3) Bestimmung von C:

NACA 0024 → 24% max. Dicke → $d_{max}/l = 0,24$ → $z^{(t)}_{max} = 0,5 \cdot d_{max}/l$
$= 0,12$

z_{max} liegt bei $\frac{dz^{(t)}}{dX} = 0$

$$\frac{dz^{(t)}}{dX} = \frac{d}{dX}\left(32C\sqrt{X(1-X)}\cdot(1-X(1-X))\right) = 32C\,\frac{d}{dX}\left(\sqrt{X(1-X)} - \sqrt{X(1-X)}\cdot X(1-X)\right)$$

$$= 32C\left(\frac{1-2X}{2\sqrt{X(1-X)}} - \frac{(1-2x)X(1-x)}{2\sqrt{X(1-X)}} - (1-2x)\sqrt{X(1-X)}\right)$$

$$= 32C\left(\frac{(1-2X)-(1-2x)X(1-X)-2(1-2x)X(1-X)}{2\sqrt{X(1-X)}}\right)$$

$$= 32C\left(\frac{(1-2X)\cdot(1-3X(1-X))}{2\sqrt{X(1-X)}}\right)$$

$$\frac{dz^{(t)}}{dX} \stackrel{!}{=} 0$$

$$(1-2X_d) \vee \left(1-3X_d(1-X_d)\right) = 0$$

$$X_d = 0{,}5 \quad \vee \quad X_d^2 - X_d + 1/3 = 0$$

$$X_d = 1/2 \pm \sqrt{1/4 - 1/3}$$

(keine reellen Lsg)

$\hookrightarrow \quad X_d = 0{,}5$

$$z^{(t)}(X_d) = 32C \sqrt{X_d(1-X_d)} \cdot \left(1 - X_d(1-X_d)\right) \stackrel{!}{=} 0{,}12$$

$$\hookrightarrow \quad 32C \sqrt{1/2(1-1/2)} \cdot \left(1 - 1/2(1-1/2)\right) \stackrel{!}{=} 0{,}12$$

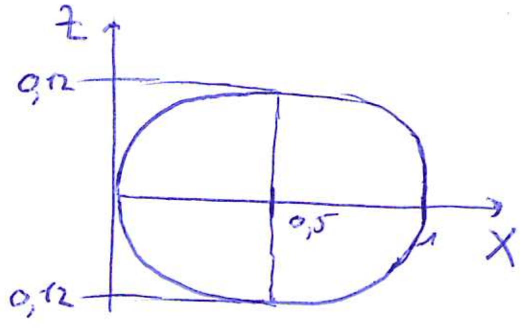
$\hookrightarrow \quad C = 0{,}01$

4] Skizze!

Winkel an Vorder u. Hinterkante

$$\tau_V = \arctan\left(\frac{dz^{(t)}(x=0)}{dX}\right) = \arctan(\infty) \Rightarrow \tau_V = 90°$$

$$\tau_H = \arctan\left(\frac{dz^{(t)}(x=1)}{dX}\right) = \arctan(\infty) \Rightarrow \tau_H = 90°$$



5] $w_a = \frac{w}{u_\infty}$?

$$w(X) = \pm 1/2\, q(X) \; ; \; q(X) = 2u_\infty \frac{dz^{(t)}}{dX}$$

$$\hookrightarrow \quad w_a(X) = \frac{w(X)}{u_\infty} = \pm \frac{dz^{(t)}}{dX}$$

$$w_a(X) = \pm 32C\left(\frac{(1-2X)\,(1-3X(1-X))}{2\sqrt{X(1-X)}}\right)$$

$$= \mp 32C\,(2X-1)\left(\frac{1-3X(1-X)}{2\sqrt{X(1-X)}}\right)$$

Transformation

$$2X-1 = \cos(\varphi)\;;\quad 2\sqrt{X(1-X)} = \sin(\varphi)\;;\quad X(1-X) = 1/4\,\sin^2(\varphi)$$

$$\Rightarrow\quad w_a(\varphi) = \mp 32C\cos(\varphi)\left(\frac{\left(1-\frac{3}{4}\sin^2(\varphi)\right)}{\sin(\varphi)}\right)$$

Alternative LSG.

$$w_a = \frac{w}{u_\infty} = \pm\frac{dz^{(t)}}{dX} = \pm\frac{dz^{(t)}}{d\varphi}\cdot\frac{d\varphi}{dX}$$

$$z^{(t)}(\varphi) = C\left(13\sin(\varphi) + \sin(3\varphi)\right) \qquad \frac{d\varphi}{dX} = \frac{-2}{\sin(\varphi)}$$

$$w_a = \frac{w}{u_\infty} = \mp 2C\left(\frac{13\cos(\varphi) + 3\cos(3\varphi)}{\sin(\varphi)}\right)$$

$$= \mp 2C\left(\frac{13\cos(\varphi) + 3\left(4\cos^3(\varphi) - 3\cos(\varphi)\right)}{\sin\varphi}\right)$$

$$= \mp 2C\,\frac{4\cos\varphi + 12\cos^3\varphi}{\sin\varphi}$$

$$= \mp 8C\cos\varphi\,\frac{(1+3\cos^2\varphi)}{\sin\varphi}$$

$$= \mp 8C\cos\varphi\left(\frac{1+3(1-\sin^2\varphi)}{\sin\varphi}\right)$$

$$= \mp 32\cos\varphi\left(\frac{1-\frac{3}{4}\sin^2\varphi}{\sin\varphi}\right)$$

Transformation:

$$\cos\varphi = 2X-1\;;\quad \sin\varphi = 2\sqrt{X(1-X)} \quad \frac{3}{4}\sin^2(\varphi) = 3X(1-X)$$

$$\Rightarrow\quad w_a(X) = \mp 32C(2X-1)\left(\frac{1-3X(1-X)}{2\sqrt{X}\,(1-X)}\right)$$

6)

$$u(X) = \frac{1}{2\pi}\int_0^1 q(X')\,\frac{dX'}{X-X'} \qquad q(X') = 2u_\infty \frac{dz^{(t)}}{dX'}$$

$$\frac{dz^{(t)}}{dX'} = \frac{dz^{(t)}}{d\varphi'}\cdot\frac{d\varphi'}{dX'}$$

Transformation:

$$X = 1/2(1+\cos\varphi) \qquad X' = 1/2(1+\cos\varphi') \qquad \frac{d\varphi'}{dX'} = \frac{-2}{\sin(\varphi')}$$

$$\int_0^1 \to \int_\pi^0 \to -\int_0^\pi$$

$$\hookrightarrow \quad u(\varphi) = -\frac{1}{2\pi}\int_0^\pi 2u_\infty \frac{dz^{(t)}}{d\varphi'}\cdot\frac{d\varphi'}{dX'}\cdot\frac{-1/2\sin(\varphi')\,d\varphi'}{1/2(1+\cos\varphi) - 1/2(1+\cos\varphi')}$$

$$= -\frac{u_\infty}{\pi}\int_0^\pi C\left(13\cos(\varphi') + 3\cos(3\varphi')\right)\cdot\frac{\cancel{2}}{\cancel{\sin(\varphi')}}\cdot\frac{-\cancel{1/2}\,\cancel{\sin(\varphi')}\,d\varphi'}{1/2(\cos\varphi - \cos\varphi')}$$

$$= -\frac{2u_\infty C}{\pi}\int_0^\pi \frac{13\cos\varphi' + 3\cos\varphi'}{\cos\varphi - \cos\varphi'}$$

$$= -\frac{2u_\infty C}{\pi}\left(13\int_0^\pi \frac{\cos\varphi'}{\cos\varphi - \cos\varphi'}\,d\varphi' + 3\int_0^\pi \frac{\cos(3\varphi')}{\cos\varphi - \cos\varphi'}\,d\varphi'\right)$$

$$= -\frac{2u_\infty C}{\pi}\left(-13\pi\frac{\sin\varphi}{\sin\varphi} - 3\pi\frac{\sin(3\varphi)}{\sin\varphi}\right)$$

$$\Rightarrow \quad u_a = \frac{u}{u_\infty} = 2C\left(13 + 3\frac{\sin(3\varphi)}{\sin\varphi}\right)$$

$\hookrightarrow$ $u_a(X)$?

$$u_a = 2C\left(13 + 3\,\frac{3\sin(\varphi) - 4\sin^3\varphi}{\sin\varphi}\right)$$

$$= 2C(13 + 9 - 12\sin^2\varphi)$$

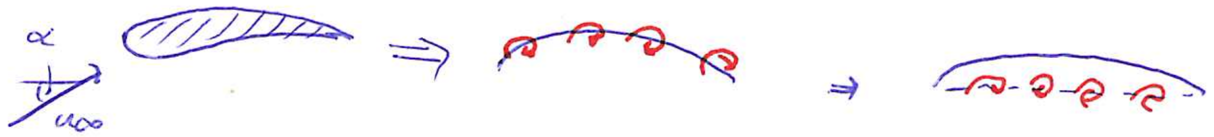
$$= 2C(48X^2 - 48X + 22)$$

Skeletttheorie (Auftriebsproblem)

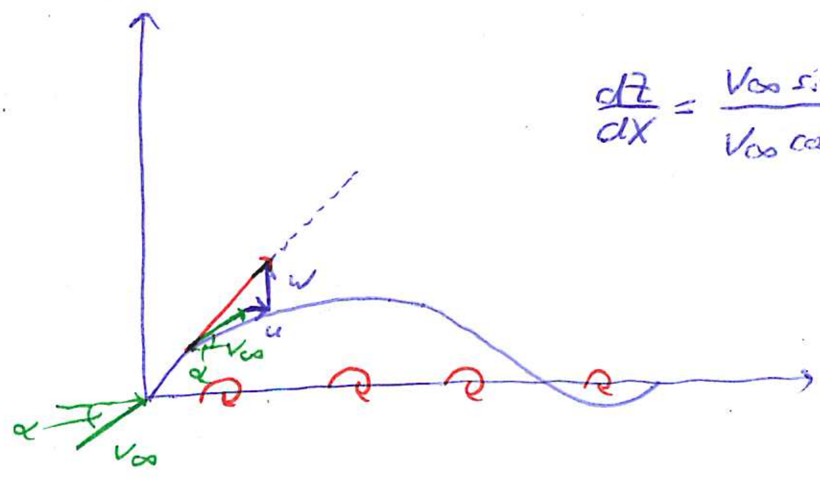
Idee: Strömung um auftriebserzeugende Profile wird überlagert von elementaren Wirbelströmungen (Wirbeldichteverteilung $\gamma(X)$) + V_∞

Voraussetzungen: • inkomp. Potentialtheorie
→ auf $Ma \leq 0{,}8$ erweiterbar mit entsprechender Korrektur für komp. Strömungen (z.B. Prandtl-Glauert Regel)

• kleine Störungen
- dünne Profile
- kleine Wölbung
- kleine Anstell-winkel



Kin. Randbed. der Skelettheorie ($V_n = 0$)



$$\frac{dZ}{dX} = \frac{V_\infty \sin\alpha + w}{V_\infty \cos\alpha + u}$$

kleine Winkel $\sin\alpha \sim \alpha$
$\cos\alpha \sim 1$

Störungen: $u \ll V_\infty$

$$\frac{dZ}{dX} = \frac{V_\infty \alpha + w}{V_\infty}$$

$$\boxed{\alpha - \frac{dZ}{dX} = -\frac{w}{V_\infty}}$$

kinematische RB d. Skeletttheorie

Störgeschw. $(u(X), w(X))$

$$u(X) = \overset{\text{oben}}{\underset{\text{unten}}{\pm}} \frac{\gamma(X)}{2}$$

$$w(X) = -\frac{1}{2\pi} \int_0^1 \frac{\gamma(X')}{X - X'} \, dX'$$

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zur Best. des Integrals → Substitution

$$X = 1/2 (1 + \cos\varphi)$$

$$X = 0 \;\hat{=}\; \varphi = \pi$$
$$X = 1 \;\hat{=}\; \varphi = 0$$

$$\int_0^1 \ldots dX' = -\int_\pi^0 \ldots \; 1/2 \sin\varphi' d\varphi'$$
$$= \int_0^\pi \ldots d\varphi'$$

Birnbaum - Ackermann Ansatz für $\gamma(\varphi)$

$$\gamma(\varphi) = 2 V_\infty \left(A_0 \tan \varphi/2 + \sum_{n=1}^{N} A_n \sin(n\varphi) \right)$$

A_0: Anstellwinkel

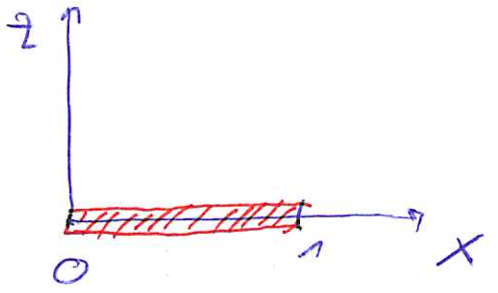
A_1: Wölbung

A_2: S-Schlag

wenn $\frac{dz}{dx} = 0 \Rightarrow A_n$ mit $n = 1, \ldots, N$ sind gleich Null

$\boxed{A1}$ (Ebene Platte in d. Skeletttheorie)

1. kin RB d. SST-Th. $\boxed{\alpha - \frac{dZ}{dX} = - \frac{w}{V_\infty}}$



$$\left.\begin{array}{l} Z(X) = 0 \\ \frac{dZ}{dX}(X) = 0 \end{array}\right\} \forall\, x \in [0,1]$$

$$\boxed{\alpha V_\infty = -w}$$

$$w = -\frac{1}{2\pi} \int_0^1 \frac{\gamma(X')}{X - X'}\, dX'$$

$$\alpha V_\infty = +\frac{1}{2\pi} \int \ldots\, dX'$$

Ansatz für $\gamma(\varphi) = 2 V_\infty A_0 \tan(\varphi/2)$

$$\alpha V_\infty = \frac{1}{2\pi} \int_{\cancel{\pi}\;0}^{\cancel{0}\;\pi} \frac{\gamma(\varphi')}{\frac{1}{2}(\cos\varphi - \cos\varphi')} \left(+\tfrac{1}{2}\sin\varphi'\right) d\varphi'$$

$$\left(X = \tfrac{1}{2}(1+\cos\varphi) \right)$$

FS: $1 - \cos\varphi'$

$$= \frac{1}{2\pi} \int_0^\pi \frac{2V_\infty A_0 \tan\frac{\varphi}{2} \sin\varphi'}{\cos\varphi - \cos\varphi'}\, d\varphi' =$$

$$= \frac{V_\infty A_0}{\pi} \int_0^\pi \frac{1 - \cos\varphi'}{\cos\varphi - \cos\varphi'}\, d\varphi'$$

$$\frac{\cos(1\cdot\varphi')}{\cos\varphi - \cos\varphi'}$$

$$\frac{\cos(0\cdot\varphi)}{\cos\varphi - \cos\varphi'}$$

$$\frac{-\pi \sin(1\cdot\varphi)}{\sin\varphi}$$

$$= \frac{-\pi \sin(0\cdot\varphi)}{\sin\varphi}$$

$\gamma(\varphi) = 2V_\infty \alpha \tan(\varphi/2)$

$$\Rightarrow \alpha V_\infty = \frac{V_\infty A_0}{\pi}(0 + \pi) = V_\infty A_0 \Rightarrow \boxed{A_0 = \alpha}$$

2. $c_p = \stackrel{o}{\underset{u}{\ominus \oplus}} \frac{\gamma(x)}{V_\infty}$ (In Aero I ohne Beweis)

$$\Delta c_p = c_{pu} - c_{po} = \frac{2\gamma(x)}{V_\infty}$$

$$c_l = \int_0^1 \Delta c_p \, dx$$

$$= \int_0^1 2 \frac{\gamma(x)}{V_\infty} dx = \int_0^\pi 4 \alpha \tan \varphi/2 \cdot (1/2 \sin\varphi) d\varphi$$

$$= 2\alpha \int_0^\pi (1-\cos\varphi) d\varphi = 2\alpha [\pi - 0]$$

$$\boxed{c_{l_{Skt.Th}} = 2\pi\alpha} \quad \boxed{c_{l_{konf.Abb}} = 2\pi \sin\alpha}$$

$$\Rightarrow \boxed{c_{l_\alpha} = 2\pi}$$

3. $c_{l_\alpha} = \frac{\partial c_l}{\partial \alpha} = 2\pi$