....
(Name, Matr.-Nr, Unterschrift)

Klausur Strömungsmechanik II

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1. Aufgabe

a) Kontinuitätsgleichung:

$$
\begin{aligned}
\nabla \cdot \vec{v} &= \vec{0} \\
\left(\vec{e_r}\frac{\partial}{\partial r} + \frac{1}{r}\vec{e_\varphi}\frac{\partial}{\partial \varphi} + \vec{e_z}\frac{\partial}{\partial z}\right) \cdot (v_r\vec{e_r} + v_\varphi\vec{e_\varphi} + v_z\vec{e_z}) &= 0 \\
\frac{\partial v_r}{\partial r} + \frac{1}{r}\vec{e_\varphi}\frac{\partial}{\partial \varphi}(v_r\vec{e_r} + v_\varphi\vec{e_\varphi} + v_z\vec{e_z}) + \frac{\partial v_z}{\partial z} &= 0 \\
\frac{\partial v_r}{\partial r} + \frac{1}{r}\vec{e_\varphi}\left(v_r\vec{e_\varphi} + \vec{e_r}\frac{\partial v_r}{\partial \varphi} + \vec{e_\varphi}\frac{\partial v_\varphi}{\partial \varphi} - v_\varphi\vec{e_r} + \frac{\partial v_z}{\partial \varphi}\vec{e_z}\right) + \frac{\partial v_z}{\partial z} &= 0 \\
\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z} &= 0
\end{aligned}
$$

Impulsgleichung in z-Richtung:

$$
\rho \cdot \left(\vec{e_z}\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z}\right) + \frac{v_\varphi}{r}\frac{\partial(v_z\vec{e_z})}{\partial \varphi}\right) = ...
$$

$$
...\rho g_z - \vec{e_z}\frac{\partial p}{\partial z} + \eta\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(v_z\vec{e_z})}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2(v_z\vec{e_z})}{\partial \varphi^2} + \frac{\partial^2(v_z\vec{e_z})}{\partial z^2}\right)
$$

$$
\rho \cdot \vec{e_z}\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} + \frac{v_\varphi}{r}\frac{\partial v_z}{\partial \varphi}\right) = ...
$$

$$
...\rho g_z - \vec{e_z}\frac{\partial p}{\partial z} + \eta\vec{e_z}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \varphi^2} + \frac{\partial^2 v_z}{\partial z^2}\right)
$$

b) Vereinfachungen:

- stationär: $\frac{\partial}{\partial t} = 0$
- ohne Gravitation: $\vec{g} = \begin{pmatrix} g_r \\ g_\varphi \\ g_z \end{pmatrix} = \vec{0}$
- rotationssymmetrisch: $\frac{\partial}{\partial \varphi} = 0$

Kontinuitätsgleichung:

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0$$

Impuls:

$$\rho \left(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \eta \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{\partial^2 v_z}{\partial z^2} \right)$$

c) dimensionslose Größen:

$$\bar{v_r} = \frac{v_r}{u_D}, \bar{v_z} = \frac{v_z}{u_D}, \overline{z} = \frac{z}{R}, \overline{r} = \frac{r}{R}, \overline{p} = \frac{p}{\rho u_D^2}, \rho, \eta = \text{konst.}$$

z-Impuls:

$$\begin{aligned}
\rho \bar{v_r} u_D \frac{\partial \left(\bar{v_z} u_D \right)}{\partial \left(\overline{r} R \right)} + \rho \bar{v_z} u_D \frac{\partial \left(\bar{v_z} u_D \right)}{\partial \left(\overline{z} R \right)} &= -\frac{\partial \left(\overline{p} \rho u_D^2 \right)}{\partial \left(\overline{z} R \right)} + \eta \left(\frac{1}{\overline{r} R} \frac{\partial}{\partial \overline{r} R} \left(\overline{r} R \frac{\partial (\bar{v_z} u_D)}{\partial \overline{r} R} \right) + \frac{\partial^2 \left(\bar{v_z} u_D \right)}{\partial \left(\overline{z} R \right)^2} \right) \\
\frac{u_D^2 \rho \bar{v_r}}{R} \frac{\partial \bar{v_z}}{\partial \overline{r}} + \frac{u_D^2 \rho \bar{v_z}}{R} \frac{\partial \bar{v_z}}{\partial \overline{z}} &= -\frac{\rho u_D^2}{R} \frac{\partial \overline{p}}{\partial \overline{z}} + \eta \frac{u_D}{R^2} \left(\frac{1}{\overline{r}} \frac{\partial}{\partial \overline{r}} \left(\overline{r} \frac{\partial \bar{v_z}}{\partial \overline{r}} \right) + \frac{1}{R^2} \frac{\partial^2 \bar{v_z}}{\partial \overline{z}^2} \right) \\
\bar{v_r} \frac{\partial \bar{v_z}}{\partial \overline{r}} + \bar{v_z} \frac{\partial \bar{v_z}}{\partial \overline{z}} &= -\frac{\partial \overline{p}}{\partial \overline{z}} + \underbrace{\frac{\eta}{\rho u_D R}}_{K} \left(\frac{1}{\overline{r}} \frac{\partial}{\partial \overline{r}} \left(\overline{r} \frac{\partial \bar{v_z}}{\partial \overline{r}} \right) + \frac{1}{R^2} \frac{\partial^2 \bar{v_z}}{\partial \overline{z}^2} \right)
\end{aligned}$$

die Strömung wird durch die Reynoldszahl Re beschrieben:

$$K = \frac{\eta}{\rho u_D R} = \frac{1}{Re}$$

2. Aufgabe

a)

$$\frac{d\tau_{\Theta\Phi}}{d\Theta} = \frac{-2\tau_{\Theta\Phi}}{\tan\Theta} \Rightarrow \frac{d\tau_{\Theta\Phi}}{\tau_{\Theta\Phi}} = \frac{-2}{\tan\Theta} d\Theta \Rightarrow \ln\tau_{\Theta\Phi} = -2\ln(\sin\Theta) + c_1'$$

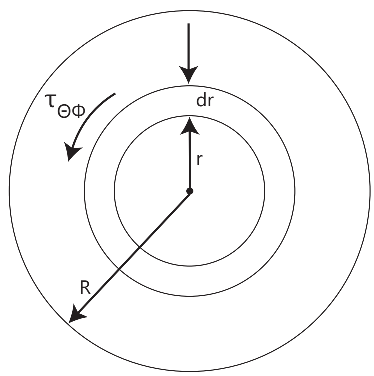
$$\Leftrightarrow \tau_{\Theta\Phi} = \mathrm{e}^{\ln\left(\frac{1}{\sin^2\Theta}\right)+c_1'} = \frac{c_1}{\sin^2\Theta}$$

Moment M wirkt auf die Platte:

$$M = \int_0^R 2\pi r r \tau_{\Theta\Phi}\Big|_{\Theta=\frac{\pi}{2}} dr, \text{ da } c_1|_{\Theta=\frac{\pi}{2}} \neq f(r)$$

$$\Leftrightarrow M = \int_0^R 2\pi r^2 c_1 dr = \frac{2}{3}\pi R^3 c_1 \Rightarrow c_1 = \frac{3M}{2\pi R^3}$$

$$\Rightarrow \tau_{\Theta\Phi} = \frac{3M}{2\pi R^3 \sin^2\Theta}$$



b)

$$\tau_{\Theta\Phi} = -\eta\left(\sin\Theta\frac{\partial}{\partial\Theta}\left(\frac{v_\Phi}{r\sin\Theta}\right) + \frac{1}{\sin\Theta}\frac{\partial v_\Theta}{\partial\Phi}\right), \quad \frac{\partial v_\Theta}{\partial\Phi} = 0, \text{ da } v_\Theta = 0$$

$$\Rightarrow \frac{3M}{2\pi R^3\sin^2\Theta} = -\eta\sin\Theta\frac{d}{d\Theta}\left(\frac{v_\Phi}{r\sin\Theta}\right) \Leftrightarrow \frac{-3M}{2\pi R^3\eta\sin^3\Theta}d\Theta = d\left(\frac{v_\Phi}{r\sin\Theta}\right)$$

$$\Rightarrow \frac{3M}{4\pi R^3\eta}\left(\frac{\cos\Theta}{\sin^2\Theta} + \frac{1}{2}\ln\left(\frac{1+\cos\Theta}{1-\cos\Theta}\right)\right) = \frac{v_\Phi}{r\sin\Theta} + c_2$$

$$\Rightarrow \frac{3M}{4\pi R^3\eta}\left(\frac{1}{\tan\Theta} + \frac{\sin\Theta}{2}\ln\left(\frac{1+\cos\Theta}{1-\cos\Theta}\right)\right) + c_2\sin\Theta = \frac{v_\Phi}{r}$$

Randbedingung:

$$\Theta = \frac{\pi}{2} \Rightarrow v_\Phi = 0 \Rightarrow c_2 = 0$$

$$\Theta = \Theta_k \Rightarrow v_\Phi = \omega r$$
$$\Rightarrow v_\Phi = \frac{3M}{4\pi R^3 \eta} \left[\frac{1}{\tan \Theta} + \frac{\sin \Theta}{2} \ln \left(\frac{1 + \cos \Theta}{1 - \cos \Theta} \right) \right] r$$
$$\Rightarrow \eta = \frac{3M}{4\pi R^3 \omega} \left[\frac{1}{\tan \Theta_k} + \frac{\sin \Theta_k}{2} \ln \left(\frac{1 + \cos \Theta_k}{1 - \cos \Theta_k} \right) \right]$$

3. Aufgabe

a)
- inkompressibel
- stationär
- zwei-dimensional
- reibungsfrei, drehungsfrei

b)

$$F(z) = u_\infty z + \frac{M}{2\pi z} + i\frac{M}{2\pi z} \text{ mit } u_\infty, M > 0$$

c)

$$\begin{aligned}
F(z) &= u_\infty r\left(cos(\varphi) + isin(\varphi)\right) + \frac{M}{2\pi r e^{i\varphi}} + i\frac{M}{2\pi r e^{i\varphi}} \\
F(z) &= u_\infty r\left(cos(\varphi) + isin(\varphi)\right) + \frac{M}{2\pi r}\left(cos(\varphi) - isin(\varphi)\right)(1+i) \\
F(z) &= \underbrace{u_\infty r cos(\varphi) + \frac{M}{2\pi r}\left(cos(\varphi) + sin(\varphi)\right)}_{Re} + i\underbrace{\left(u_\infty r sin(\varphi) + \frac{M}{2\pi r}\left(cos(\varphi) - sin(\varphi)\right)\right)}_{Im}
\end{aligned}$$

$$\begin{aligned}
\Phi(r,\varphi) &= u_\infty r cos(\varphi) + \frac{M}{2\pi r}\left(cos(\varphi) + sin(\varphi)\right) \\
\Psi(r,\varphi) &= u_\infty r sin(\varphi) + \frac{M}{2\pi r}\left(cos(\varphi) - sin(\varphi)\right)
\end{aligned}$$

d)

$$\begin{aligned}
v_r &= \frac{\partial\Phi}{\partial r} = u_\infty cos(\varphi) - \frac{M}{2\pi r^2}\left(cos(\varphi) + sin(\varphi)\right) \\
v_\varphi &= \frac{1}{r}\frac{\partial\Phi}{\partial\varphi} = -u_\infty sin(\varphi) + \frac{M}{2\pi r^2}\left(cos(\varphi) - sin(\varphi)\right)
\end{aligned}$$

e) Staupunkt: $v_r = v_\varphi = 0$

$$\begin{aligned}
v_r = 0 : u_\infty cos(\varphi) &= \frac{M}{2\pi r^2}\left(cos(\varphi) + sin(\varphi)\right) \\
r^2 &= \frac{M}{2\pi u_\infty}\left(1 + tan(\varphi)\right)
\end{aligned}$$

$$
\begin{aligned}
v_\varphi = 0 : u_\infty sin(\varphi) &= \frac{M}{2\pi r^2}\left(cos(\varphi) - sin(\varphi)\right) \\
r^2 \text{ einsetzen: } u_\infty sin(\varphi) &= \frac{M}{2\pi}\frac{2\pi u_\infty}{M}\frac{cos(\varphi) - sin(\varphi)}{1 + tan(\varphi)} \\
1 + tan(\varphi) &= \frac{1}{tan(\varphi)} - 1 \\
tan^2(\varphi) + 2tan(\varphi) - 1 &= 0 \\
tan(\varphi) &= -1 \pm \sqrt{2} \\
\varphi &= tan^{-1}(-1 \pm \sqrt{2}) \\
\rightarrow r &= \sqrt{\frac{\pm\sqrt{2}M}{2\pi u_\infty}}
\end{aligned}
$$

somit ergeben sich folgende Koordinaten der Staupunkte:

$$
\begin{aligned}
\varphi_{s1} &= tan^{-1}(-1 + \sqrt{2}) = tan^{-1}\left(\frac{\pi}{8}\right), r_{s1} = \sqrt{\frac{\sqrt{2}M}{2\pi u_\infty}} \\
\varphi_{s2} &= tan^{-1}\left(\frac{9\pi}{8}\right), r_{s2} = \sqrt{\frac{\sqrt{2}M}{2\pi u_\infty}}
\end{aligned}
$$

Alternativer Lösungsweg:

$$
\begin{aligned}
v_\varphi = 0 : u_\infty sin(\varphi) &= \frac{M}{2\pi r^2}\left(cos(\varphi) - sin(\varphi)\right) \\
r^2 \text{ einsetzen: } u_\infty sin(\varphi) &= \frac{M}{2\pi}\frac{2\pi u_\infty}{M}\frac{cos(\varphi) - sin(\varphi)}{1 + tan(\varphi)} \\
sin(\varphi) &= \frac{1}{1 + \frac{sin(\varphi)}{cos(\varphi)}}\left(cos(\varphi) - sin(\varphi)\right) \\
0 &= sin(\varphi)cos(\varphi) + sin^2(\varphi) - cos^2(\varphi) + sin(\varphi)cos(\varphi) \\
sin(2\varphi) &= cos(2\varphi)
\end{aligned}
$$

$$
\rightarrow \varphi = \frac{\pi}{8}, \frac{5\pi}{8} \quad , \quad \frac{9\pi}{8}, \frac{13\pi}{8}
$$

$$
\rightarrow tan\left(\frac{\pi}{8}\right) = tan\left(\frac{9\pi}{8}\right) > 0 \quad , \quad tan\left(\frac{5\pi}{8}\right) = tan\left(\frac{13\pi}{8}\right) < -1
$$

somit ergeben sich dieselben Koordinaten der Staupunkte:

$$
\begin{aligned}
\varphi_{s1} &= tan^{-1}\left(\frac{\pi}{8}\right), r_{s1} = \sqrt{\frac{\sqrt{2}M}{2\pi u_\infty}} \\
\varphi_{s2} &= tan^{-1}\left(\frac{9\pi}{8}\right), r_{s2} = \sqrt{\frac{\sqrt{2}M}{2\pi u_\infty}}
\end{aligned}
$$

4. Aufgabe

a) Aus der Eulergleichung

$$u_a(x)\frac{du_a(x)}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$$

folgt mit $p = konst.$, dass $u_a(x) = u_\infty = konst.$

b) 1. Randbedingung: Haftbedingung: $\frac{y}{\delta} = 0 \rightarrow u = 0$

2. Randbedingung: Grenzschichtrand: $\frac{y}{\delta} = 1 \rightarrow u = u_\infty$

3. Randbedingung: x-Impulsgleichung an der Wand, kein Druckgradient:

$$\begin{aligned} \cancel{u}\frac{\partial u}{\partial x} - v_A \left.\frac{\partial u}{\partial y}\right|_{y=0} &= -\frac{1}{\rho}\frac{\cancel{\partial p}}{\cancel{\partial x}} + \nu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} \\ -v_A \left.\frac{\partial u}{\partial y}\right|_{y=0} &= \nu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} \end{aligned}$$

aus 1. folgt $a_0 = 0$
aus 2. folgt $1 = a_1 + a_2$
mit

$$\begin{aligned} \frac{\partial u}{\partial y} &= u_a\left(\frac{a_1}{\delta} + \frac{2a_2y}{\delta^2}\right) \rightarrow \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{u_a a_1}{\delta} \\ \frac{\partial^2 u}{\partial y^2} &= u_a\frac{2a_2}{\delta^2} = \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} \end{aligned}$$

aus 3. folgt

$$-v_A\frac{u_a a_1}{\delta} = \nu\frac{u_a 2a_2}{\delta^2} \rightarrow a_1 = -\frac{\nu 2a_2}{\delta v_A}$$

in 2. einsetzen liefert

$$1 = -\frac{\nu 2a_2}{\delta v_A} + a_2 \rightarrow a_2 = \frac{1}{1 - \frac{2\nu}{\delta v_A}}$$

$$\rightarrow a_1 = 1 - a_2 = \frac{-\frac{2\nu}{\delta v_A}}{1 - \frac{2\nu}{\delta v_A}} = -\frac{1}{\frac{\delta v_A}{2\nu} - 1}$$

mit $\nu = \frac{\eta}{\rho}$ ergibt sich für das Geschwindigkeitsprofil

$$\frac{u(x,y)}{u_\infty} = -\frac{1}{\frac{\delta(x)v_A\rho}{2\eta} - 1}\frac{y}{\delta(x)} + \frac{1}{1 - \frac{2\eta}{\delta(x)v_A\rho}}\left(\frac{y}{\delta(x)}\right)^2.$$

c)

$$
\begin{aligned}
\tau_w &= \eta \left.\frac{\partial u}{\partial y}\right|_{y=0} \\
\text{mit} \qquad \frac{\partial u}{\partial y} &= u_\infty \left(-\frac{1}{\frac{\delta^2(x) v_A \rho}{2\eta} - \delta(x)} + \frac{2}{\delta(x) - \frac{2\eta}{v_A \rho}} \frac{y}{\delta(x)} \right) \\
\rightarrow \tau_w &= -\frac{u_\infty \eta}{\frac{\delta^2(x) v_A \rho}{2\eta} - \delta(x)}
\end{aligned}
$$

d) Mithilfe einer Grenzschichtabsaugung kann eine Grenzschichtablösung, die in den meisten Anwendungsfällen vermieden werden soll, verzögert oder sogar verhindert werden.

5. Aufgabe

a) der Kesseldruck p_K entspricht dem Ruhedruck p_0
Verhältnis von $\frac{p}{p_0} = \frac{p}{p_K}$ mit der Energiegleichung, angewendet auf den Ruhezustand: $h_0 = h + \frac{u^2}{2}$

$$\begin{aligned}
\text{mit } h = c_p T \quad &\Rightarrow \quad c_p T_0 = c_p T + \frac{u^2}{2} \\
\text{mit } c_p = \frac{\gamma R}{\gamma - 1} \quad &\Rightarrow \quad \frac{\gamma R T_0}{\gamma - 1} = \frac{\gamma R T}{\gamma - 1} + \frac{u^2}{2} \\
& \quad\quad \frac{T_0}{T} = 1 + \frac{u^2}{2}\frac{\gamma - 1}{\gamma R T} = 1 + \frac{\gamma - 1}{2} M^2 \\
\text{mit} \quad \tfrac{p}{p_0} &= \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma-1}} \\
\text{folgt} \quad \tfrac{p}{p_K} &= \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-\gamma}{\gamma-1}}
\end{aligned}$$

Kesseldruck p_K bestimmen:

$$\begin{aligned}
p_K &= \frac{p_K}{p_1} \cdot \frac{p_1}{p_2} \cdot p_2 \qquad \text{mit } p_2 = p_a \\
p_K &= \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \cdot \left(1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)\right)^{-1} \cdot p_a
\end{aligned}$$

b)

$$\begin{aligned}
M_1^* \cdot M_2^* &= 1 \leftrightarrow \frac{u_1}{c_1^*} \cdot \frac{u_2}{c_2^*} = 1 \qquad \text{mit } c_1^* = c_2^* = c^* \\
\rightarrow u_1 u_2 &= c^{*2} \qquad \text{mit } c^{*2} = \gamma R T^* \\
\rightarrow u_2 &= \frac{\gamma R T^*}{M_1 \sqrt{\gamma R T_1}} = \frac{\sqrt{\gamma R T_0}}{M_1} \cdot \frac{T^*}{T_0} \cdot \sqrt{\frac{T_0}{T_1}}
\end{aligned}$$

mit $\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$ aus a) und $\frac{T^*}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^{*2}\right)^{-1} = \frac{2}{\gamma + 1}$ folgt

$$u_2 = \frac{\sqrt{\gamma R T_0}}{M_1} \cdot \frac{2}{\gamma + 1} \cdot \sqrt{1 + \frac{\gamma - 1}{2} M_1^2}$$

c) Reynoldszahl $Re = \frac{u \rho L}{\eta}$ mit $u = M\sqrt{\gamma R T}$
erweitern mit $\rho_0 = \frac{p_K}{R T_0}$ und η_0

$$Re = \frac{M\sqrt{\gamma RT} \cdot \rho_0 \frac{\rho}{\rho_0} \cdot L}{\eta_0 \frac{\eta}{\eta_0}}$$

$$Re = \frac{M\sqrt{\gamma RT} \cdot \frac{p_K}{RT_0} \cdot \left(\frac{T}{T_0}\right)^{\frac{1}{\gamma-1}} \cdot L}{\eta_0 \left(\frac{T}{T_0}\right)^{\frac{3}{4}}}$$

erweitern mit $\sqrt{T_0}$

$$Re = \frac{M\sqrt{\gamma RT_0} \cdot \sqrt{\frac{T}{T_0}} \cdot \frac{p_K}{RT_0} \cdot \left(\frac{T}{T_0}\right)^{\frac{1}{\gamma-1}} \cdot L}{\eta_0 \left(\frac{T}{T_0}\right)^{\frac{3}{4}}}$$

$$\rightarrow Re_1 = \frac{M_1 p_K \sqrt{\gamma} \cdot \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\frac{1}{2}} \cdot \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\frac{1}{\gamma-1}} \cdot L}{\eta_0 \sqrt{RT_0} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\frac{3}{4}}}$$

$$Re_1 = \frac{M_1 p_K \sqrt{\gamma} L}{\eta_0 \sqrt{RT_0}} \cdot \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{1}{4} - \frac{1}{\gamma-1}}$$

6. Aufgabe

a)

$$Ma = Ma' \Rightarrow \frac{v}{\sqrt{\gamma R_L T}} = \frac{v'}{\sqrt{\gamma R_L T'}} \Rightarrow v = v'$$

b) An der Platte $(y = 0)$ ist:

$$\frac{\partial v}{\partial x} = 0, \quad \frac{\partial u}{\partial y} \neq 0 \Rightarrow \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)_{y=0} \neq 0$$

$\Rightarrow$ Widerspruch zur drehungsfreien Strömung: $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

c) Flaches Wasser: $H/\lambda \ll 1$ und mit $tanhx \approx x$ sofern $x \to 0$ folgt

$$c = \sqrt{gH}$$

d) Die Normalkomponente der Geschwindigkeit stromab des Stoßes ist kleiner als die Normalkomponente der Geschwindigkeit stromauf. Da sich die tangentialen Geschwindigkeitskomponenten stromauf und stromab des Stoßes nicht unterscheiden, werden die Stromlinien zum Stoß hin umgelenkt.