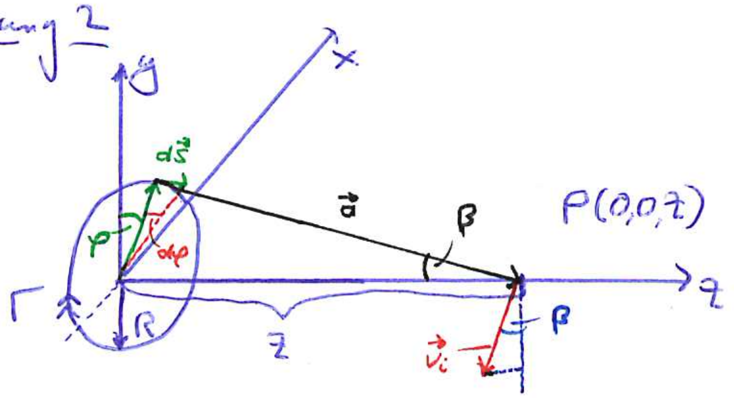
Übung 2



$$d\vec{v}_i = -\frac{\Gamma}{4\pi} \frac{\vec{a} \times d\vec{s}}{\|\vec{a}\|^3}$$

$$\vec{v}_i = -\frac{\Gamma}{4\pi} \oint_l \frac{\vec{a} \times d\vec{s}}{\|\vec{a}\|^3}$$

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1. Mögl.: $\|\vec{v}_i\| = \frac{\Gamma}{4\pi} \oint \frac{\|\vec{a} \times d\vec{s}\|}{\|\vec{a}\|^3}$

$$\|\vec{a} \times d\vec{s}\| = \|\vec{a}\| \cdot \|d\vec{s}\| \cdot \sin \underset{\pi/2}{\alpha} = a \cdot \underbrace{ds}_{R\,d\varphi} = a \cdot R \cdot d\varphi$$

$a = \sqrt{R^2 + z^2}$

$$\|\vec{v}_i\| = \frac{\Gamma}{4\pi} \oint_0^{2\pi} \frac{\cancel{a} R\, d\varphi}{a^{\cancel{3}2}} = \frac{\Gamma}{4\pi} \oint_0^{2\pi} \frac{R}{a^2}\, d\varphi$$

$$= \frac{\Gamma R}{4\pi a^2} \cdot 2\pi = \frac{\Gamma R}{2a^2} = \frac{\Gamma R}{2(R^2+z^2)}$$

wegen symmetrie $v_{ix}, v_{iy} = 0$

$$v_{iz} = -\|\vec{v}_i\| \cdot \sin\beta$$
$$= -\|\vec{v}_i\| \cdot \frac{R}{a}$$
$$= -\frac{\Gamma R}{2(R^2+z^2)} \cdot \frac{R}{\sqrt{R^2+z^2}} = \frac{-\Gamma R^2}{2(R^2+z^2)^{3/2}}$$

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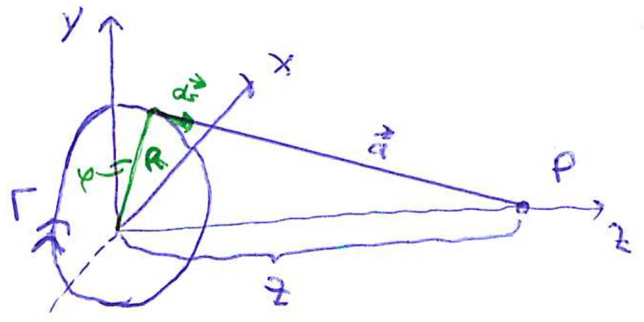
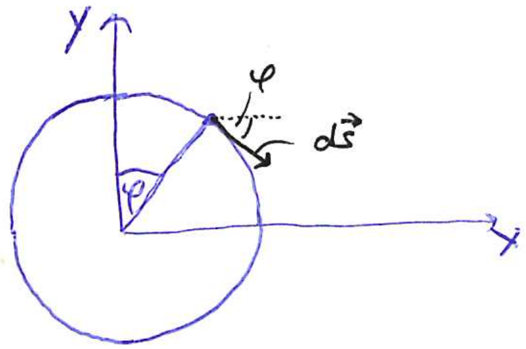
2. Mögl.: $\vec{v}_i = \frac{-\Gamma}{4\pi} \oint_l \frac{\vec{a} \times d\vec{s}}{\|\vec{a}\|^3}$



$$\vec{a} = \begin{pmatrix} -R\sin\varphi \\ -R\cos\varphi \\ z \end{pmatrix}$$

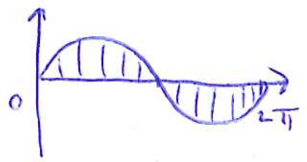


$$d\vec{s} = \begin{pmatrix} R\,d\varphi\cos\varphi \\ -R\,d\varphi\sin\varphi \\ 0 \end{pmatrix}$$

$$\vec{a} \times d\vec{s} = \begin{pmatrix} -R\sin\varphi \\ -R\cos\varphi \\ z \end{pmatrix} \times \begin{pmatrix} R\,d\varphi\cos\varphi \\ -R\,d\varphi\sin\varphi \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} z R\,d\varphi\sin\varphi \\ z R\,d\varphi\cos\varphi \\ R^2\sin^2\varphi\,d\varphi + R^2\cos^2\varphi\,d\varphi \end{pmatrix} = \begin{pmatrix} zR\sin\varphi\,d\varphi \\ zR\cos\varphi\,d\varphi \\ R^2\,d\varphi \end{pmatrix}$$

$$\vec{V}_i = -\frac{\Gamma}{4\pi \underbrace{\sqrt{R^2+z^2}^{\,3}}_{a^3}} \left(\vec{e}_x \cdot \left[\oint_0^{2\pi} zR\,\overset{0}{\cancel{\sin\varphi}}\,d\varphi \right] + \vec{e}_y \left[\oint_0^{2\pi} z\cdot R\,\overset{0}{\cancel{\cos\varphi}}\,d\varphi \right] + \vec{e}_z \cdot \left[\oint_0^{2\pi} R^2\,d\varphi \right] \right)$$



$$\vec{V}_i = \frac{-\Gamma R^2 \cdot 2\pi}{4\pi\,(R^2+z^2)^{3/2}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

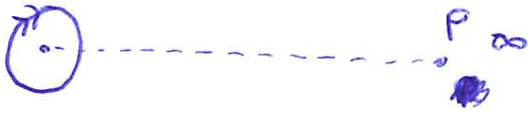
$$= -\frac{\Gamma R^2}{2\,(R^2+z^2)^{3/2}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Grenzwerte: $b = 0$

$$V_{iz} = \frac{-\Gamma R^2}{2\,(R^2)^{3/2}} = -\frac{\Gamma}{2R}$$

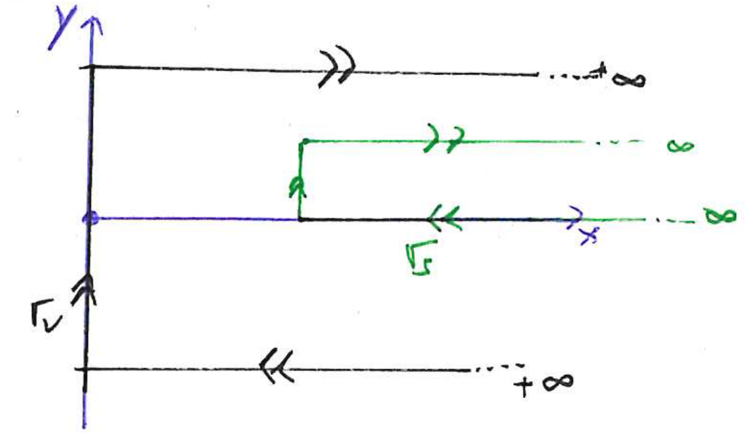
- $b \to \infty$

$\hookrightarrow V_{iz} = 0$



A3

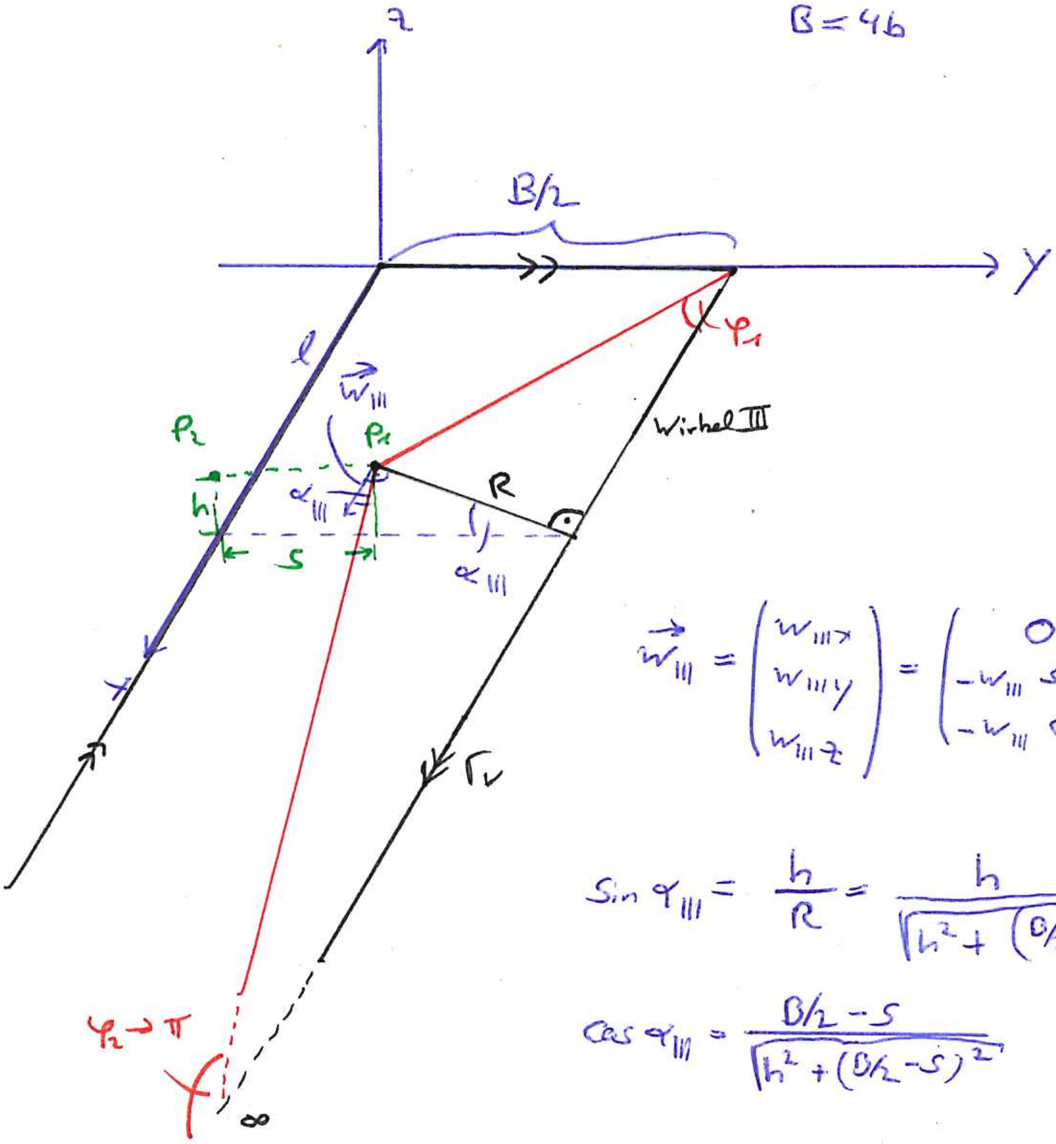
1)



2)

$$w_{III} = \frac{\Gamma_v}{4\pi R}\left(\cos\varphi_1 - \cos\varphi_2\right)$$

geg: $h = b$
$l = 5b$
$s = b$
$B = 4b$



$$\vec{w}_{III} = \begin{pmatrix} w_{III\,x} \\ w_{III\,y} \\ w_{III\,z} \end{pmatrix} = \begin{pmatrix} 0 \\ -w_{III}\sin\alpha_{III} \\ -w_{III}\cos\alpha_{III} \end{pmatrix}$$

$$\sin\alpha_{III} = \frac{h}{R} = \frac{h}{\sqrt{h^2 + \left(B/2 - s\right)^2}} = \frac{b}{\underbrace{\sqrt{2}\,b}_{R}}$$

$$\cos\alpha_{III} = \frac{B/2 - s}{\sqrt{h^2 + \left(B/2 - s\right)^2}}$$

$$w_{III} = \frac{\Gamma_v}{4\pi R}\left(\cos\varphi_1 + 1\right)$$

Punkt P1

$$w_{III} = \frac{\Gamma_V}{4\pi\sqrt{2}b}\Bigg(\underbrace{\frac{l}{\sqrt{l^2+h^2+(B/2-s)^2}}}_{\cos\varphi_1}+1\Bigg)$$

$$= \frac{\Gamma_V}{4\pi\sqrt{2}b}\left(\frac{5b}{\sqrt{25b^2+2b^2}}+1\right)$$

$$= \frac{\Gamma_V}{4\pi\sqrt{2}b}\left(\frac{5}{\sqrt{27}}+1\right)$$

$$\vec{w}_{III,P_1} = 0\cdot\vec{e}_x - \frac{\Gamma_V}{4\pi\sqrt{2}b}\left(\frac{5}{\sqrt{27}}+1\right)\cdot\frac{b}{\sqrt{2}b}\cdot\vec{e}_y$$

$$- \frac{\Gamma_V}{4\pi\sqrt{2}b}\left(\frac{5}{\sqrt{27}}+1\right)\frac{1}{\sqrt{2}}\vec{e}_z$$

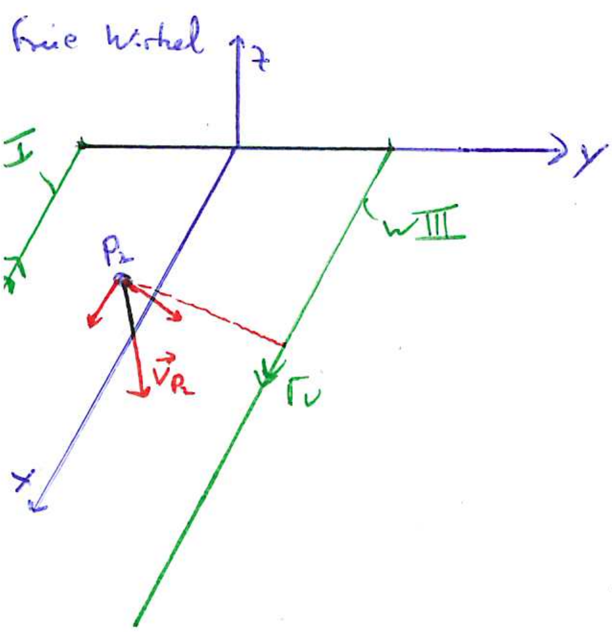
Punkt P2 (analog → $s=0$) $R=\sqrt{5}b$

$$w_{III} = \frac{\Gamma_V}{4\pi\sqrt{5}b}\left(\frac{5}{\sqrt{30}}+1\right)$$

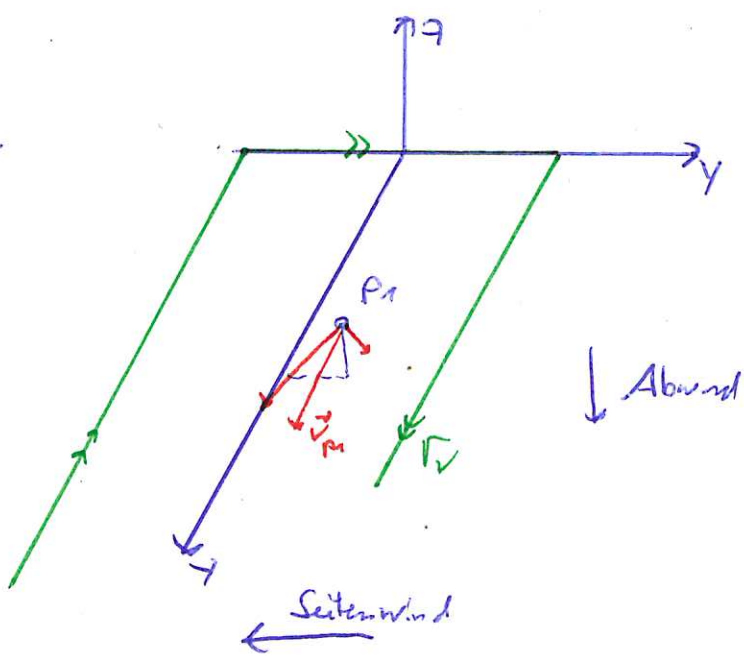
$$\vec{w}_{III,P_2} = 0\cdot\vec{e}_x - \frac{\Gamma_V}{4\pi\sqrt{5}b}\left(\frac{5}{\sqrt{30}}+1\right)\frac{1}{\sqrt{5}}\vec{e}_y$$

$$- \frac{\Gamma_V}{4\pi\sqrt{5}b}\cdot\left(\frac{5}{\sqrt{30}}+1\right)\frac{1}{\sqrt{5}}\cdot 2\,\vec{e}_z$$

3. Fluglage



↳ Abwind ↓



geb. Wirbel: