turbulent boundary layers

good agreement between theory of laminar boundary layers and experiments until $x < x_{krit}$

$x < x_{krit}$: boundary layer becomes unstable and turbulent

- increased momentum exchange

  $\rightarrow$ larger shear stresses

  $\rightarrow$ larger friction $\rightarrow$ increased drag

- increased momentum exchange (mixing)

Reynolds averaging $\rightarrow$ Navier-Stokes equations
$Re \gg 1 \rightarrow$ turbulent boundary layer equations

turbulent boundary layers

conti: $$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

x-momentum: $$\rho \bar{u} \frac{\partial \bar{u}}{\partial x} + \rho \bar{v} \frac{\partial \bar{u}}{\partial y} = - \underbrace{\frac{\partial \bar{p}}{\partial x}}_{= \frac{d\bar{p}}{dx}} + \eta \frac{\partial^2 \bar{u}}{\partial y^2} \underbrace{- \rho \frac{\partial \overline{u'v'}}{\partial y}}_{\text{apparent stress}}$$

y-momentum: $$\frac{\partial \bar{p}}{\partial y} = 0$$

- unknown

$\rightarrow$ closure problem

$\rightarrow$ turbulence modelling (mixing layer)

turbulent flat plate

Approximation of the velocity profile

$$\frac{\bar{u}}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

$\rightarrow$ Computation of displacement thickness δ_1
momentum thickness δ_2

But: $\tau|_{\text{wall}} \neq \eta \frac{\partial u}{\partial y}|_{y=0} = \infty$

Assumption: the flow over a flat plate is similar
to a turbulent flow in a pipe

$$\rightarrow \lambda = \frac{0.316}{\sqrt[4]{Re}} \rightarrow \lambda = \frac{8\tau_w}{\rho \bar{u}_m^2} \rightarrow \tau_w$$

turbulent flat plate

$$\rightarrow \frac{\tau_w}{\rho U^2} = \frac{d\delta_2}{dx} \rightarrow \frac{\delta(x)}{x} = \frac{0.37}{\left(\frac{Ux}{\nu}\right)^{\frac{1}{5}}} = \frac{0.37}{\sqrt[5]{Re_x}}$$

$$\rightarrow \delta_{\text{turb}} \sim x^{4/5}$$

$$\rightarrow \delta_{\text{lam}} \sim x^{1/2}$$

16.1

A flat plate is flown against parallel to the surface with air.

$u_\infty = 45\ m/s \qquad \nu = 1,5 \cdot 10^{-5}\ m^2/s$

Determine

a) the transition point for $Re_{crit} = 5 \cdot 10^5$,

b) the velocity in the point $x = 0,1\ m,\ y = 2 \cdot 10^{-4}\ m$ using the Blasius solution. What is the coordinate y with the same velocity at $x = 0,15\ m$?

Sketch

c) the distribution of the boundary layer thickness $\delta(x)$ and a velocity profile for $x < x_{crit}$ and $x > x_{crit}$.

d) the wall shear stress as a function of x for $dp/dx < 0$, $dp/dx = 0$ and $dp/dx > 0$.

16.1

flat plate $\Longrightarrow$ Blasius is valid until Re_{crit}

$$u_\infty = 45\, m/s \qquad \nu = 1,5 \cdot 10^{-5}\, m^2/s$$

a)

$$\text{Re}_{crit} = 5 \cdot 10^5 = \frac{\rho u_\infty x_{krit}}{\eta} = \frac{u_\infty x_{crit}}{\nu}$$

$$x_{crit} = \text{Re}_{crit} \frac{\nu}{u_\infty} = 0.167m$$

b)

$$u\left(x = 0.1m\,; y = 2 \cdot 10^{-4} m\right) =?$$

$x = 0.1m < x_{crit} \rightarrow$ laminar boundary layer

$$\Longrightarrow \text{Blasius} \qquad \frac{u}{u_\infty} = g(\xi)$$

$$\xi = \frac{y}{\delta(x)} \qquad \text{with} \quad \delta(x) = \sqrt{\frac{\nu x}{u_\infty}}$$

$$\Rightarrow \frac{u}{u_\infty} = f\left(y\sqrt{\frac{u_\infty}{\nu x}}\right) \rightarrow \qquad \text{similar profiles}$$

$$\xi = y\sqrt{\frac{u_\infty}{\nu x}} = \frac{y}{x}\sqrt{\frac{u_\infty x}{\nu}} = \frac{y}{x}\sqrt{\mathrm{Re}_x}$$

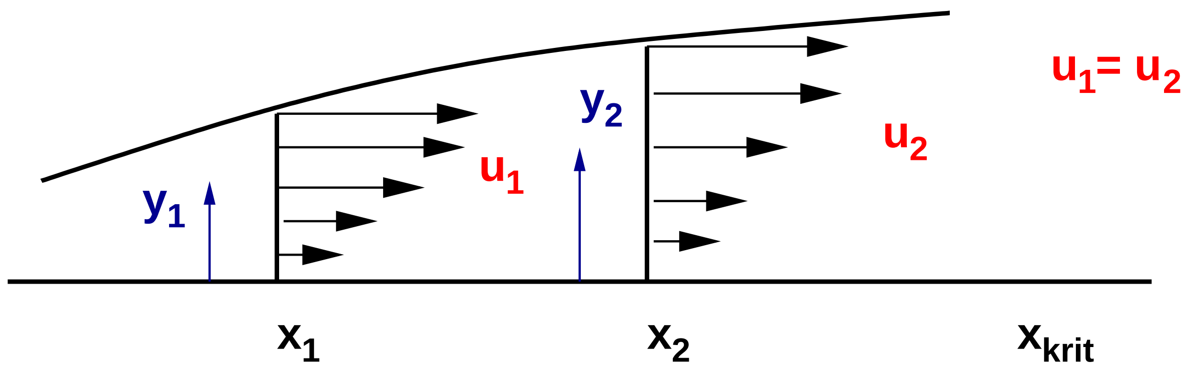
$$x = 0.1m \rightarrow \mathrm{Re}_x = 0.3 \cdot 10^6 \; ; \; y = 2 \cdot 10^{-4} m \rightarrow \xi = 1.095$$

$$\Rightarrow \text{diagram} \quad \frac{u}{u_\infty} = 0.36 \Rightarrow u(x,y) = 16.2\frac{m}{s}$$

16.1

$$u\,(x = 0.15m\,;\ y = ?) = u\left(x = 0.1m\,; y = 2 \cdot 10^{-4} m\right)$$

$x = 0.15$: the flow is still laminar



$$\mathrm{Re}_2 = \frac{u_\infty x_2}{\nu} = 4.5 \cdot 10^5$$

$$\frac{u_1}{u_\infty} = \frac{u_2}{u_\infty} \Rightarrow \xi_1 = \xi_2 = 1.095 \quad \text{self similar solution}$$

$$\xi = \frac{y}{x}\sqrt{\mathrm{Re}_x} \rightarrow \boxed{y = \frac{\xi x}{\sqrt{\mathrm{Re}_x}} = 2.45 \cdot 10^{-4} m > y_1}$$

c) sketch of $\delta(x)$

laminar: $\mathcal{O}$ (inertia) = $\mathcal{O}$ (friction)

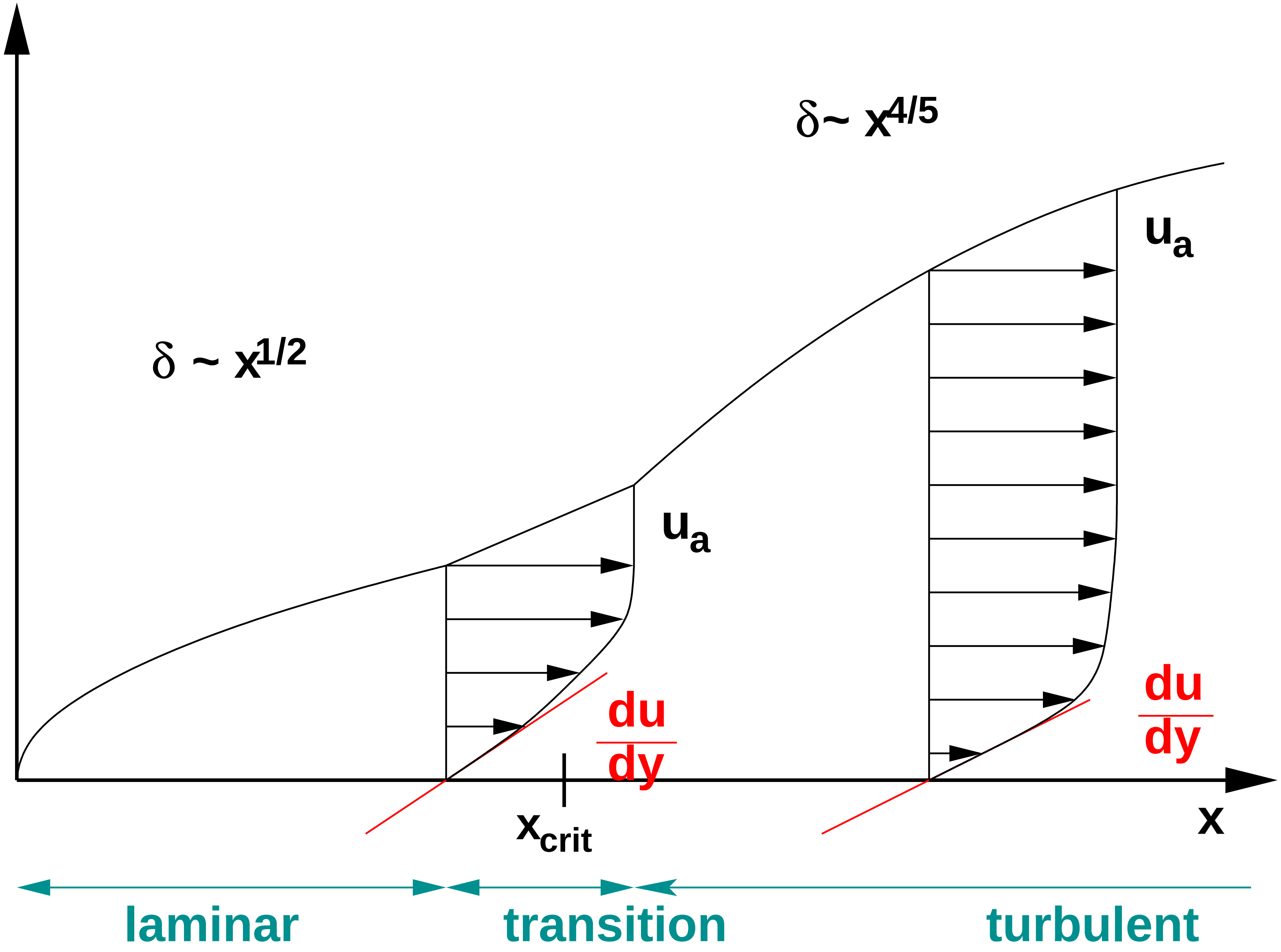
$$\rightarrow \frac{\delta}{x} = \mathcal{O}\left(\frac{1}{\sqrt{\mathrm{Re}_x}}\right) \Longrightarrow \delta(x) \sim \sqrt{x}$$

turbulent: high frequent oscillations

$$\mathcal{O}(\text{inertia}) \neq \mathcal{O}(\text{friction})$$

good agreement with experimental results

$$\left(\frac{u}{u_\infty}\right) = \left(\frac{y}{\delta}\right)^{1/7} \Longrightarrow \frac{\delta}{x} = \frac{0.37}{\sqrt[5]{\mathrm{Re}_x}} \Longrightarrow \delta(x) \sim x^{4/5}$$


δ ~ x1/2
δ~ x4/5
ua
ua
du
dy
du
dy
xcrit
x
laminar
transition
turbulent

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}^{\text{laminar}} < \left.\frac{\partial u}{\partial y}\right|_{y=0}^{\text{turbulent}}$$

$$|\tau_w|_{\text{laminar}} < |\tau_w|_{\text{turbulent}}$$

due to the energy input to the near wall layer by turbulence