3.2

Determine for the velocity field

$$u = u_0 \cos(\omega t) \quad v = -v_0 \sin(\omega t)$$

with $u_0 / \omega = v_0 / \omega = 1\, m$

a) the streamlines for $\omega t = 0, \pi / 2, \pi / 4,$

b) the pathlines,

c) the pathline for the particle, that is at $t = 0\, s$ in $x = 0\, m,\ y = 1\, m$!

3.2

a) $$\frac{dy}{dx} = \frac{v}{u} = -\frac{v_0}{u_0}\tan(\omega t)$$

Integration : $$y = \left[-\frac{v_0}{u_0}\tan(\omega t)\right] x + c$$

Straight lines with slope 0, -1, -∞

b)

$$x(t) = \int u \, dt + c_1 = \frac{u_0}{\omega}\sin(\omega t) + c_1$$

$$y(t) = \int v \, dt + c_2 = \frac{v_0}{\omega}\cos(\omega t) + c_2$$

$$\Longrightarrow \left(\frac{\omega}{u_0}\right)^2 (x - c_1)^2 + \left(\frac{\omega}{v_0}\right)^2 (y - c_2)^2 = 1$$

Circles with radius 1 m

c) Circle around the origin

A 2-dimensional frictionless flow field can be described for time $t > t_0$ can be described by the following velocity components:

$$u(y,t) = U_0 e^{-At} y$$
$$v(t) = V_0 e^{-Bt}$$

1. Determine the function $x = f(y)$ of that streamline which passes through the location $(0,1)$ at time $t_1 = \frac{1}{A-B}$. First, give the definition of the streamline for a 2-dimensional flow.
2. Determine the x-component of the path line $x(t)$ of a fluid particle that was at location $(0,0)$ at time $t_2 = 0$.

1. A streamline is defined as:

$$\frac{dx}{dy} = \frac{u}{v}$$

$$\frac{dx}{dy} = \frac{U_0}{V_0}\,\frac{e^{-At}}{e^{-Bt}}\,y$$

$$dx = \frac{U_0}{V_0}\,\frac{e^{-At}}{e^{-Bt}}\,y\,dy$$

$$x = \frac{1}{2}\frac{U_0}{V_0}\,e^{(A-B)t}y^2 + C_1$$

At $t = \frac{1}{A-B}$: $x = 0, y = 1$:

$$C1 = -\frac{1}{2}\frac{U_0}{V_0}\,e^{-1}$$

Inserting results

$$x = \frac{1}{2}\frac{U_0}{V_0}\frac{1}{e}(y^2 - 1)$$

2. $y = \int v(t)dt$

$$y = -\frac{V_0}{B}\, e^{-Bt} + C_2$$

At $t = 0$: $x = y = 0$:

$$C_2 = \frac{V_0}{B}$$

Inserting:

$$y = -\frac{V_0}{B}\,(e^{-Bt} - 1)$$

$$x = \int u(y,t)dt$$

$$x = \int U_0\, e^{-At}(-\frac{V_0}{B}\,(e^{-Bt} - 1))dt$$

$$x = \int -U_0\frac{V_0}{B}e^{-(A+B)t} + C_2U_0e^{-At}dt$$

$$x = \frac{U_0V_0}{B(A+B)}e^{-(A+B)t} - \frac{C_2U_0}{A}e^{-At} + C_3$$

At $t = 0 : x = y = 0$:

$$C_3 = \frac{U_0 V_0}{B}\left(\frac{1}{A} - \frac{1}{A+B}\right)$$

Insert:

$$x = \frac{U_0 V_0}{B}\left[\frac{1}{A+B}e^{-(A+B)t} - \frac{1}{A}e^{-At} + \left(\frac{1}{A} - \frac{1}{A+B}\right)\right]$$