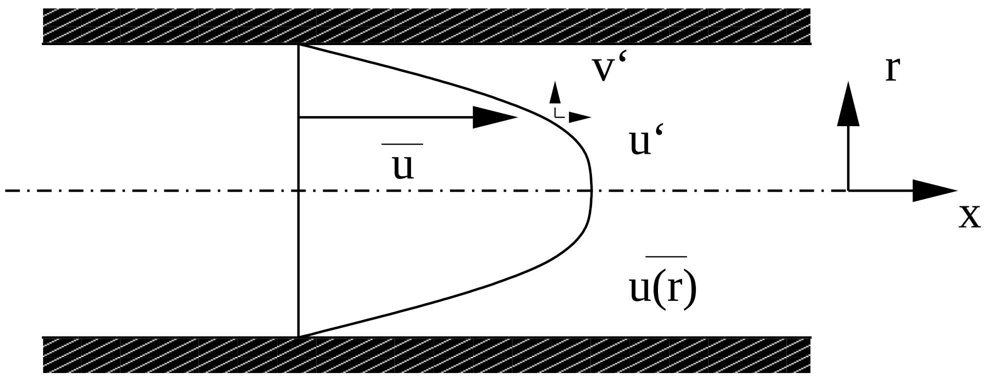
Turbulent flows

Reynolds averaging: splitting of the turbulent velocity $\vec{v}$ in an average value $\vec{\overline{v}}$ and a fluctuation $\vec{v'}$

$$\vec{v} \quad = \quad \vec{\overline{v}} \quad + \quad \vec{v'}$$

total vector — time average — fluctuation

Example: Pipe



fully turbulent

symmetric flow

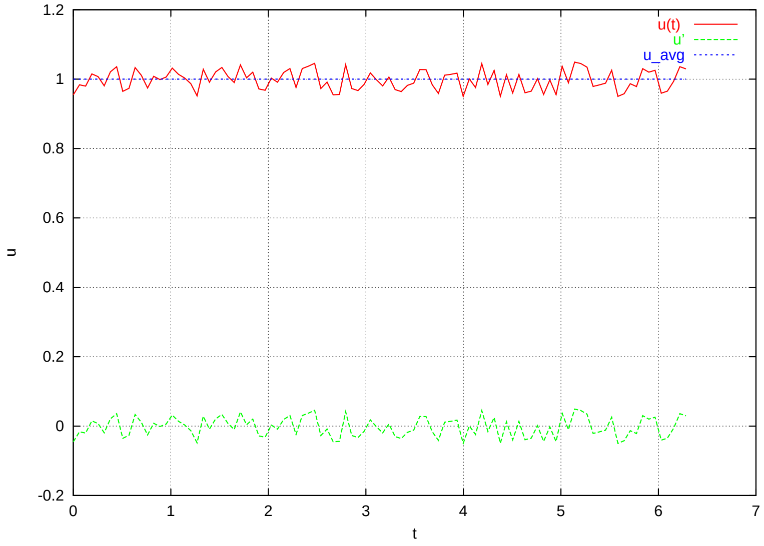
Turbulent flows

$$u(r,\phi,x,t) = \overline{u}(r) + u'(r,\phi,x,t)$$
$$v(r,\phi,x,t) = v'(r,\phi,x,t)$$

definition:

$$\overline{u} = \frac{1}{T}\int_T u(x,y,z,t)dt$$

$$\rightarrow \overline{u} = \overline{u}(x,y,z) \neq f(t) \qquad u' = u - \overline{u}$$



Computational rules

$$\overline{f'} = 0$$ average of the fluctuation

$$\overline{\overline{f}} = \underbrace{\overline{f}}_{\text{konst} \neq f(t)}$$ average of the average

$$\overline{f+g} = \frac{1}{T}\int_T (f+g)dt = \frac{1}{T}\int_T f\ dt + \frac{1}{T}\int_T g\ dt = \overline{f} + \overline{g}$$

$$\overline{f\ \overline{g}} = \overline{f}\ \overline{g} : \quad \overline{g} \neq \overline{g}(t) \rightarrow \frac{1}{T}\int_T f\ \overline{g}\ dt = \frac{1}{T}\overline{g}\int_T f\ dt = \overline{f}\ \overline{g}$$

$$\overline{\frac{\partial f}{\partial x}} = \frac{\partial \overline{f}}{\partial x}$$ average of the derivative

Computational rules

$$\overline{f\ g} = \frac{1}{T}\int_T f g\ dt = \frac{1}{T}\int_T (\overline{f} + f')(\overline{g} + g')\ dt$$

$$= \frac{1}{T}\int_T (\overline{f}\overline{g} + f'\overline{g} + \overline{f}g' + f'g')dt$$

$$= \overline{f}\overline{g} + \overline{g}\underbrace{\frac{1}{T}\int_T f'\ dt}_{=0} + \overline{f}\underbrace{\frac{1}{T}\int_T g'\ dt}_{=0} + \overline{f'\ g'}$$

$$= \overline{f}\overline{g} + \underline{\underline{\overline{f'\ g'}}}$$

usually $\neq 0$, z. B.. $f = g \rightarrow \overline{f'^2} \neq 0$

level of turbulence

turbulent intensity

$$\text{Tu} = \frac{1}{\overline{u_\infty}}\sqrt{\frac{1}{3}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})}$$

3-D, incompressible, unsteady momentum equation

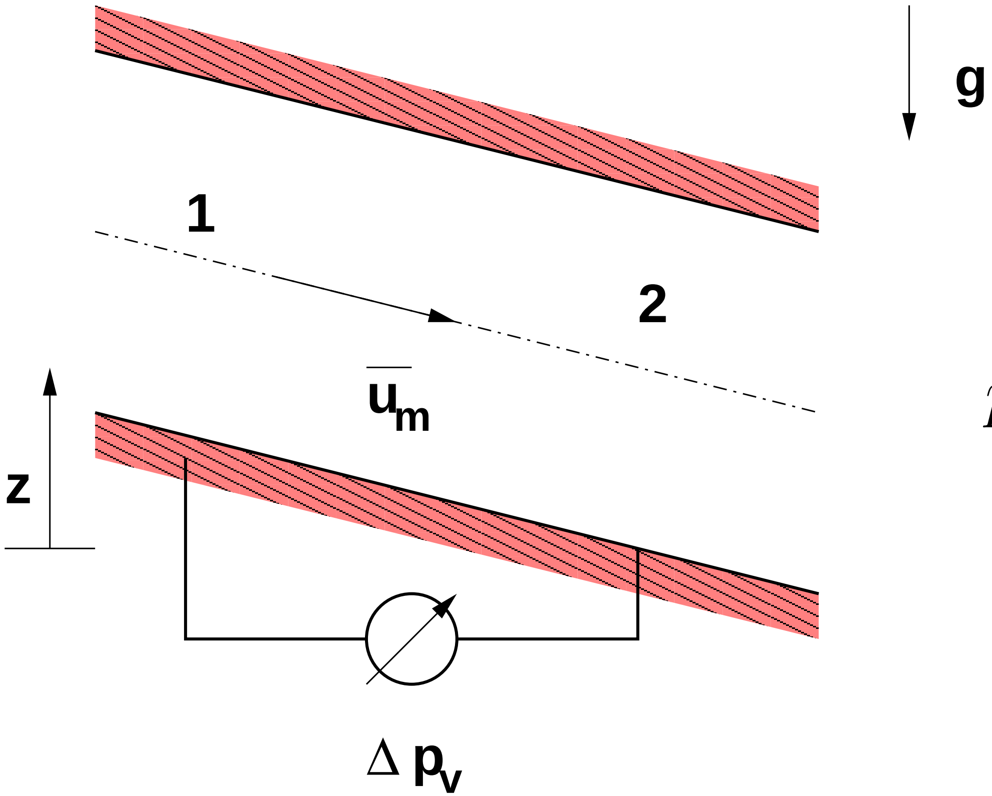
konvective term: $\frac{\partial v_k v_j}{\partial x_k}$ z. B.: $\frac{\partial uv}{\partial x}; \frac{\partial vw}{\partial y}$

for turbulent flows: average of the complete equation

$$\rightarrow \overline{\frac{\partial v_k v_j}{\partial x_k}} = \frac{\partial}{\partial x_k} = (\overline{v_k}\,\overline{v_j} + \underbrace{\overline{v'_k v'_j}}_{\text{additional term}})$$

$-\rho\overline{v'_k v'_j}$ turbulent shear stress tensor

Bernoulli equation (Energy equation) for pipe flows with loss of the total pressure



$$p_{01} = p_{02} + \underbrace{\Delta p_v}$$

Total pressure loss

$$p_1 + \frac{\rho}{2}\overline{u_{m_1}}^2 + \rho g z_1 = p_2 + \frac{\rho}{2}\overline{u_{m_2}}^2 + \rho g z_2 + \Delta p_v$$

$$\Delta p_v = \Sigma(\xi_i + \lambda_i \frac{L_i}{D_i})\frac{\rho}{2}\overline{u_{m_i}}^2$$

$\zeta_i \triangleq$ pressure loss coefficient for special places,
where losses occur
(inlet, unsteady enlargement of cross section, ellbow, ...)

$\lambda_i \triangleq$ loss coefficient in straight pipes

$\overline{u_{m_i}} \triangleq$ average velocity

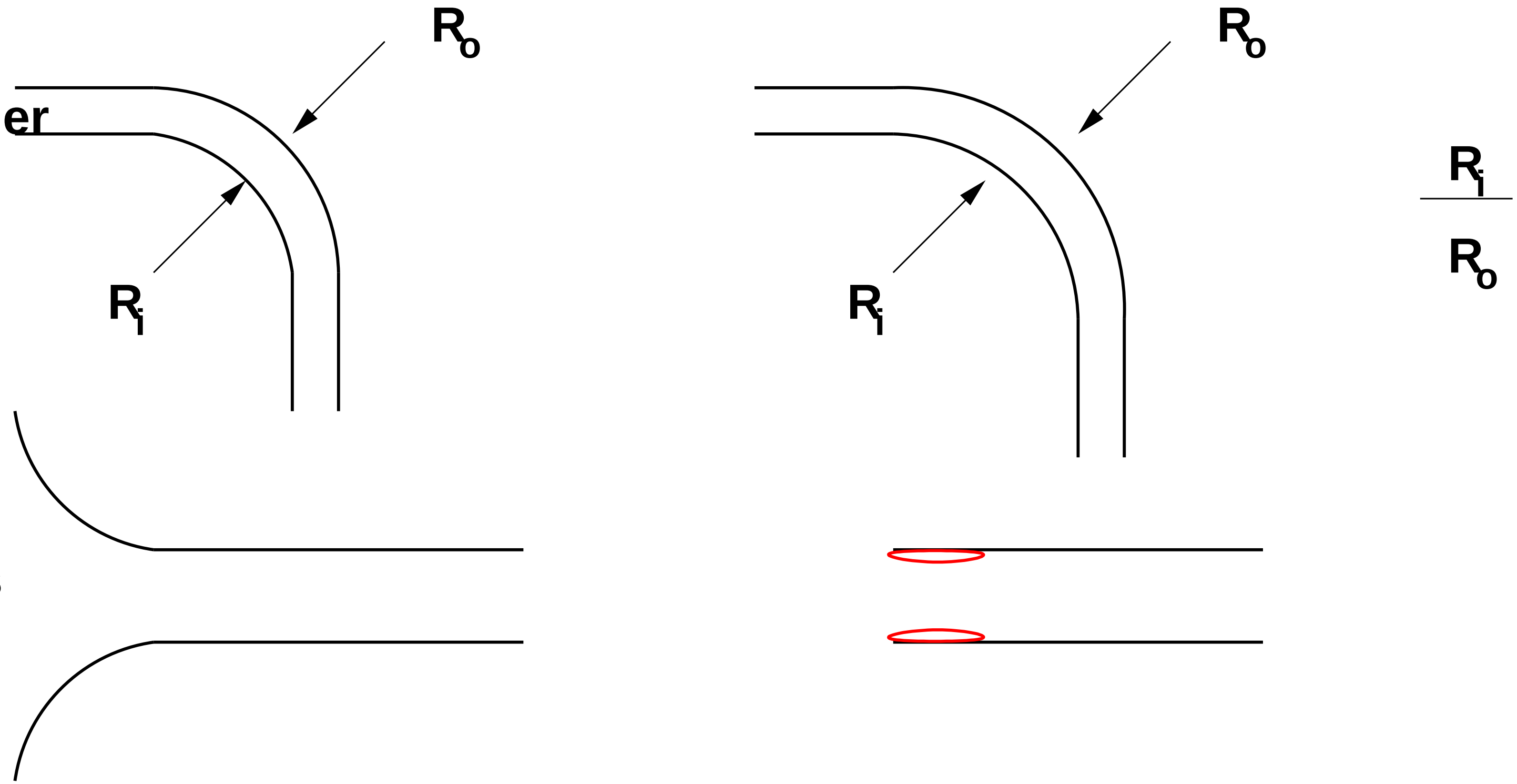
examples for pressure loss coefficients

uasually: determine ζ from experiments

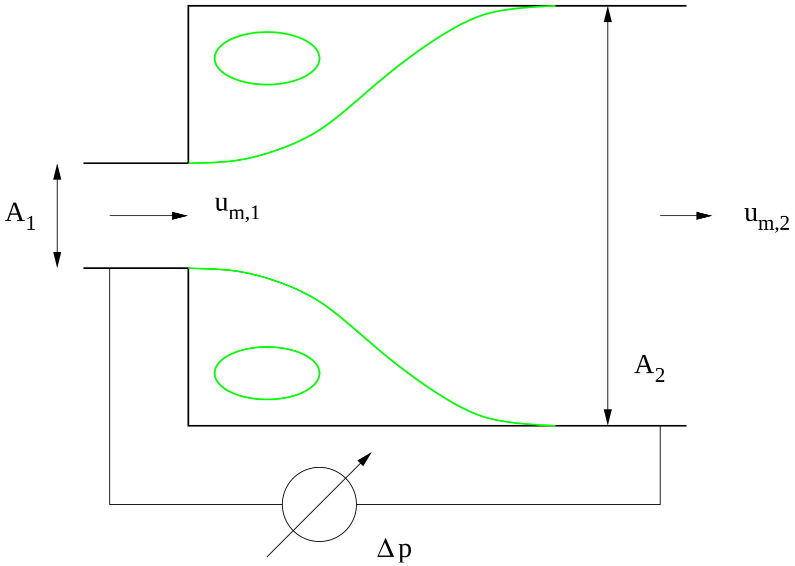
$$\xi = \xi(\text{Re, geometry})$$

Krümmer

Einlass



unsteady enlargement of cross section



Carnot eaution

$$\zeta_E = \frac{\Delta p}{\frac{\rho}{2} u_{m_1}^2} = (1 - \frac{A_1}{A_2})^2$$

laminar flow, inlet, circulare pipes

$\rightarrow 1.12 \leq \zeta_e \leq 1.45$ experimental

pressure loss coefficients for pipes (smooth pipes)

$$\text{Re} = \frac{\overline{u}\rho D}{\eta}$$

- laminar: (Re ≤ 2.300) $\lambda = \frac{C}{\text{Re}}$

  $C = 64$ for circular cross-sections (Hagen-Poisseuille)

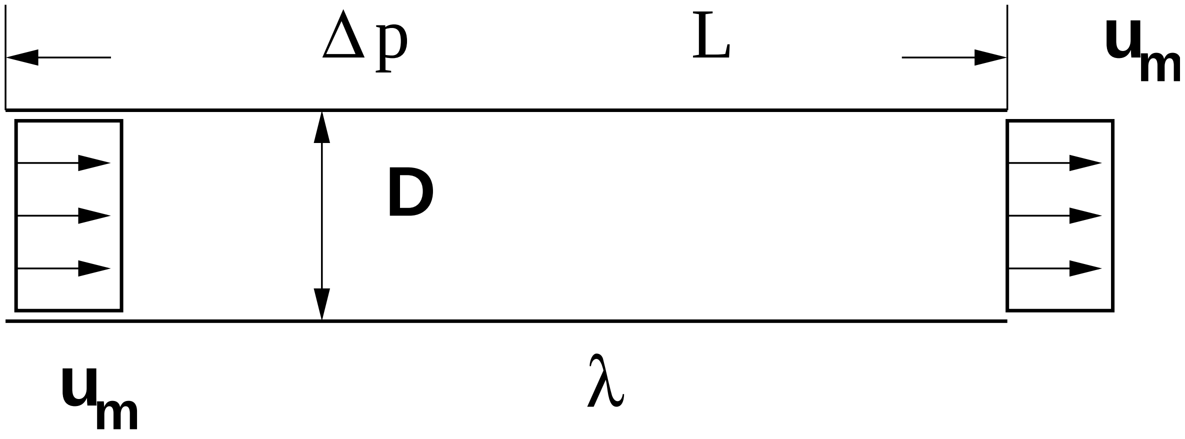
- turbulent: Blasius ($2.300 \leq$ Re $\leq 10^5$)

  $\lambda = \frac{0.316}{\sqrt[4]{\text{Re}}}$

  iterative solution: Prandtl: $\frac{1}{\sqrt{\lambda}} = 2\log(\text{Re}\sqrt{\lambda}) - 0.8$

Reference velocity

viscous effects in the pipe



$$\Delta p = \lambda \frac{L}{D} \frac{\rho}{2} \overline{u_m^2}$$

average pipe velocity

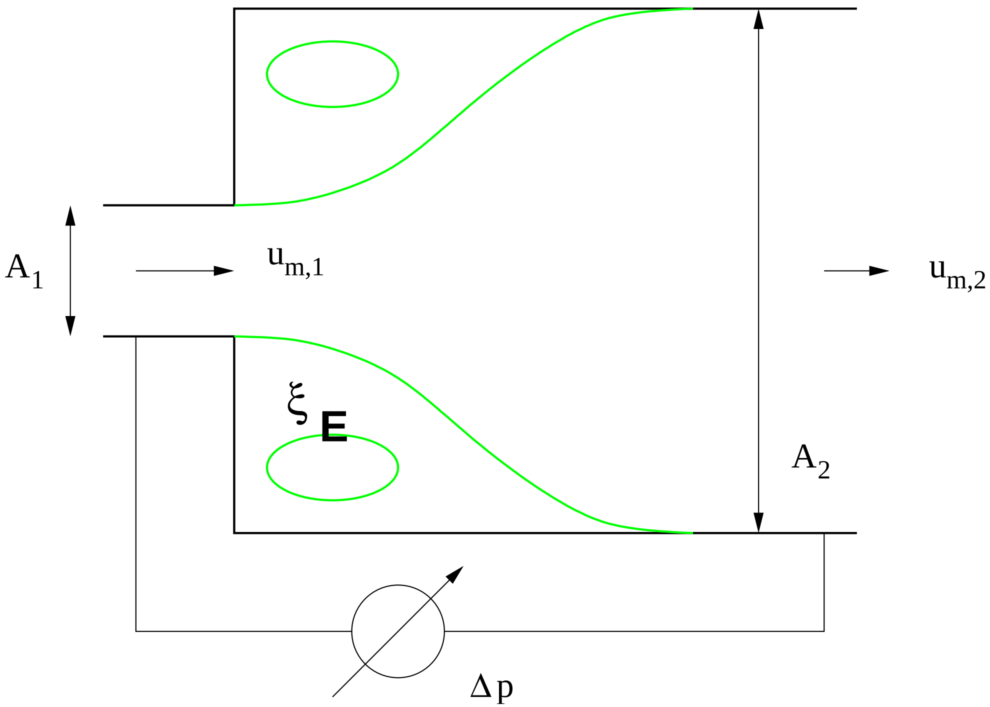
inlet

$$\Delta p_v = \xi_e \frac{\rho}{2} \overline{u_m^2}$$

average pipe velocity

Reference velocity

unsteady change of cross section

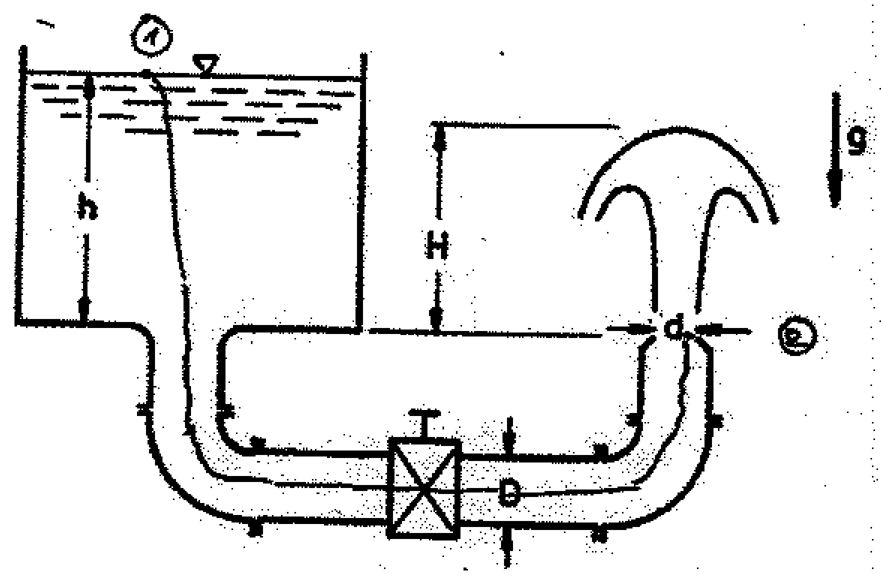


$$\Delta p_v = \xi_E \frac{\rho}{2} \overline{u_{m_1}^2}$$

incoming velocity

typical problem (losses)

6.8 . Die Zuleitung eines Springbrunnens besteht aus vier geraden Rohrstücken der Gesamtlänge L, zwei Krümmern (Verlustbeiwert ζ_K) und einem Ventil (ζ_V).



h = 10 m D = 0,05 m L = 4 m $\zeta_K = 0{,}25$
$\zeta_V = 4{,}5$ $\lambda = 0{,}025$

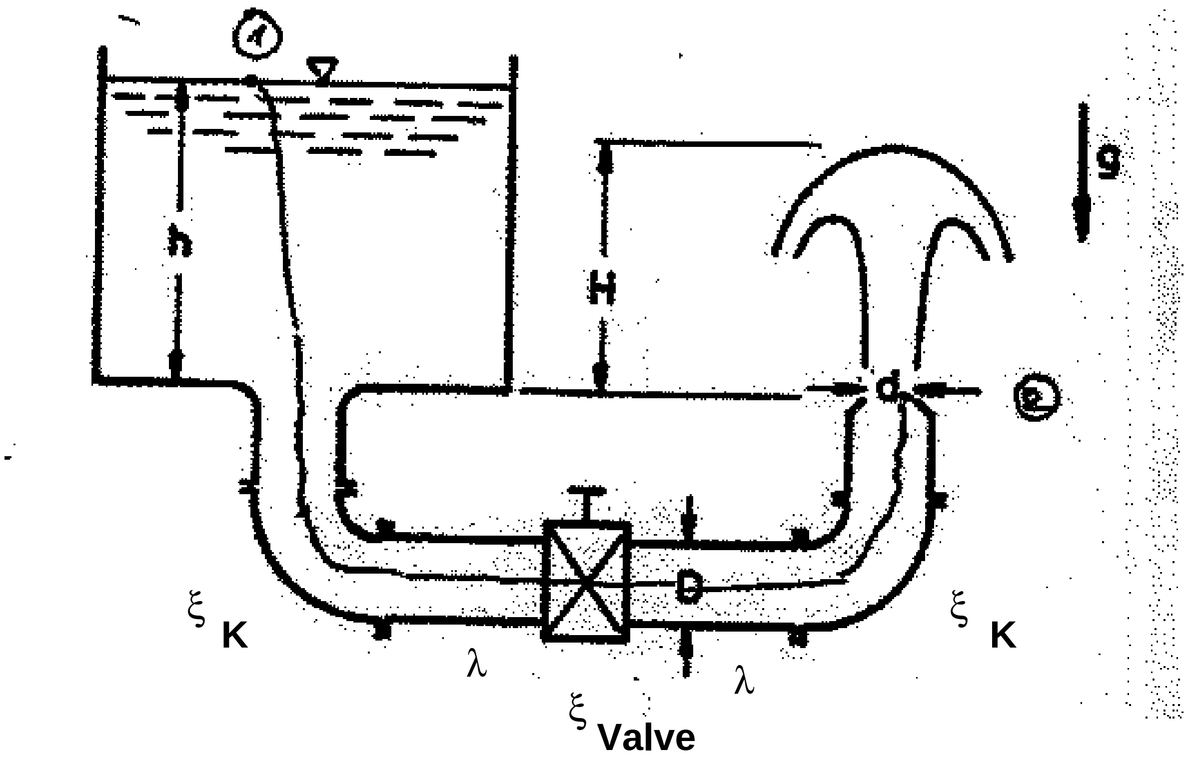
Bestimmen Sie für verlustbehaftete und verlustfreie Strömung den Volumenstrom und die Höhe H für

a) d = D/2,

b) d = D!

Hinweis: Nehmen Sie an, daß die Strömung in der Düse und im Einlauf verlustfrei und in den Rohrstücken ausgebildet ist!

typical problem (losses)



Bemerkung:

- mechanical losses are known (ζ, λ)
- the flow in the inlet and in the nozzle is lossfree
- the flow in the pipes is fully developed

typical problem (losses)

$\longrightarrow$ Bernoulli

$$p_{01} = p_{02} + \Delta p_v$$

total available energie

still existing total energy in '2'

transformed $\rightarrow$ inner energy total pressure loss

$$p_0 = p + \frac{\rho}{2}u^2 + \rho g z$$

$$\Delta p_v = \Sigma(\xi_i + \lambda_i \frac{L_i}{D_i})\frac{\rho}{2}\overline{u_{m_i}}^2$$

typical problem (losses)

Bernoulli from ’d’ $\rightarrow$ ’H’ ($u_H = 0$)

$$p_a + \frac{\rho}{2} u_d^2 = p_a + \rho g H$$

$$\rightarrow H = \frac{u_d^2}{2g} \rightarrow \quad \text{unknown} \quad u_d\,?$$

extended Bernoulli

$$p_{01} = p_a + \rho g h = p_a + \tfrac{\rho}{2} u_d^2 + \tfrac{\rho}{2} u_{mD}^2 \underbrace{(2\xi_K + \xi_v + \lambda \frac{L}{D})}_{K}$$

nozzle velocity — pipe velocity

continuity: $u_{mD} A_D = u_d A_d \rightarrow u_{mD} = u_d \left(\frac{d}{D}\right)^2$

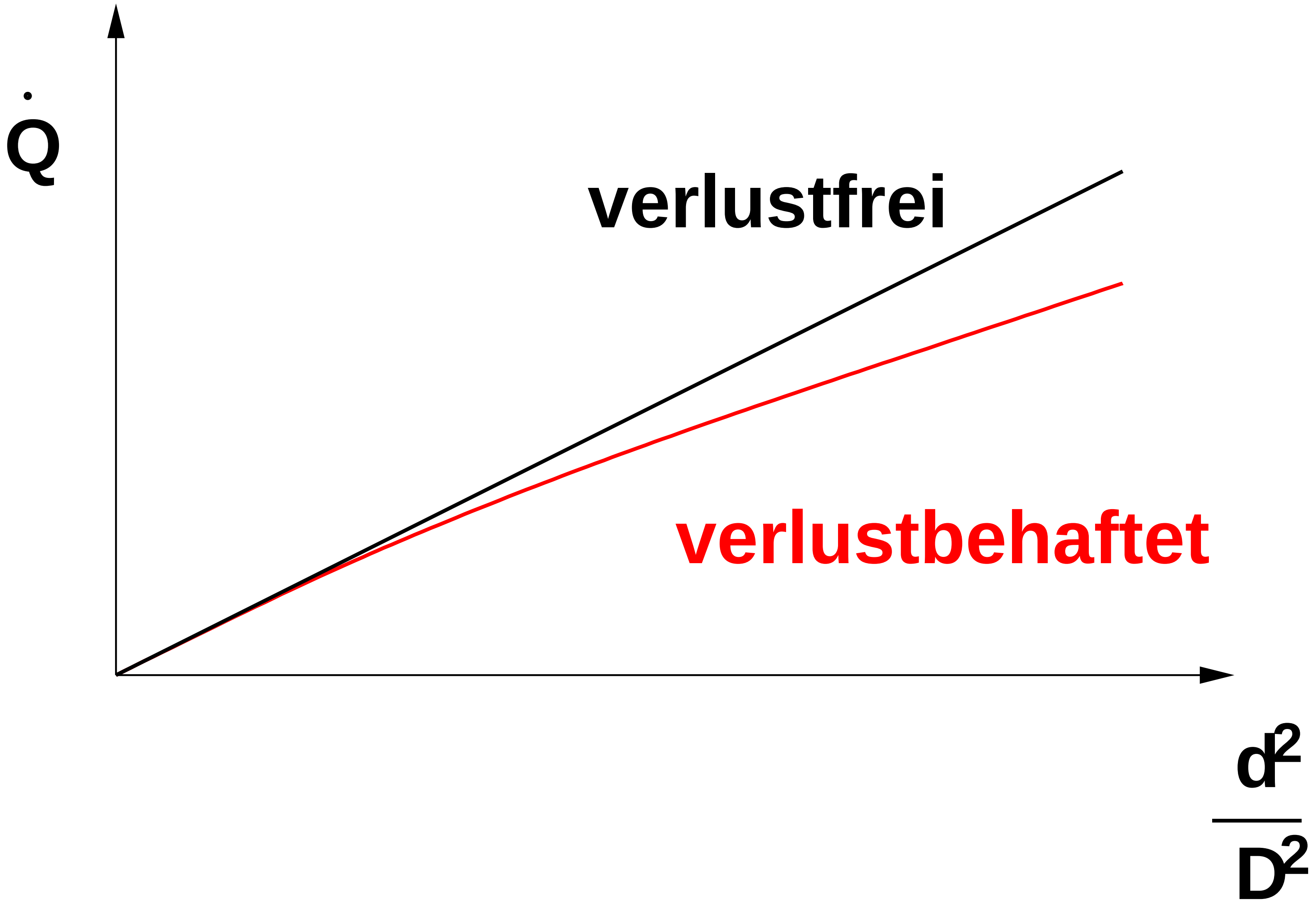
typical problem (losses)

lossfree	with losses
$\xi_K = \xi_v = \lambda = 0$	$\rho g h = \frac{\rho}{2} u_d^2 + \rho 2 u_d^2 \left(\frac{d}{D}\right)^4 K$
$u_d = \sqrt{2gh}$	$u_d = \sqrt{\frac{2gh}{1+\left(\frac{d}{D}\right)^4 K}}$

Volume flux $\dot{Q} = \frac{\pi}{4} d^2 u_d$

$\dot{Q} = \frac{\pi}{4}\sqrt{2gh} D^2 \frac{d^2}{D^2}$	$\dot{Q} = \frac{\pi}{4}\sqrt{2gh} D^2 \frac{d^2}{D^2} \frac{\left(\frac{d}{D}\right)^2}{\sqrt{1+\left(\frac{d}{D}\right)^4 K}}$
$\dot{Q} \sim \left(\frac{d}{D}\right)^2$	

typical problem (losses)



typical problem (losses)

ceiling of the fountain

$$H = \frac{u_d^2}{2g}$$

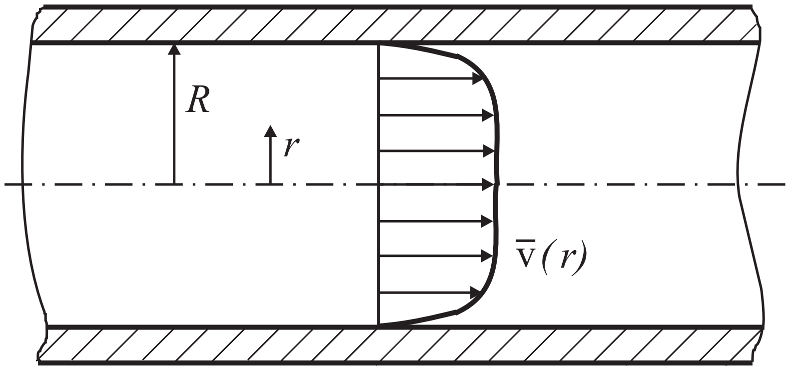
no losses	with losses
$H = h$	$H = \frac{h}{1+\left(\frac{d}{D}\right)^4 K}$
	influence of $\frac{d}{D}$
	$\frac{d}{D} \downarrow \rightarrow H \uparrow$

10.2

The velocity profile in a fully developed flow in a pipe with a smooth surface can be approximated with the potential law:

$$\frac{\bar{v}}{\bar{v}_{max}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}, \text{ mit } n = n(Re).$$

Re	n
$1 \cdot 10^5$	7
$6 \cdot 10^5$	8
$1.2 \cdot 10^6$	9
$2 \cdot 10^6$	10



10.2

a) Use the continuity equation to compute the relation between the average velocity $\bar{v}_m$ and the maximum velocity $\bar{v}_{max}$, i. e. $\frac{\bar{v}_m}{\bar{v}_{max}} = f(n)$.

b) At what position $\frac{r}{R}$ is $\bar{v}(r/R) = \bar{v}_m$?

c) How can the results of a) and b) be used, if the volume flux shall be measured?

The ratio between the average and the maximum velocity is

$$\frac{\bar{v}_m}{\bar{v}_{max}} = 2 \int_0^1 \xi(1 - \xi)^{\frac{1}{n}} \, d\xi = \frac{2n^2}{(n+1)(2n+1)} \quad \text{mit} \quad \xi = \frac{r}{R} \; .$$

The integral is solved using partial integration. The average velocity is at a distance

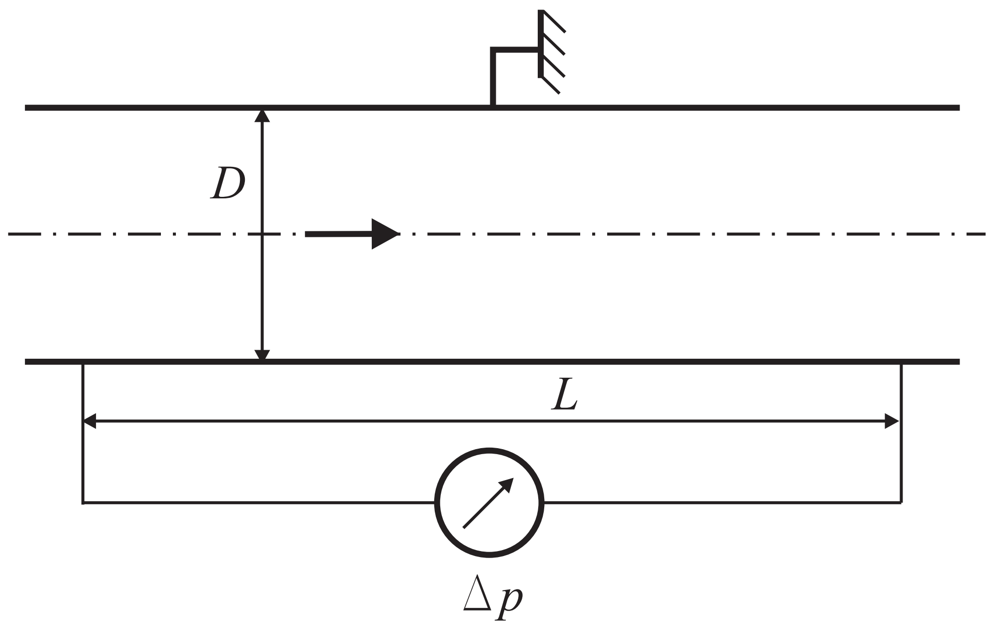
$$\frac{r_m}{R} = 1 - \left(\frac{\bar{v}_m}{\bar{v}_{max}}\right)^n$$

see table.

Re	n	$\bar{v}_m/\bar{v}_{max}$	r_m/R
$1 \cdot 10^5$	7	0.8166	0.7577
$6 \cdot 10^5$	8	0.8366	0.76
$1.2 \cdot 10^6$	9	0.8526	0.762
$2 \cdot 10^6$	10	0.8658	0.7633

Measuring $\bar{v}(r)$ at a distance $R - r_m$ from the wall, and with the known $\bar{v}_{max}$ the average velocity can be determined, and the volume flux $\dot{V} = v_m \, \pi \, R^2$ can be computed.

The pressure decrease Δp along L is measured in a fully developed pipe flow with the volume flux $\dot{V}$.

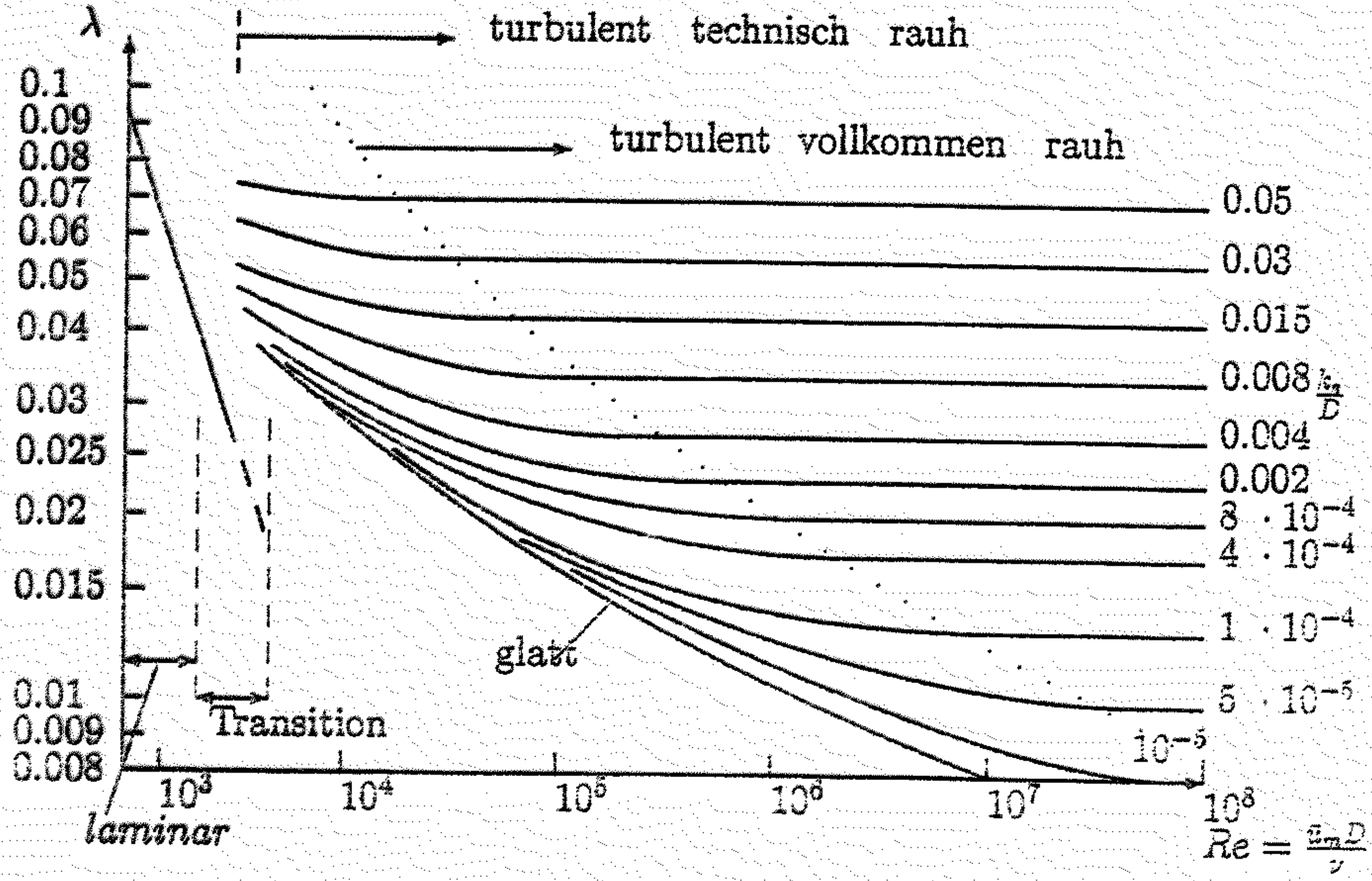


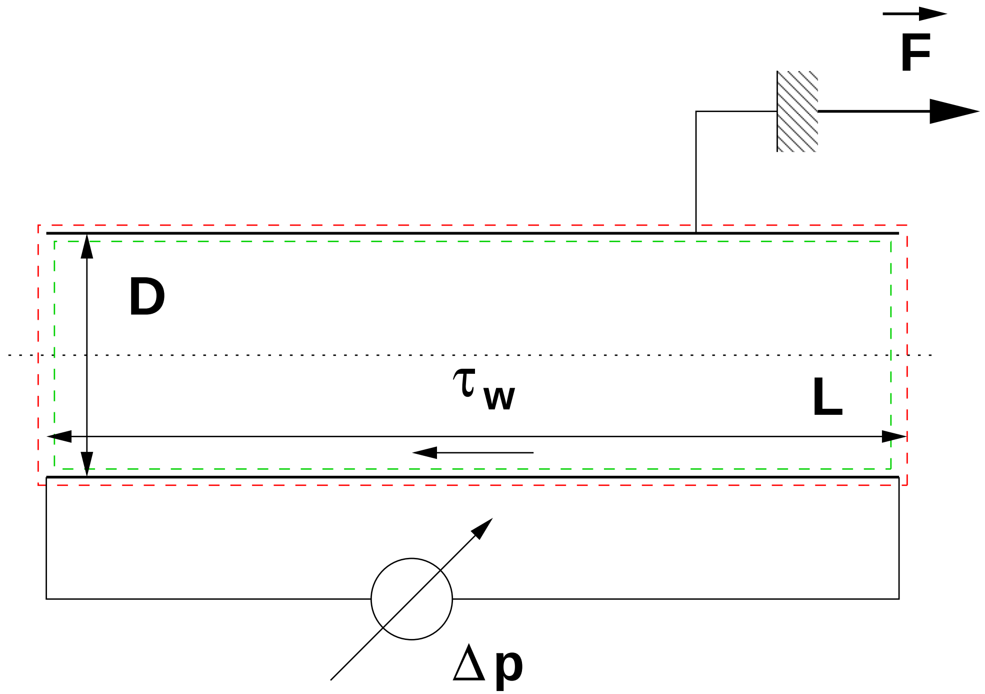
$\dot{V} = 0,393\ m^3/s \quad L = 100\ m \quad D = 0,5\ m \quad \Delta p = 12820\ N/m^2 \quad \rho = 900\ kg/m^3$
$\eta = 5 \cdot 10^{-3}\ Ns/m^2$

10.6

Determine

a) the skin-friction coefficient,
b) the equivalent roughness of the pipe,
c) the wall shear stress and the force of the support.
d) What is the pressure decrease, if the pipe is smooth?


λ
turbulent technisch rauh
turbulent vollkommen rauh
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
0.05
0.03
0.015
0.008 $\frac{k_s}{D}$
0.004
0.002
8 · 10⁻⁴
4 · 10⁻⁴
1 · 10⁻⁴
5 · 10⁻⁵
10⁻⁵
glatt
Transition
laminar
10³
10⁴
10⁵
10⁶
10⁷
10⁸
$Re = \frac{\bar{u}_m D}{\nu}$



a)

$$\Delta p = \lambda \frac{L}{D} \frac{\rho}{2} \bar{u}_m^2$$

$$\dot{V} = \bar{u}_m \frac{\pi D^2}{4}$$

$$\Longrightarrow \quad \lambda = \frac{\pi^2 \Delta p D^5}{8 \rho L \dot{V}^2} = 0,0356$$

b)

$$Re = \frac{\rho \, \bar{u}_m \, D}{\eta} = 1,8 \cdot 10^5$$

$$\frac{k_s}{D} = 0,0083 \quad \text{(from Moody diagram)}$$

$$\Longrightarrow \quad k_s = 4,2 \, mm$$

c)

momentum equation for the inner control surface:

$$\Delta p \, \frac{\pi \, D^2}{4} - \tau_W \, \pi \, D \, L = 0$$

$$\Longrightarrow \quad \tau_W = \Delta p \, \frac{D}{4L} = 16 \; N/m^2$$

momentum equation for the outer control surface:

$$F = -\Delta p \frac{\pi D^2}{4} = -2517\ N$$

d) $$\lambda = 0,016 \quad \text{(from diagram)}$$

$$\Longrightarrow \quad \Delta p = 5,8 \cdot 10^3\ N/m^2$$