Aerodynamik I
Übung I

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AIA, Raum 127

Vorlesung Di: 10:30-12:00
Übung Mo: 8:15-9:00

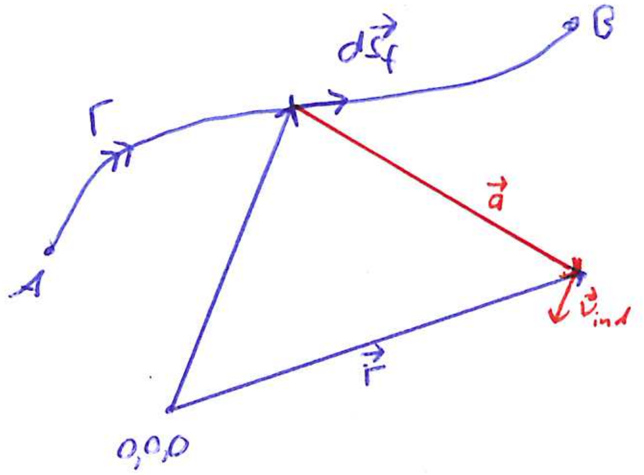
Sprechstunde: Mo: 9:30-10:15, Raum 104, Herr Wegmann

Klausur: Mo.: 2.9.19

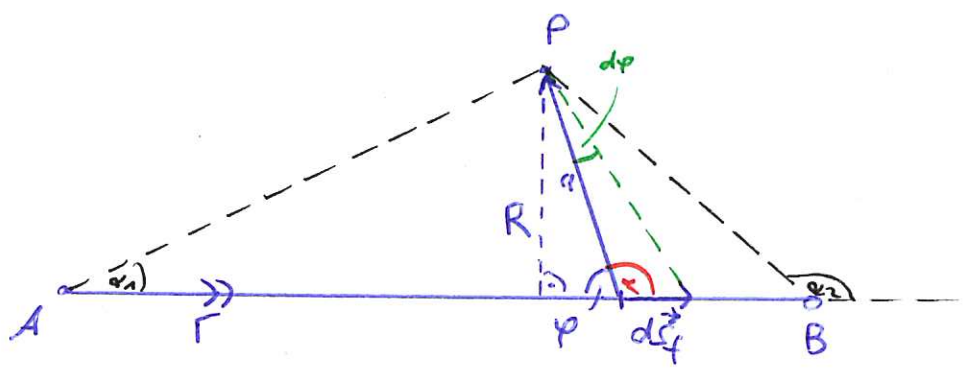
Biot-Savart:

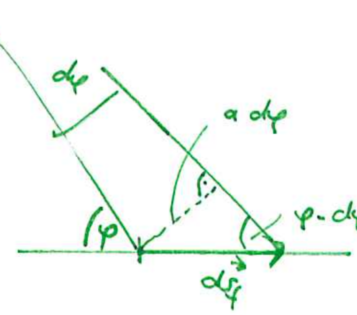
$$\vec{V} = -\frac{\Gamma}{4\pi}\oint_l \frac{\vec{a}\times d\vec{s}_f}{\|\vec{a}\|^3}$$

Berechnung des durch das Wirbelsystem induzierte Geschwindigkeitsfeldes das der Potentialtheorie genügt



A1 Stabwirbel





$$\|\vec{V}\| = \frac{\Gamma}{4\pi}\oint_l \frac{\|\vec{a}\times d\vec{s}_f\|}{\|\vec{a}\|^3} \qquad \|\vec{a}\times d\vec{s}_f\| = \|\vec{a}\|\cdot\|d\vec{s}_f\|\cdot\sin\alpha$$

$$\|\vec{v}\| = \frac{\Gamma}{4\pi}\oint_l \frac{\sin\alpha \, \|d\vec{s}_f\|}{\|\vec{a}\|^2}$$

$$\frac{R}{a} = \sin\varphi \quad \Big| \; d\varphi \text{ klein} \Rightarrow \frac{a \cdot d\varphi}{ds_f} = \sin\varphi$$

$$\Rightarrow \boxed{\frac{R}{a} = \frac{a\, d\varphi}{ds_f}}$$

$$\varphi = \pi - \alpha \quad \Rightarrow \quad d\varphi = -d\alpha$$

$$ds_f = \frac{-a^2}{R}\, d\alpha$$

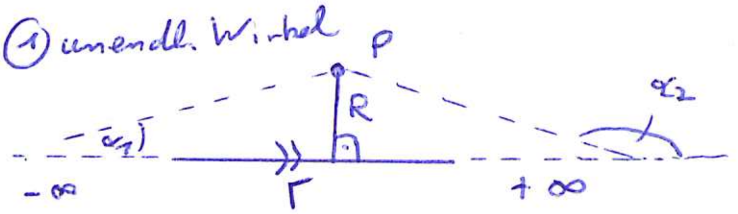
$$\|\vec{v}\| = \frac{\Gamma}{4\pi}\oint_l \frac{\|d\vec{s}_f\|}{\|\vec{a}\|^2}\sin\alpha = \frac{\Gamma}{4\pi}\oint_l \frac{\sin\alpha \, d\alpha}{R}$$

$$= \frac{\Gamma}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\sin\alpha\, d\alpha}{R} = \frac{\Gamma}{4\pi}\left(-\frac{\cos\alpha}{R}\right)\Bigg|_{\alpha_1}^{\alpha_2}$$

$$\Rightarrow \boxed{v_{ind} = \frac{\Gamma}{4\pi R}(\cos\alpha_1 - \cos\alpha_2)}$$

A2:

(1) unendl. Wirbel



$$v = \frac{\Gamma}{4\pi R}(\cos\alpha_1 - \cos\alpha_2)$$

$\alpha_1 \to 0$
$\alpha_2 \to \pi$

$$v = \frac{\Gamma}{2\pi R}$$

Abstand $\perp$
+ Vorzeichen aus phys. Verständnis

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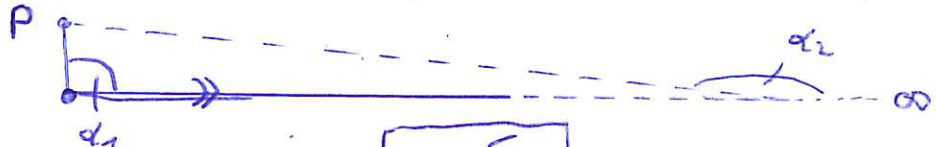
(2) halbunendl. Wirbel



$\alpha_1 = \pi/2 \; ; \; \alpha_2 \to \pi \quad \Rightarrow \quad \boxed{v = \frac{\Gamma}{4\pi R}}$

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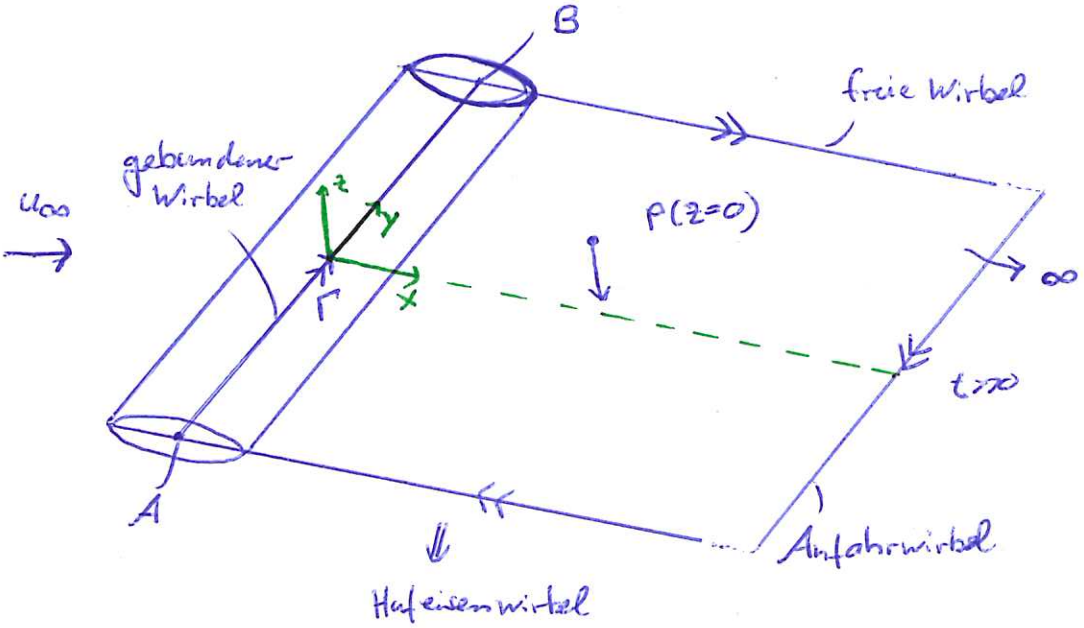
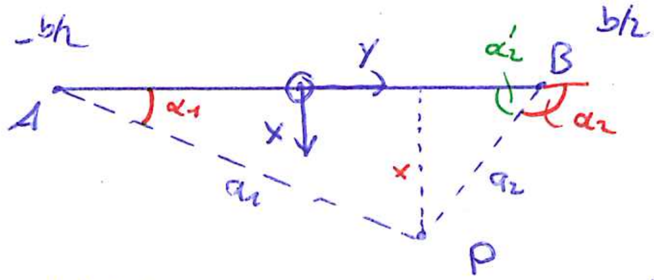
(3) kreisförmiger Wirbel $\Rightarrow$ nächste

(4) gebundener Wirbel:



$$\frac{d\Gamma}{dt} = 0$$

$$|w| = \frac{\Gamma}{4\pi R}(\cos\alpha_1 - \cos\alpha_2)$$



$$\cos\alpha_1 = \frac{b/2 + y}{a_1}$$

$$a_1 = \sqrt{x^2 + (b/2 + y)^2}$$

$$\cos\alpha_2' = \frac{b/2 - y}{a_2}$$

$$\alpha_2' = \pi - \alpha_2$$

$$a_2 = \sqrt{(b/2 - y)^2 + x^2}$$

$$\cos\alpha_2' = \cos(\pi - \alpha_2) =$$

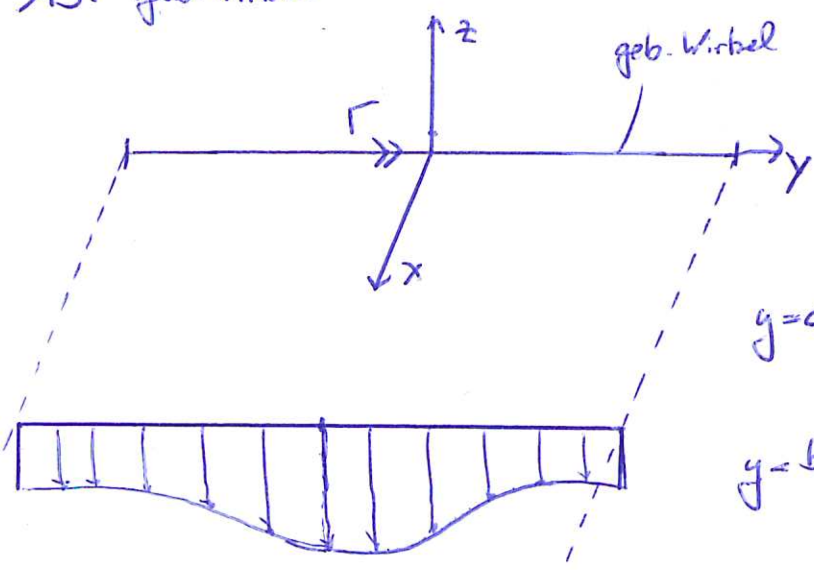
$$= -\cos\alpha_2$$

$$\cos\alpha_2 = \frac{-(b/2 - y)}{a_2}$$

$$|w| = \frac{\Gamma}{4\pi x}\left(\frac{b/2 + y}{\sqrt{x^2 + (b/2 + y)^2}} + \frac{b/2 - y}{\sqrt{(b/2 - y)^2 + x^2}}\right)$$

$$\vec{v}_P(x, y, 0) = \begin{pmatrix} 0 \\ 0 \\ -|w| \end{pmatrix}$$

A3: geb. Wirbel



$$y=0: \frac{\Gamma}{4\pi x} \frac{b}{\sqrt{(b/2)^2+x^2}}$$

$$y=b/2: \frac{\Gamma}{4\pi x} \frac{b}{\sqrt{b^2+x^2}}$$

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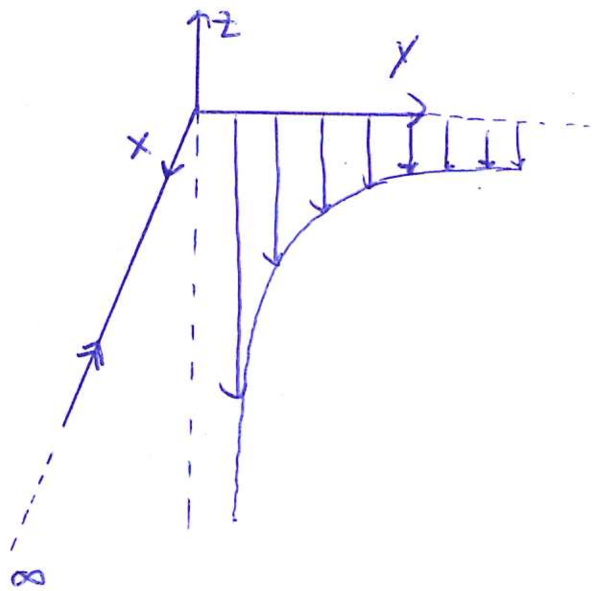
freie Wirbel



$$|w| = \frac{\Gamma}{4\pi R}$$