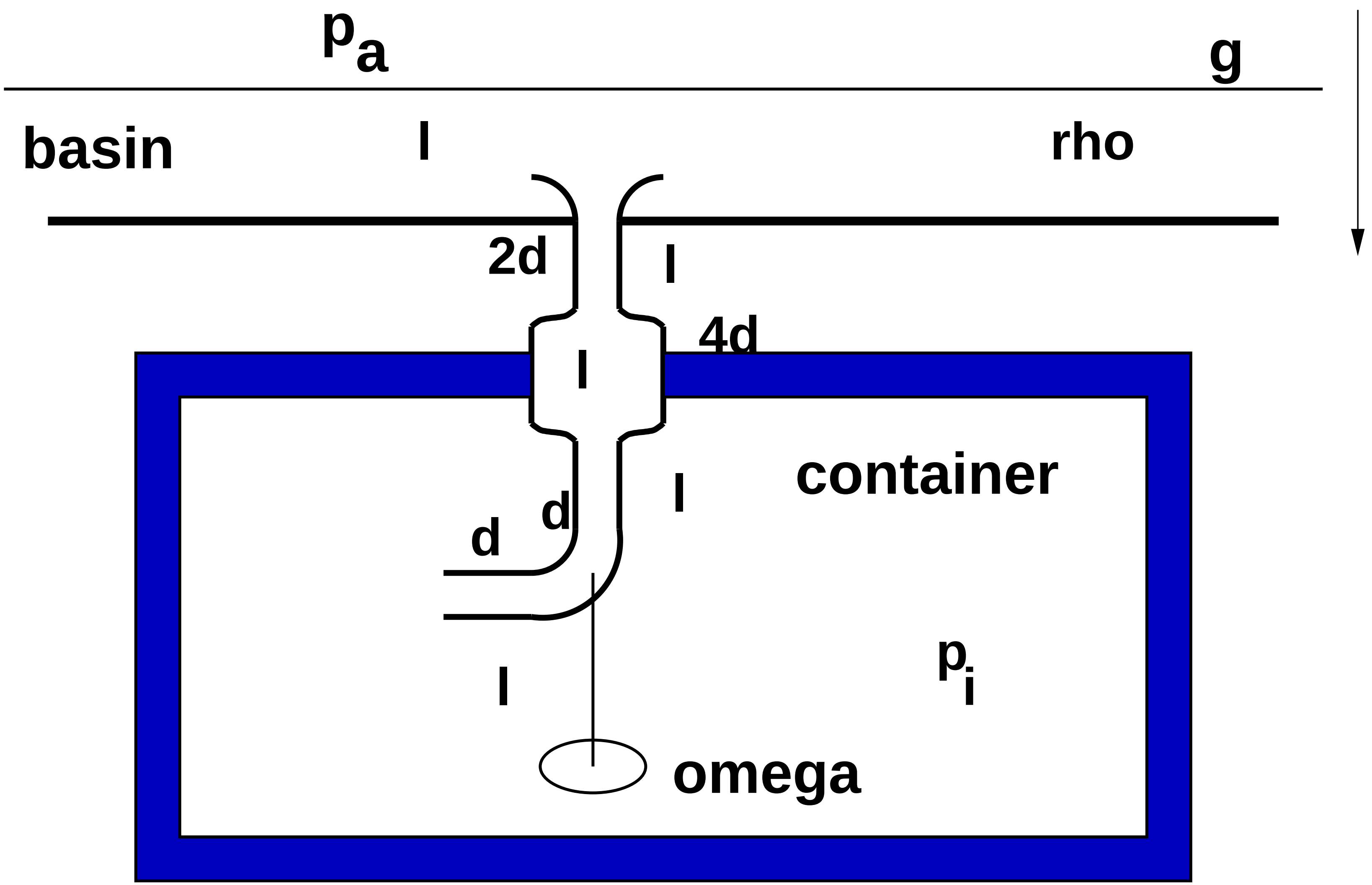
Example



A large basin is connected with a closed container.

Example

The inlet is well rounded. The connection consists of a rounded duct. Each part of duct with length l has a friction coefficient λ. The duct is moving at constant angular velocity ω.

At the angular velocity ω_0 the mass flux is zero.

a) Compute the container pressure p_i.

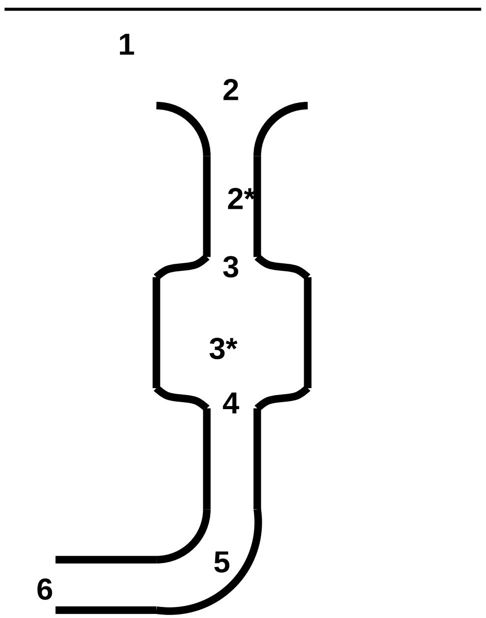
b) Compute the volume flux $\dot{Q}$ through the pipe for $\omega = 2 \cdot \omega_0$

c) Sketch the mechanical energies for a streamline beginning at the surface

Given: $l, d, g, \rho, \omega_0, p_a, \lambda$, Hint: $\dfrac{\partial p}{\partial r} = \rho \omega^2 r$

example

a) no mass flux at $\omega = \omega_0$



$$p_5 = p_a + 4\rho g l$$

$$p_6 = p_5 + \tfrac{1}{2}\rho(\omega_0 l)^2$$

$$p_i = p_6 = p_a + 4\rho g l + \tfrac{1}{2}\rho\omega_0^2 l^2$$

b) Bernoulli in rotating system [$\omega = 2\omega_0$]: “1“ $\longrightarrow$ “6“

$$p_a + 4\rho g l \;=\; p_i + \tfrac{1}{2}\rho v_6^2 - \tfrac{1}{2}\rho(2\omega_0 l)^2 +$$

$$+\tfrac{1}{2}\rho\left(v_2^{*2}\lambda\frac{l}{2d} + v_3^{*2}\lambda\frac{l}{4d} + v_6^{*2}\lambda\frac{2l}{d}\right)$$

example

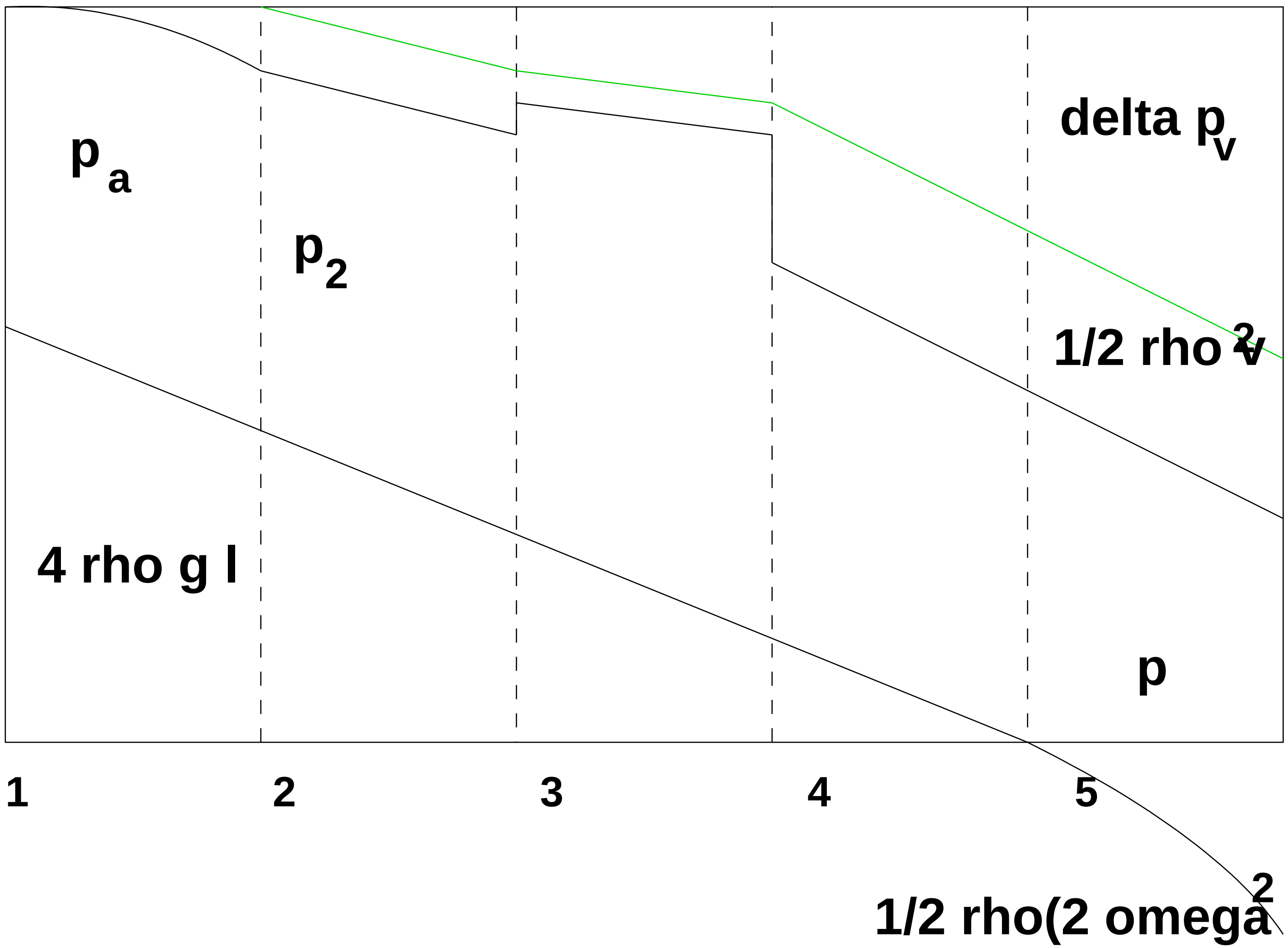
Continuity: $v_2^* = v_6(\frac{d}{2d})^2 = v_3^*(\frac{4d}{2d})^2$

$$v_2^* = \frac{1}{4}v_6 \; ; \; v_3^* = \frac{1}{16}v_6$$

$$0 = -\frac{3}{2}\rho\omega_0^2 l^2 + \frac{1}{2}\rho v_6^2 \left[1 + \lambda\frac{l}{d}(2 + \frac{1}{32} + \frac{1}{1024})\right]$$

$$\longrightarrow v_6 = \sqrt{\frac{3\omega_0^2 l^2}{1 + \lambda\frac{l}{d}\frac{2081}{1024}}} \; ; \; \dot{Q} = \frac{\pi d^2}{4}v_6$$

example



example

Water is flowing in the sketched configuration from a large container through a well rounded inlet into a pipe. At point 3 is a narrowing such that no water flows through through the thin pipe. At point 4 the water flows into the open air.

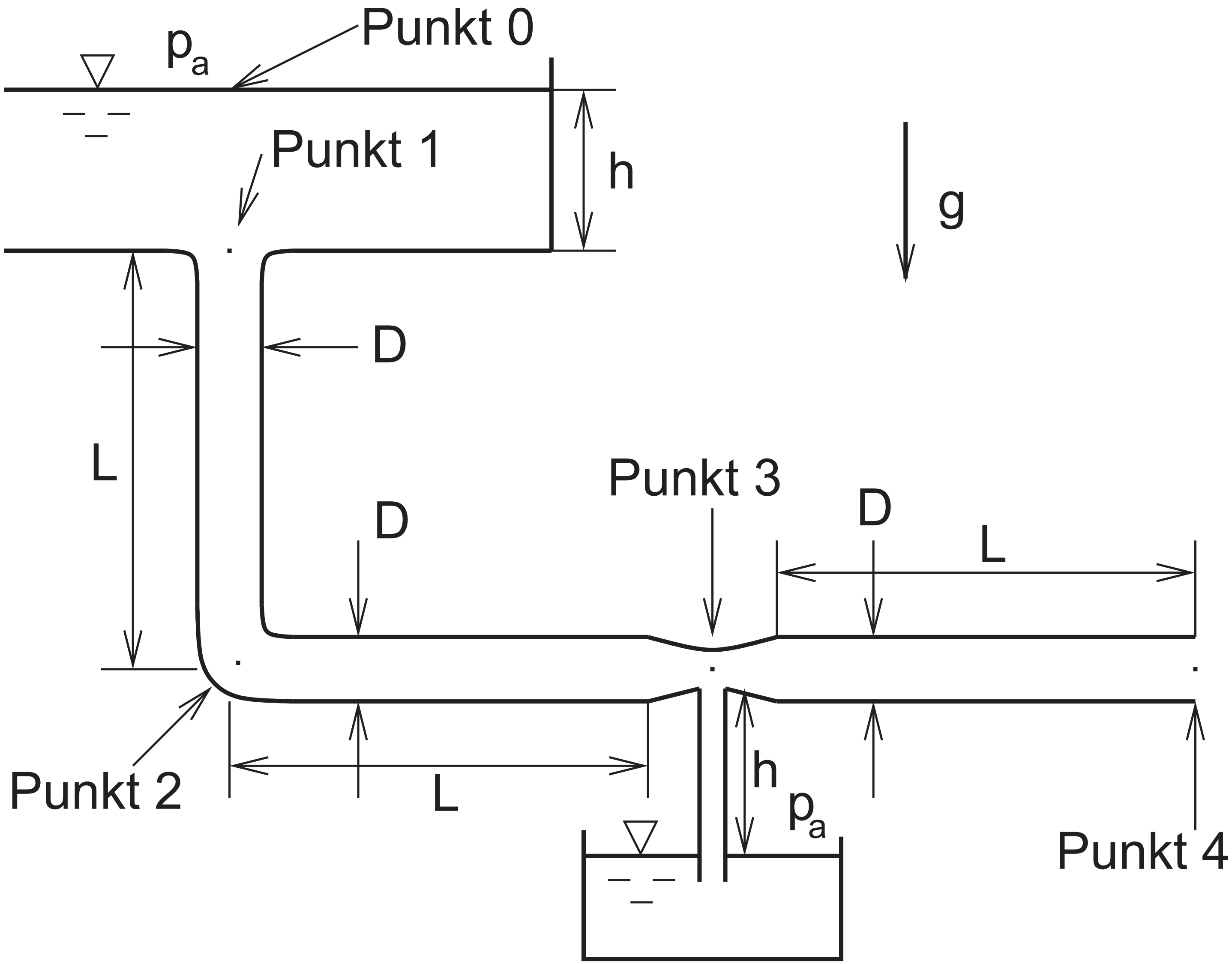
1. Determine the ratio A_4/A_3 for a loss-free flow.
2. Determine the ratio A_4/A_3 if the pipe loss coefficient is λ in all sections of length L. There are no other losses.

Given: L, h, D, λ

Hint:

- Check the units and signs of your results!

example



example

1. Conti:

$$v_3 = v_4 \frac{A_4}{A_3}$$

Bernoulli 0 $\rightarrow$ 4:

$$p_a + \varrho g(L+h) = p_4 + \frac{\varrho}{2} v_4^2$$

mit $p_4 = p_a$:

$$\Rightarrow v_4 = \sqrt{2g(L+h)}$$

Bernoulli 0 $\rightarrow$ 3:

$$p_a + \varrho g(L+h) = p_3 + \frac{\varrho}{2} v_3^2$$

no suction:

$$p_3 = p_a - \varrho g h$$

$$\Rightarrow \varrho g(L+h) = -\varrho g h + \frac{\varrho}{2} v_4^2 (\frac{A_4}{A_3})^2$$

$$\Rightarrow (\frac{A_4}{A_3})^2 = \frac{2g(L+2h)}{v_4^2} \Rightarrow \frac{A_4}{A_3} = \sqrt{\frac{L+2h}{L+h}} = f(L,h) \quad [-]$$

example

2. Bernoulli with losses 0 $\rightarrow$ 4:

$$p_a + \varrho g(L+h) = p_4 + \frac{\varrho}{2}v_4^2 + \Delta p_v$$

with $\Delta p_v = \frac{\varrho}{2}v_4^2\lambda\frac{3L}{D}$ and $p_4 = p_a$:

$$\Rightarrow \varrho g(L+h) = \frac{\varrho}{2}v_4^2(1+\lambda\frac{3L}{D})$$

$$v_4^2 = \frac{2g(L+h)}{1+\lambda\frac{3L}{D}}$$

Bernoulli with losses 0 $\rightarrow$ 3:

$$p_a + \varrho g(L+h) = p_3 + \frac{\varrho}{2}v_4^2(\frac{A_4}{A_3})^2 + \frac{\varrho}{2}v_4^2\lambda\frac{2L}{D}$$

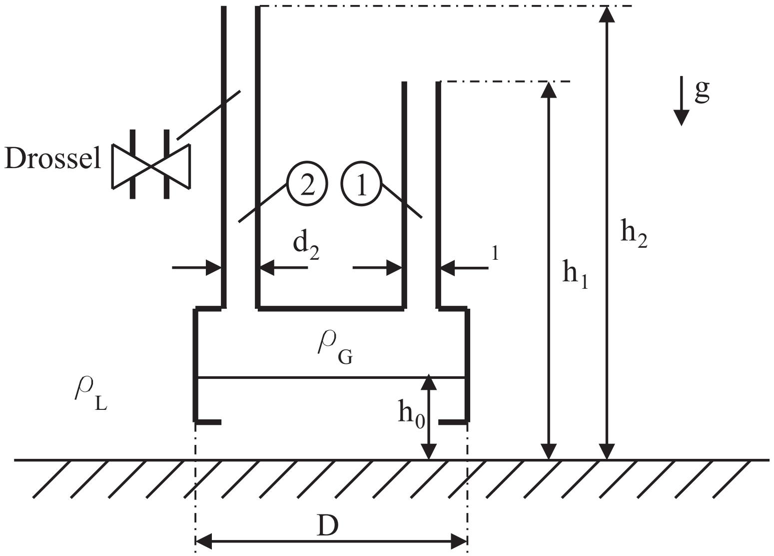
with $p_3 = p_a - \varrho g h$:

$$\Rightarrow 2g(L+2h) = v_4^2((\frac{A_4}{A_3})^2 + \lambda\frac{2L}{D})$$

$$\frac{A_4}{A_3} = \sqrt{\frac{2g(L+2h)}{v_4^2} - \lambda\frac{2L}{D}} \Rightarrow \frac{A_4}{A_3} = \sqrt{\frac{(L+2h)(1+\lambda\frac{3L}{D})}{(L+h)} - \lambda\frac{2L}{D}} \quad [-]$$

example

Exhaust gas (ρ_G = const) flows out of a large industrial furnace ($D \gg d_{1,2}$) stationary through two chimneys with the diameters d_1 and d_2 and the heights h_1 und h_2 into the atmosphere. The air coming from outside (ρ_L = const, $\rho_L > \rho_G$) stands up to the height h_0.



example

1. Determine the ratio d_1/d_2, when the volume fluxes in the two chimneys.
2. A choke with the loss coefficient ζ_{Dr} is mounted in chimney 2. Determine the loss coefficient ζ_{Dr} for the condition $d_1 = d_2$ and $\dot{Q}_1 = \dot{Q}_2$

Given:

$h_0, \quad h_1, \quad h_2$

Hint:

The only losses are in the choke.

example

1. Hydrostatic basic eq. and Bernoulli from 0 to k:

outside: $p_{a_0} = p_{a_k} + \rho_L g(h_k - h_0)$ with $k = 1, 2$

inside: $p_{a_0} = p_{a_k} + \frac{\rho_G}{2} v_{G_k}^2 + \rho_G g(h_k - h_0)$

$$\rho_L g(h_k - h_0) = \frac{\rho_G}{2} v_{G_k}^2 + \rho_G g(h_k - h_0)$$

$$v_{G_k} = \sqrt{2g(h_k - h_0)(\frac{\rho_L}{\rho_G} - 1)}$$

Conti: $v_{G_2}\frac{\pi d_2^2}{4} = v_{G_1}\frac{\pi d_1^2}{4}$

$$\frac{v_{G_1}}{v_{G_2}} = \frac{d_2^2}{d_1^2} \quad \Rightarrow \quad \frac{d_1}{d_2} = \sqrt{\frac{v_{G_2}}{v_{G_1}}}$$

$$\frac{d_1}{d_2} = \sqrt[4]{\frac{h_2 - h_0}{h_1 - h_0}}$$

example

2. Bernoulli from $\boxed{0}$ to $\boxed{2}$:

outside: $p_{a_0} = p_{a_2} + \rho_L g(h_2 - h_0)$

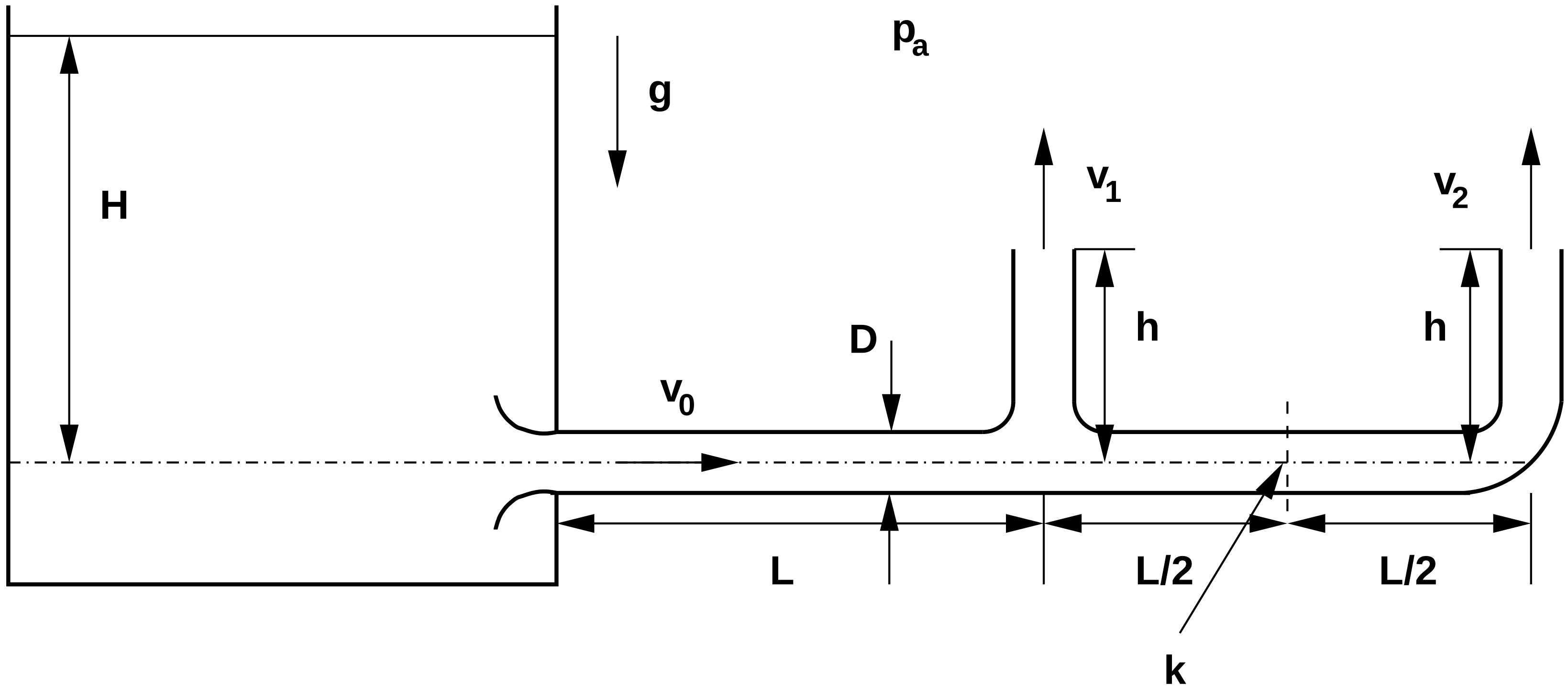
inside: $p_{a_0} = p_{a_2} + (1 + \zeta_{Dr})\frac{\rho_G}{2} v_{G_2}^2 + \rho_G g(h_2 - h_0)$

$d_1 = d_2 \quad , \quad \dot{V}_1 = \dot{V}_2 \quad \Rightarrow \quad v_{G_2} = v_{G_1}$

$$\zeta_{Dr} = \frac{h_2 - h_1}{h_1 - h_0}$$

example

An irrigation system consists of 2 fountains and is feeded by a large tank. The flow in the pipes with the diameter D is lossy. In the inlet and in the bends no additional losses occur.



example

1. Determine the velocities v_0, v_1, and v_2.
2. Determine the static pressure in point ”k”.
3. The system shall be changed such that the exit velocities are the same ($v_1 = v_2$). The length L and the height h cannot be changed. Name 2 possibilities of costructive steps to reach the same velocity in both pipes. Substantiate your answer. (No calculation)

Given: $L, \quad H, \quad h, \quad \rho, \quad g, \quad D, \quad \lambda, \quad p_a$

example

a)

Bernoulli from tank surface to point 1:

$$p_a + \varrho g H = p_a + \varrho g h + \frac{1}{2}\varrho v_1^2 (1 + \lambda \frac{h}{D}) + \frac{1}{2}\varrho v_0^2 \lambda \frac{L}{D}$$

Bernoulli from tank surface to point 2:

$$p_a + \varrho g H = p_a + \varrho g h + \frac{1}{2}\varrho v_2^2 (1 + \lambda \frac{L+h}{D}) + \frac{1}{2}\varrho v_0^2 \lambda \frac{L}{D}$$

Conti:

$$v_0 D^2 = v_1 D^2 + v_2 D^2$$

$$v_1^2 (1 + \lambda \frac{h}{D}) = v_2^2 (1 + \lambda \frac{L+h}{D})$$

$$\Rightarrow \quad v_2 = v_1 \sqrt{\frac{1 + \lambda \frac{h}{D}}{1 + \lambda \frac{h+L}{D}}} = v_1 C_1 \Rightarrow v_0 = v_1 (1 + C_1)$$

example

Plug in Bernoulli from surface to point 1:

$$p_a + \varrho g H = p_a + \varrho g h + \frac{1}{2}\varrho v_1^2 (1 + \lambda \frac{h}{D} + (1 + C_1)^2 \lambda \frac{L}{D})$$

$$\Rightarrow \;\; v_1 = \sqrt{\frac{2g(H-h)}{1 + \lambda \frac{h}{D} + (1 + C_1)^2 \lambda \frac{L}{D}}}$$

$$\Rightarrow \;\; v_0 = (1 + C_1) v_1$$

$$\Rightarrow \;\; v_2 = C_1 v_1$$

example

b)

Bernoulli from surface of the tank to point k:

$$p_a + \varrho g H = p_k + \frac{1}{2}\varrho v_0^2 \lambda \frac{L}{D} + \frac{1}{2}\varrho v_2^2 (1 + \lambda \frac{L}{2D})$$

$$p_k = p_a + \varrho g H - \frac{1}{2}\varrho (1 + C_1)^2 v_1^2 \lambda \frac{L}{D} - \frac{1}{2}\varrho C_1^2 v_1^2 (1 + \lambda \frac{L}{2D})$$

$$p_k = p_a + \varrho g h + \frac{1}{2}\varrho v_2^2 (\lambda \frac{L}{2D} + \lambda \frac{h}{D})$$

example

c)

The losses in the stand pipes should be the same.

- pump in the longer pipe
- decrease λ with a better surface quality.
- reduce the losses by using a larger cross section and accelerate at the end in a lossfree nozzle.