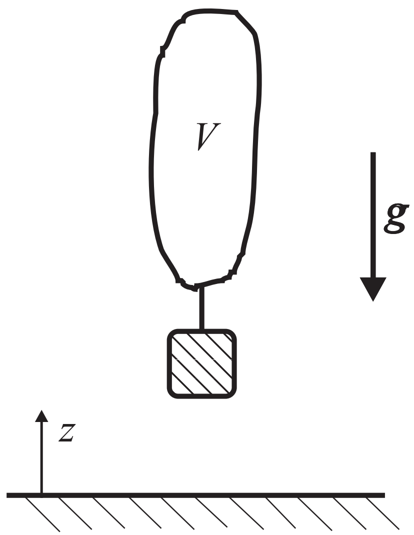
5.10

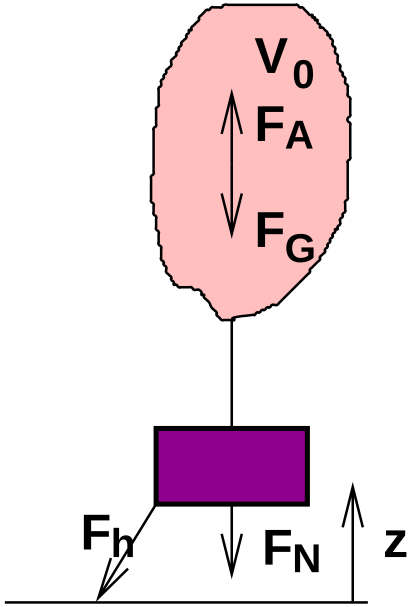
A wheather balloon with mass m and initial volume V_0 ascends in an isothermal atmosphere. Its envelope is loose up to the achievement of the maximal volume V_1.



$p_0 = 10^5\ N/m^2 \quad \rho_0 = 1,27\ kg/m^3 \quad m = 2,5\ kg \quad V_0 = 2,8\ m^3 \quad V_1 = 10\ m^3$
$R = 287\ Nm/kgK \quad g = 10\ m/s^2$

a) What is the necessary force to hold down the balloon before launch?
b) In what altitude the balloon reaches its amximum volume V_1?
c) What ceiling reaches the balloon?

a) before launch



$$\Sigma F_z = 0 = F_A - F_G - F_N - F_H$$
$$F_H = F_A - (F_N + F_G) =$$
$$= \rho_L(z=0)V_0 g - mg =$$
$$= (\rho_0 V_0 - m)g) = 10.6 \text{ N}$$

b) z for $V = V_1$

perfectly loose for $V < V_1$
the envelope can change its volume

$$m_G = const = \rho_G V = \frac{p_G}{R_G T_G} V$$
$$p_i = p_a$$

The movement is quite slow: $\longrightarrow T_i = T_a$

Assumption: isothermal atmosphere $\longrightarrow$ scale height relation

$$V = \frac{m_G R_G T_G}{p_G} \sim \frac{1}{p_G} = \frac{1}{p_L}$$

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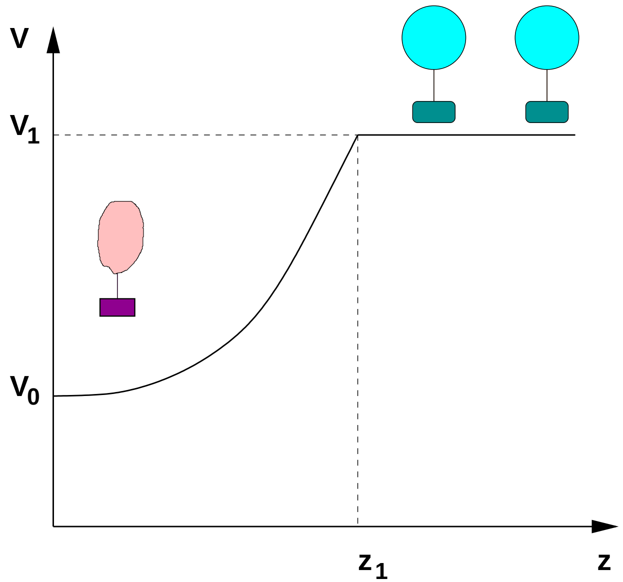


$$z = 0 \longrightarrow V = V_0$$

$$V(z) = V_0 \mathrm{e}^{\frac{gz}{R_L T_0}}$$

$$V_1 = V(z = z_1) = V_0 \mathrm{e}^{\frac{gz_1}{R_L T_0}}$$

$$\longrightarrow z_1 = \ln \frac{V_1}{V_0} \frac{R_L T_0}{g} \qquad \frac{p_0}{\rho_0} = R_L T_0$$

$$\longrightarrow \boxed{z_1 = \frac{p_0}{\rho_0 g} \ln \frac{V_1}{V_0}} = 10.0 \text{ km}$$

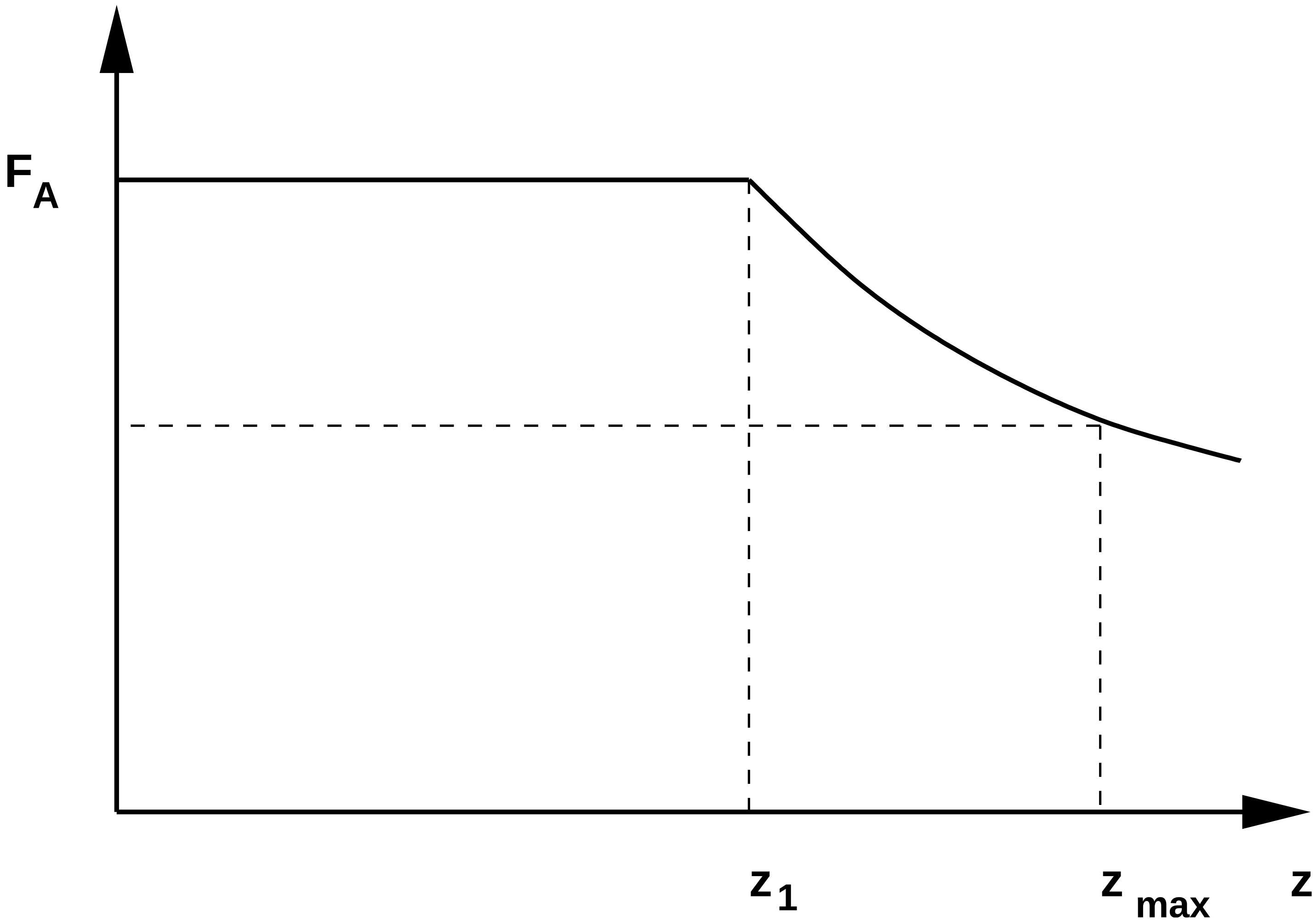
c)

$$z \leq z_1 : F_A = \rho_L V g = \frac{\cancel{p_L}}{R_L T_0} \frac{m_g R_G T_G g}{\cancel{p_G}} = const$$

$\longrightarrow$ The lift force onto a loose balloon is constant.
$(T_L = T_G, g = const)$

$$F_A(z \leq z_1) = \rho_0 V_0 g = \rho_L(z_1) V_1 g$$
$$F_A(z > z_1) = \rho_L(z) V_1 g$$

$$F_A(z > z_1) = F_A(z \leq z_1) \cdot \frac{\rho_L(z)}{\rho_L(z_1)} =$$
$$= F_A(z \leq z_1) \cdot \mathrm{e}^{-\frac{g(z-z_1)}{R_L T_0}}$$



F_A
z_1
z_{max}
z

5.10

ceiling: $\Sigma F_z = 0 \longrightarrow mg = F_A$

$$= mg - \rho(z_{max}) V_1 g$$

$$\longrightarrow \rho_0 \mathrm{e}^{-\frac{g z_{max}}{R_L T_0}} = \frac{m}{V_1}$$

$$z_{max} = \frac{R_L T_0}{g} \ln \frac{V_1 \rho_0}{m} \frac{p_0}{\rho_0 g} \ln \frac{V_1 \rho_0}{m} =$$

$$= 12.8\,\mathrm{km}$$