\partial x} > 0$$

basically

- pressure gradient in the outer flow influences the velocity distribution of the boundary layer
- equilibrium of forces is determined by inertia-, pressure- and viscous forces
- at the boundary layer edge: inertia >> friction
- at the wall: friction >> inertia

pressure gradient (At the wall: $\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2}\Big|_{y=0}$)

$\frac{\partial p}{\partial x} > 0$: decelerated flow, seperation can occur
positive curvature at the wall
turning point in the profile between 0 and δ

$\frac{\partial p}{\partial x} = 0$ flat plate (asymptotic: $\tau \to \infty$)
turning point is at the wall

$\frac{\partial p}{\partial x} < 0$ accelerated ~~flow~~ flow; negative curvature for the whole profile

Boundary layer separation

A boundary layer can separate, if $\frac{\partial p}{\partial x} > 0$

x-momentum at the wall

$$\eta \frac{\partial^2 u}{\partial y^2}\Big|_{y=0} = \frac{\partial p}{\partial x}$$

relation between outer pressure gradient and wall shear stress

accelerated flow

$$\frac{\partial u}{\partial x} > 0 \Rightarrow \frac{\partial p}{\partial x} < 0 \Rightarrow \frac{\partial^2 u}{\partial y^2}\Big|_{y=0} < 0$$

$\Rightarrow$ curvature is negative

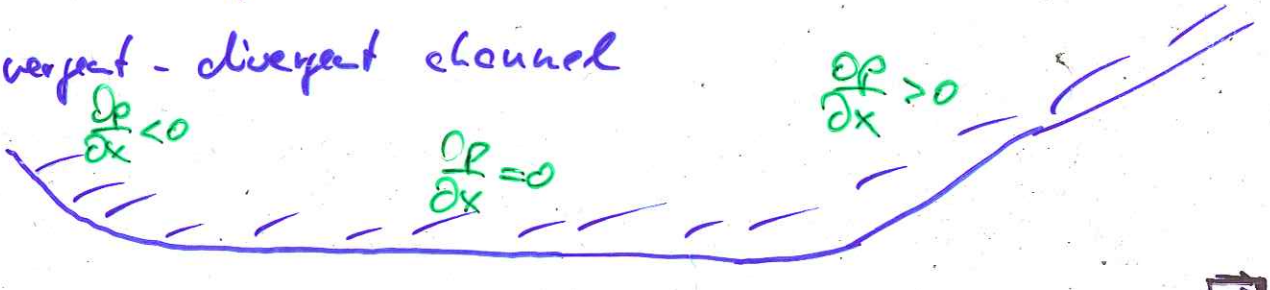
decelerated flow

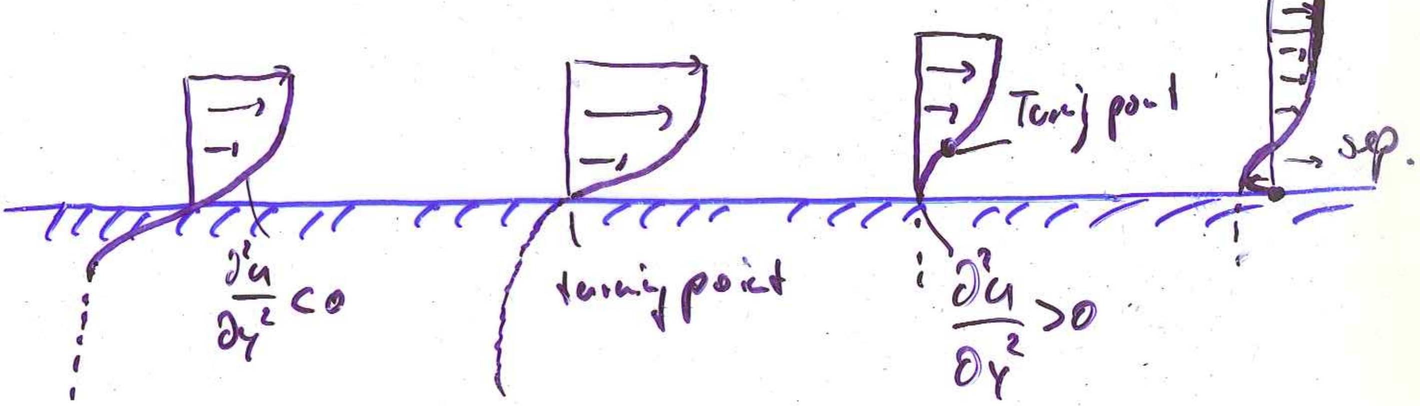
$$\frac{\partial u}{\partial x} < 0 \Rightarrow \frac{\partial p}{\partial x} > 0 \Rightarrow \frac{\partial^2 u}{\partial y^2}\Big|_{y=0} > 0$$

$\Rightarrow$ curvature is positive

$\Rightarrow$ backward flow is possible

convergent - divergent channel





1.) small acceleration · friction > pressure

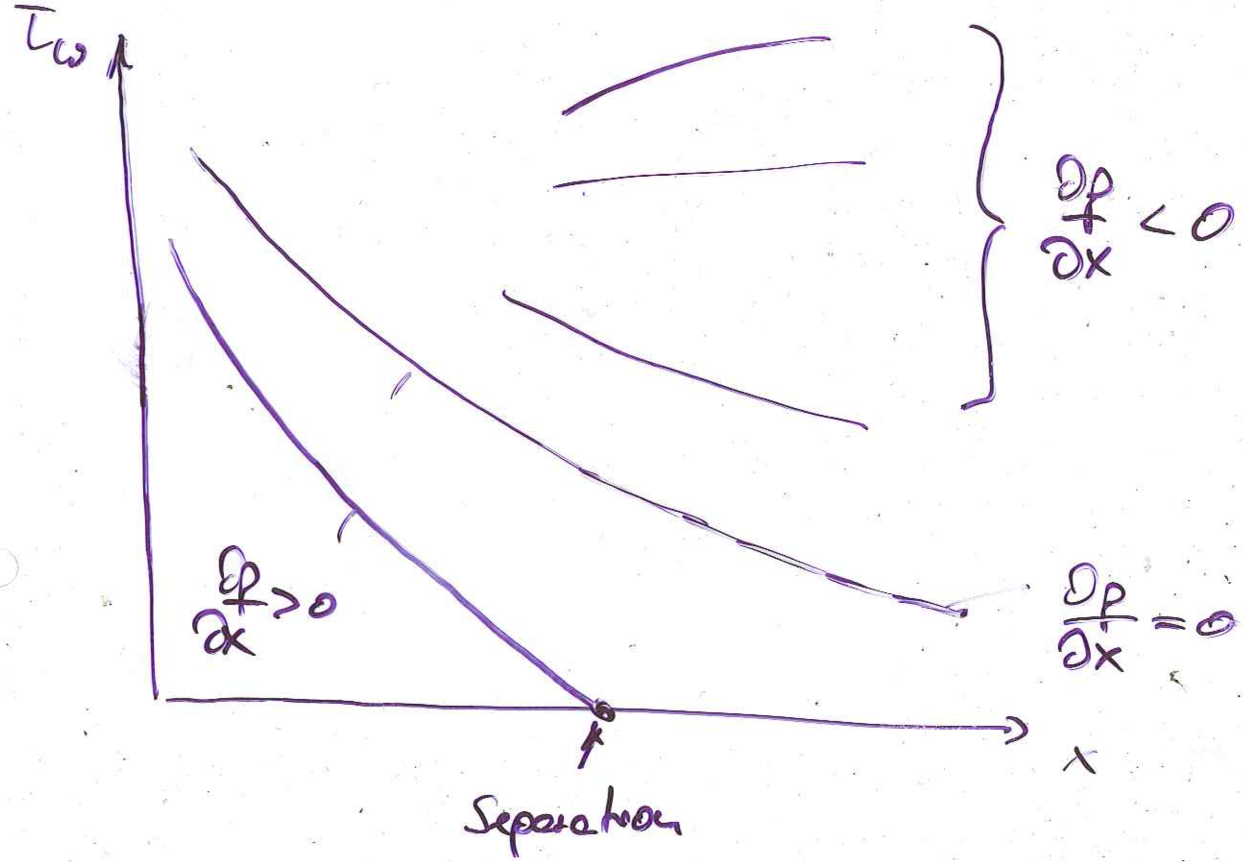
$\rightarrow \tau_w$ decreases $\rightarrow \dfrac{\partial \tau_w}{\partial x} < 0$

2.) equilibrium of friction and pressure

$$\rightarrow \frac{\partial \tau_w}{\partial x} = 0$$

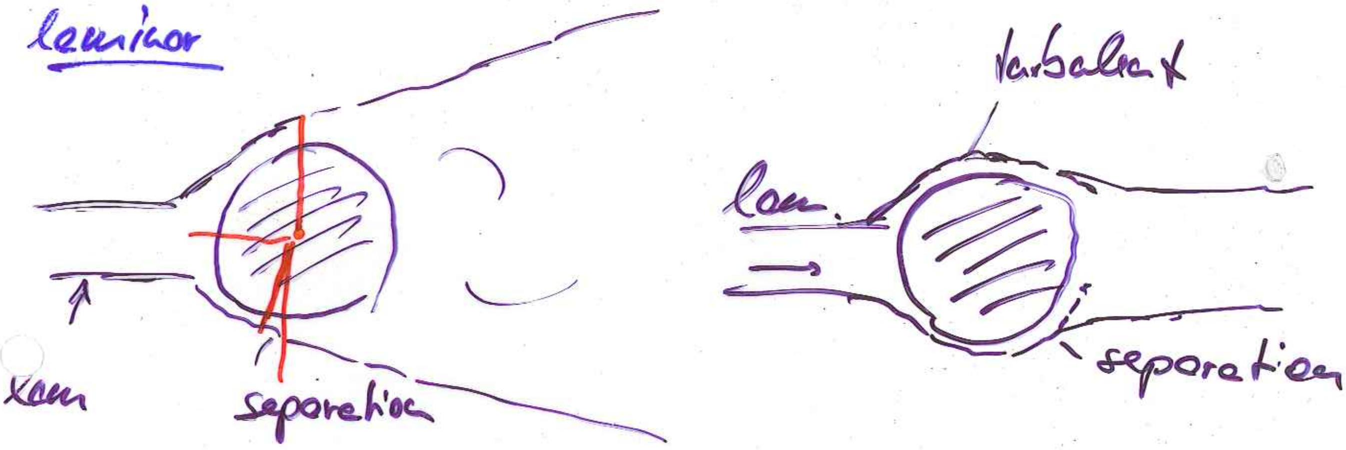
3.) pressure > friction

$\Rightarrow \tau_w$ increases $\rightarrow \dfrac{\partial \tau_w}{\partial x} > 0$



At the seperation point : $\tau_w = 0 \Rightarrow \frac{\partial u}{\partial y}\Big|_{y=0} = 0$

Influence of turbulence on the seperation (circular cylinder)



transition from laminar → turbulent

- kinetic energy in the wall region is larger (turbulent momentum exchange)
- boundary layer can override larger pressure gradients
- separation point is further downstream
- the wake becomes smaller
- pressure drag becomes smaller
- but : viscous drag increases

[17.1] divergent channel, polynomial of 3rd order

$$\frac{u(x,y)}{U(x)} = a_0(x) + a_1(x)\frac{y}{\delta(x)} + x \cdot a_2(x)\left(\frac{y}{\delta(x)}\right)^2 + a_3(x)\left(\frac{y}{\delta(x)}\right)^3$$

necessary: 4 coefficients
$\Rightarrow$ 4 boundary conditions

1.) no slip : $\frac{y}{\delta(x)} = 0 \Rightarrow u = 0 \Rightarrow a_0 = 0$

2.) b. l. edge : $\frac{y}{\delta(x)} = 1 \Rightarrow u = U$

$$1 = a_1(x) + x \cdot a_2 + a_3(x)$$

3.) pressure-wall-stress-relation
at the wall : $\frac{y}{\delta(x)} = 0 \rightarrow \eta \frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial x}$ unknown

$\Rightarrow$ look at the seperation : $x = x_a \rightarrow \tau_\omega = 0$

$$\tau_\omega = \eta \frac{\partial u}{\partial y}\Big|_{y=0} = \eta \frac{U}{\delta} \frac{\partial(u/U)}{\partial(y/\delta)}\Big|_{y/\delta=0} \Rightarrow \frac{\partial(u/U)}{\partial(y/\delta)} = 0$$

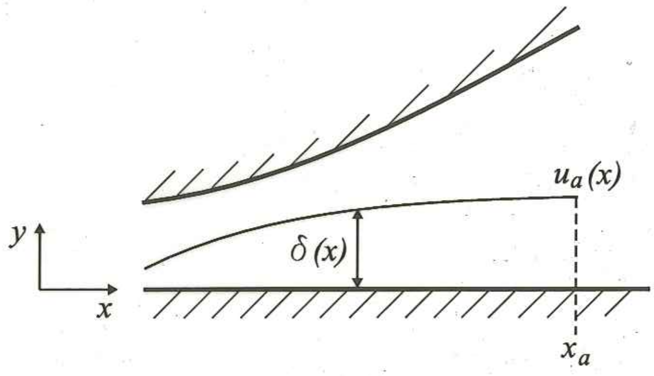
$$0 = a_1(x) + \underbrace{2x a_2 \left(\frac{y}{\delta}\right)}_{=0} + \underbrace{3 a_2(x)\left(\frac{y}{\delta}\right)^2}_{=0}$$

$\Rightarrow a_1(x) = 0$

17.1 The lower border of a divergent channel is formed by a flat plate. At $x = x_a$ the flow separates. The velocity profile is described with a polynomial of third order:

$$\frac{u(x,y)}{u_a(x)} = a_0(x) + a_1(x)\frac{y}{\delta(x)} + xa_2\left(\frac{y}{\delta(x)}\right)^2 + a_3(x)\left(\frac{y}{\delta(x)}\right)^3$$

Given: x_a



Determine the velocity profile $\dfrac{u(x_a, y/\delta(x_a))}{u_a(x_a)}$ at the separation point.
Sketch three velocity profiles for $x < x_a$; $x = x_a$; $x > x_a$.

4.) at the b.l. edge

$$\frac{y}{\delta} = 1 \Rightarrow \frac{\partial u}{\partial y}\Big|_{y/\delta = 1} = 0$$

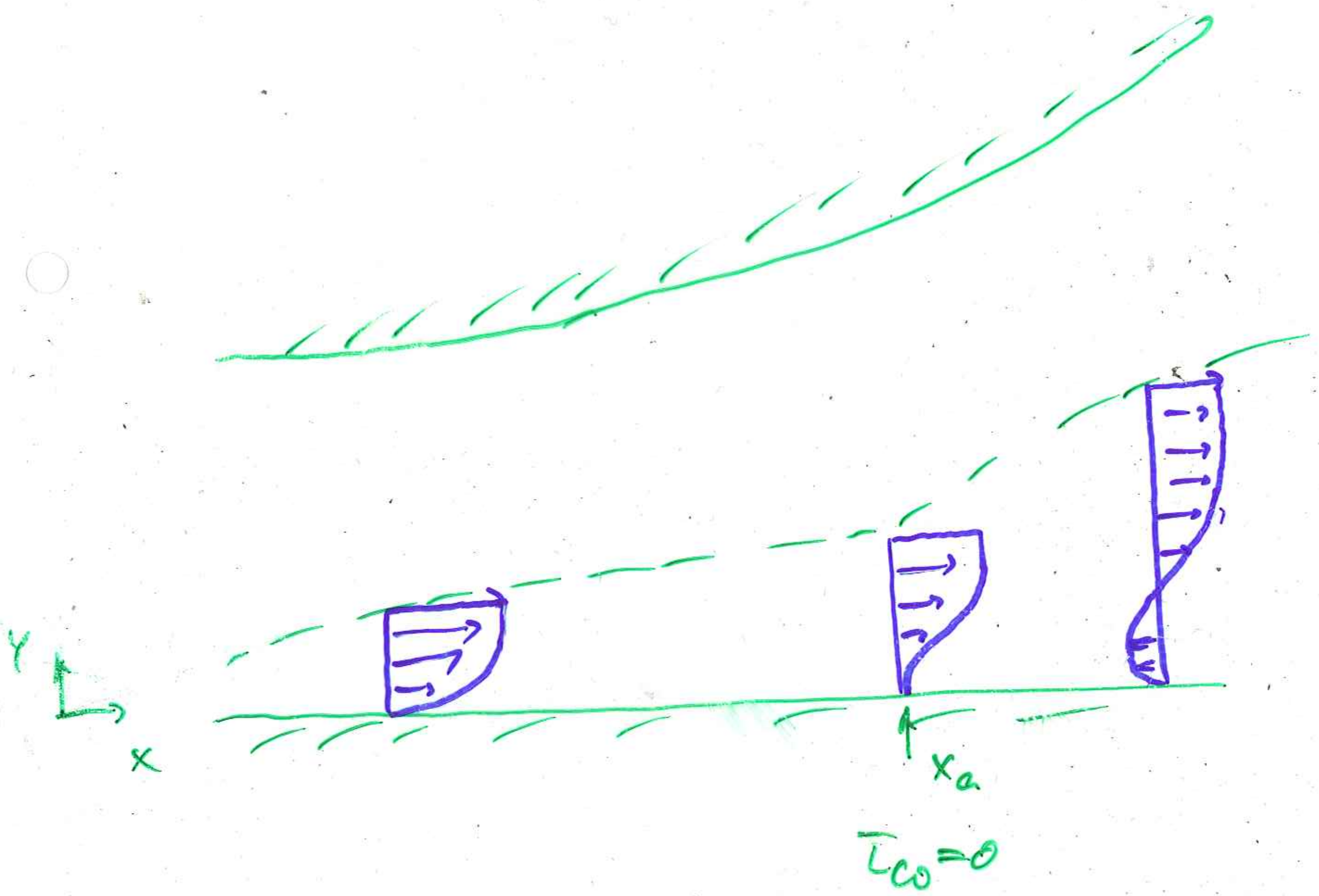
$$\Rightarrow 0 = 2x a_2 + 3a_3(x)$$

at the separation point $x = x_a$

$$0 = 2x_a \cdot a_2 + 3a_3(x_a) \rightarrow a_3(x_a) = -\frac{2}{3} a_2 x_a$$

$$a_2 = \frac{3}{x_a} \qquad a_3(x_a) = -2$$

$$\frac{u\left(x_a, \frac{y}{\delta(x_a)}\right)}{U(x_a)} = 3\left(\frac{y}{\delta(x_a)}\right)^2 - 2\left(\frac{y}{\delta(x_a)}\right)^3$$



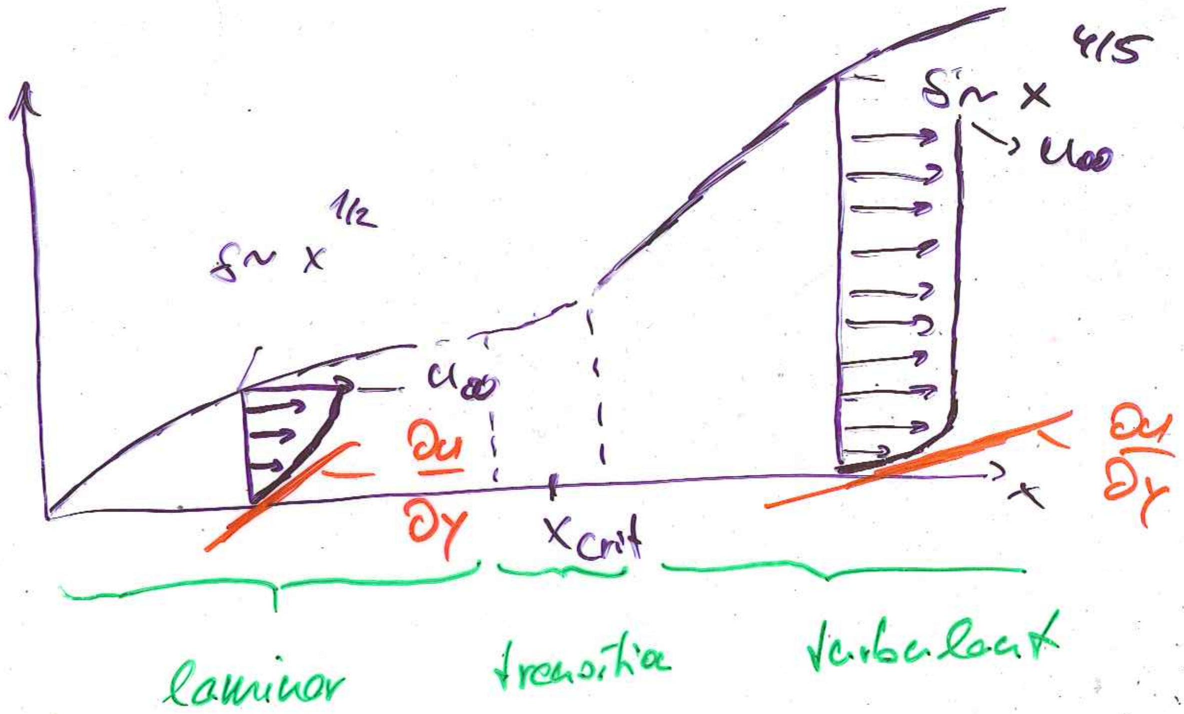
<u>turbulent</u>: high frequent oscillations

$$o(\text{inertia}) \neq o\left(\rho \frac{u_\infty}{L}\right)$$

from experimental data good approximation

$$\left(\frac{u}{u_\infty}\right) = \left(\frac{y}{\delta}\right)^{1/7} \Rightarrow \frac{\delta}{x} = \frac{0.37}{\sqrt[5]{Re_x}}$$

$$\boxed{\delta(x) \sim x^{4/5}}$$



$$\left.\frac{\partial u}{\partial y}\right|_{y=0}^{lam} < \left.\frac{\partial u}{\partial y}\right|_{y=0}^{turb}$$

$$|\tau_w|_{lam} < |\tau_w|_{turb}$$

Energy addition in the wall region

turbulent boundary layer over a flat plate

Approximation for the velocity profile

$$\frac{\bar{u}}{\bar{U}} = \left(\frac{y}{\delta}\right)^{1/7}$$

→ compute displacement thickness δ_1

momentum thickness δ_2

! $$\tau|_{wall} \neq \eta \frac{\partial u}{\partial y}\Big|_{y=0} = \infty$$

Assumption: flow over a flat plate is similar to a pipe flow

$$\Rightarrow \quad \lambda = \frac{0.316}{\sqrt[4]{Re}} \rightarrow \lambda = \frac{8\tau_w}{\rho \bar{u}_m^2} \rightarrow \tau_w$$

⇒ v. Kármán integral equation

$$\frac{\tau_w}{\rho U^2} = \frac{d\delta_2}{dx} \rightarrow \frac{\delta(x)}{x} = \frac{0.37}{\left(\frac{U \cdot x}{\nu}\right)^{1/5}} = \frac{0.37}{\sqrt[5]{Re_x}}$$

$$\delta_{turb} \sim x^{4/5}$$

$$\delta_{lam} \sim x^{1/2}$$