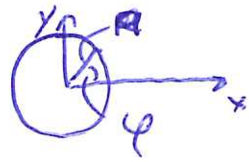
A1) $\zeta = z + \frac{a^2}{z}$ Kreis in z-Ebene



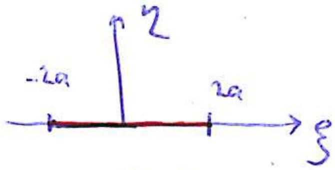
$z = x + iy$

$= a\cos\varphi + ia\sin\varphi$

$= a \cdot e^{i\varphi}$

$$\zeta = a\,e^{i\varphi} + \frac{a^2}{a e^{i\varphi}} = a\left(e^{i\varphi} + e^{-i\varphi}\right) = 2a\cos\varphi + i\cdot 0$$

$\zeta = \xi + i\eta \Rightarrow \xi = 2a\cos\varphi ;\ \eta = 0$



$-2a < \xi < 2a \Rightarrow l = 4a$

$-l/2 < \xi < l/2$ Profillänge

2) Gesucht $w_\zeta(\zeta)$

$$F(z) = u_\infty \left(z + \frac{a^2}{z}\right) + i v_\infty \left(-z + \frac{a^2}{z}\right) + \frac{i\Gamma}{2\pi} \ln(z)$$

$$w_\zeta = \frac{dF}{d\zeta} = \underbrace{\frac{dF}{dz}}_{w_z(z)} \cdot \frac{dz}{d\zeta} \Rightarrow \frac{dF}{dz} = (u_\infty - i v_\infty) - (u_\infty + i v_\infty) \frac{a^2}{z^2} + \frac{i\Gamma}{2\pi z}$$

$$\zeta = z + \frac{a^2}{z} \qquad \frac{dz}{d\zeta} = \frac{1}{\left(\frac{d\zeta}{dz}\right)} = \frac{z^2}{z^2 - a^2}$$

$$\Rightarrow w_\zeta = u_\infty - i v_\infty \left(\frac{z^2 + a^2}{z^2 - a^2}\right) + \frac{i\Gamma}{2\pi} \left(\frac{z}{z^2 - a^2}\right)$$

w_ζ muss Fkt von ζ sein!!!

Umformen üben!

ges. für $w_\zeta(\zeta)$: $\frac{z^2 + a^2}{z^2 - a^2}$ u. $\frac{z}{z^2 - a^2}$

$$\zeta = z + \frac{a^2}{z} = \frac{z^2 + a^2}{z}$$

$$\zeta^2 = \frac{(z^2 + a^2)^2}{z^2} = \frac{z^4 + 2z^2 a^2 + a^4 \overbrace{- 2z^2 a^2 + 2z^2 a^2}^{=0}}{z^2}$$

$$= \frac{(z^2 - a^2)^2}{z^2} + 4a^2$$

$$\frac{z^2 - a^2}{z} = \pm \sqrt{\zeta^2 - 4a^2}$$

$$\frac{z^2 + a^2}{\cancel{z}} \cdot \frac{\cancel{z}}{z^2 - a^2} = \frac{\zeta}{\pm\sqrt{\zeta^2 - 4a^2}}$$

$$w_\zeta(\zeta) = u_\infty - i v_\infty \left[\frac{\pm \zeta}{\sqrt{\zeta^2 - 4a^2}}\right] + \frac{i\Gamma}{2\pi} \frac{\pm 1}{\sqrt{\zeta^2 - 4a^2}}$$

$$= u_\infty \mp \frac{\left(\zeta v_\infty - \frac{\Gamma}{2\pi}\right) i}{\sqrt{\zeta^2 - 4a^2}}$$

$$w_\zeta(\zeta) = u_\infty \overset{\text{oben}}{\underset{\text{unten}}{\mp}} \frac{\left(v_\infty - \frac{\Gamma}{2\pi\zeta}\right) i}{\sqrt{1 - \frac{4a^2}{\zeta^2}}} = u - iv$$

$\zeta = \xi + i\eta \Rightarrow$ Geschw. auf der Platte : $\eta = 0$

$$w_{Platte}(\xi) = u_\infty \mp \frac{i\left(V_\infty \xi - \frac{\Gamma}{2\pi}\right)}{\sqrt{\xi^2 - (l/2)^2}}$$

$l = 4a$

$$w_{Platte}(\xi) = u_\infty \underset{\text{unten}}{\overset{\text{oben}}{\ominus \oplus}} \frac{V_\infty \xi - \frac{\Gamma}{2\pi}}{\sqrt{(l/2)^2 - \xi^2}}$$

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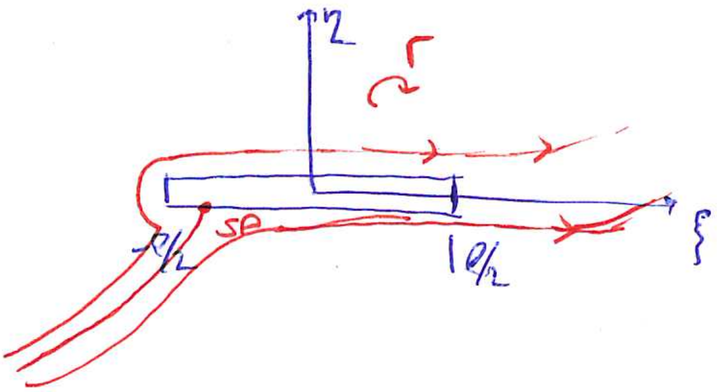
(3) Best. v. Γ, sodass d. Kuttasche Abflussbed. gilt $\Rightarrow$ Glattes Abfließen an d. HK

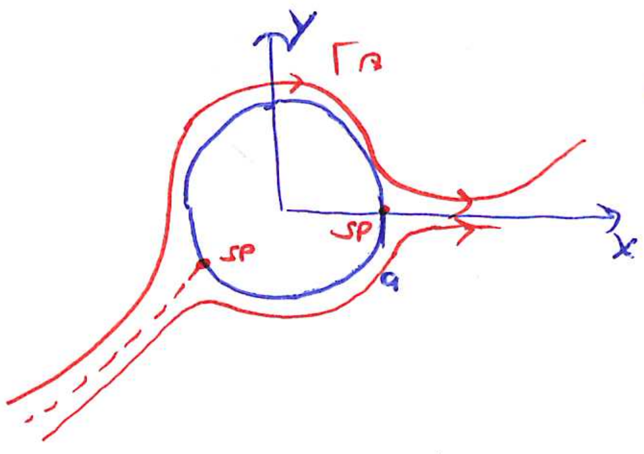
$w_\zeta(HK) \neq 0$!!!

ζ-Ebene



$w_\zeta(HK) \neq 0$

z-Ebene



$w_z(HK) \stackrel{!}{=} 0$

Var I : ζ-Ebene:

Hk: $\xi = \ell/2$

$$w_{\zeta_{\text{Platte}}}(Hk) = u_\infty \mp \frac{v_\infty \ell/2 - \frac{\Gamma}{2\pi}}{\sqrt{(\ell/2)^2 - (\ell/2)^2}}$$

Damit $w_\xi(Hk)$ endlich bleibt muss d. Zähler Null sein $\Rightarrow \boxed{\Gamma = \pi v_\infty \ell}$

Var II : z-Ebene:

$w_z(Hk) = 0$ Hk: $z = a$

$$w_z(z=a) = (u_\infty - iv_\infty) - (u_\infty + iv_\infty)\frac{\cancel{a^2}}{\cancel{a^2}} + \frac{i\Gamma}{2\pi a} = 0$$

$$\hookrightarrow \Gamma = 4a\pi v_\infty = \underline{\ell \pi v_\infty}$$

Γ einsetzen

$$w_{\zeta_{\text{Platte}}} = u_\infty \mp \frac{v_\infty \xi - \frac{v_\infty \ell}{2}}{\sqrt{(\ell/2)^2 - \xi^2}}$$

$$= u_\infty \pm \frac{v_\infty(\ell/2 - \xi)}{\sqrt{(\ell/2)^2 - \xi^2}}$$

$$= u_\infty \pm v_\infty \frac{\sqrt{\ell/2 - \xi}\sqrt{\ell/2 - \xi}}{\sqrt{((\ell/2) - \xi)(\ell/2 + \xi)}}$$

$$= u_\infty \overset{\text{oben}}{\underset{\text{unten}}{\pm}} v_\infty \sqrt{\frac{\ell/2 - \xi}{\ell/2 + \xi}}$$

$w_\xi(\text{Nase}) \Rightarrow \infty$

$\xi = -\ell/2$

4.) c_p und Δc_p:

$$c_p = \frac{p - p_\infty}{\underbrace{\rho/2\, w_\infty^2}_{(u_\infty^2 + v_\infty^2)}}$$

$\Rightarrow$ reibungsfrei + inkompressibel

Bernoulli

$$p_\infty + \rho/2\, w_\infty^2 = p(\xi) + \rho/2\, w_\xi^2$$

$$c_p = 1 - \left(\frac{w_\xi}{w_\infty}\right)^2 \qquad w_{\xi\,\text{platte}} = \underbrace{u_\infty}_{w_\infty \cos\alpha} \pm \underbrace{v_\infty}_{w_\infty \sin\alpha} \sqrt{\frac{l/2 - \xi}{l/2 + \xi}}$$

$$w_{\xi\,\text{Platte}} = w_\infty \Big(\underbrace{\cos\alpha}_{a} \pm \underbrace{\sin\alpha \sqrt{\frac{l/2 - \xi}{l/2 + \xi}}}_{b} \Big)$$

$$= w_\infty (a \pm b)$$

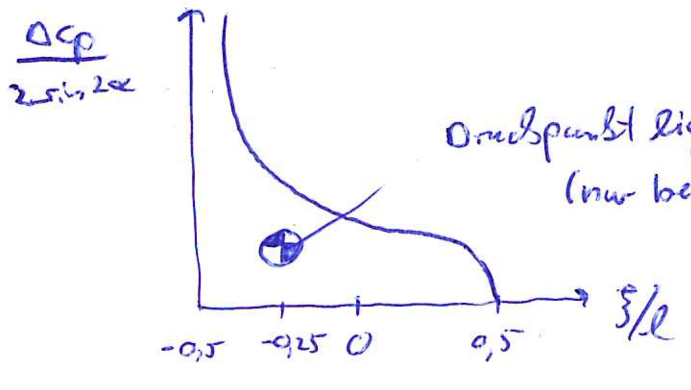
$$c_p = 1 - \left(\cos\alpha \overset{o.}{\underset{u.}{\pm}} \sin\alpha \sqrt{\frac{l/2 - \xi}{l/2 + \xi}} \right)^2$$

$$\Delta c_p = c_{p_u} - c_{p_o} = 1 - \frac{w_{\xi u}^2}{w_\infty^2} - \left(1 - \frac{w_{\xi o}^2}{w_\infty^2} \right) =$$

$$= \frac{w_{\xi o}^2 - w_{\xi u}^2}{w_\infty^2}$$

$$= \frac{\cancel{w_\infty^2} (a+b)^2 - \cancel{w_\infty^2} (a-b)^2}{\cancel{w_\infty^2}} = \underline{4ab}$$

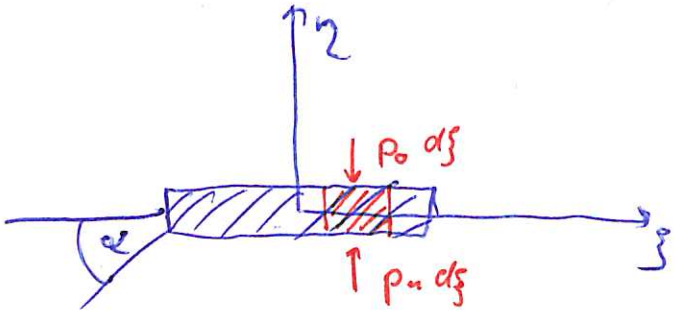
$$\Delta c_p = 4 \cos\alpha \sin\alpha \sqrt{\frac{l/2 - \xi}{l/2 + \xi}} = 2 \sin 2\alpha \sqrt{\frac{l/2 - \xi}{l/2 + \xi}}$$



Druckpunkt liegt bei $l/4$
(nur bei d. Ebenen Platte)

5) Best. der Kräfte:

reibungsfrei! ⇒ $\oint$ Druckverteilung



$$d\hat F_\eta = (p_u - p_o)\, d\xi = \Delta c_p \cdot \rho/2\, w_\infty^2\, d\xi$$

$$\hat F_\eta = \rho/2\, w_\infty^2\, 4 \sin\alpha \cos\alpha \int_{-l/2}^{l/2} \sqrt{\frac{l/2-\xi}{l/2+\xi}}\, d\xi$$

Koordinatentransformation ⇒ Polarkoordinaten

$$\xi = l/2 \cos\varphi \qquad \Big| \quad \xi = -l/2 \ \hat{=}\ \varphi = \pi$$
$$d\xi = -l/2 \sin\varphi\, d\varphi \qquad \Big| \quad \xi = l/2 \ \hat{=}\ \varphi = 0$$

$$\hat F_\eta = 2\rho\, w_\infty^2 \sin\alpha \cos\alpha \left(-l/2 \int_\pi^0 \sqrt{\frac{1-\cos\varphi}{1+\cos\varphi}} \cdot \sin\varphi\, d\varphi\right)$$

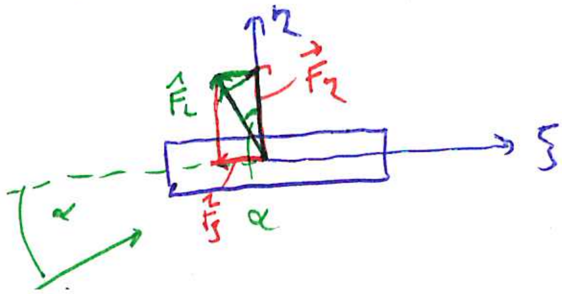
NR: $$\sqrt{\frac{1-\cos\varphi}{1+\cos\varphi}} = \sqrt{\frac{(1-\cos\varphi)(1-\cos\varphi)}{(1-\cos\varphi)\,(1+\cos\varphi)}} = \frac{1-\cos\varphi}{\sqrt{(1-\cos^2\varphi)}} = \frac{1-\cos\varphi}{\sin\varphi}$$

$$\hat F_\eta = \rho\, w_\infty^2 \sin\alpha \cos\alpha\, l \int_0^\pi \cancel{\sin\varphi}\, \frac{1-\cos\varphi}{\cancel{\sin\varphi}}\, d\varphi$$

$$\boxed{\hat F_\eta = l \rho\, w_\infty^2 \sin\alpha \cos\alpha\, \pi}$$

$\hat F_\eta$ ist nicht die Auftriebskraft!!!

⇒ Auftriebskraft steht senkrecht zur Anströmung



$$\boxed{\hat{F}_L = \frac{\hat{F}_{y_2}}{\cos\alpha}}$$

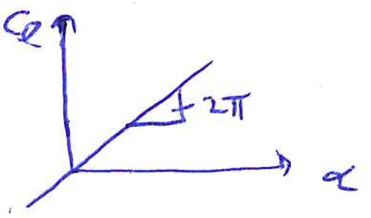
$$\hat{F}_L = \pi l \rho w_\infty^2 \sin\alpha$$

$$\Gamma = \pi V_\infty l = \pi w_\infty \sin\alpha \, l$$

$$\hookrightarrow \boxed{\hat{F}_L = \Gamma \rho w_\infty}$$ – Satz v. Kutta-Zhukowski

$$c_L = \frac{\hat{F}_L}{\rho/_2 \, w_\infty^2 l} = 2\pi \sin\alpha \approx \underline{\underline{2\pi\alpha}}$$

$$\frac{\partial c_L}{\partial \alpha} = 2\pi \cos\alpha \approx \underline{\underline{2\pi}}$$



(6) siehe Punkt 3

Übung 7

Blasiussche Formel

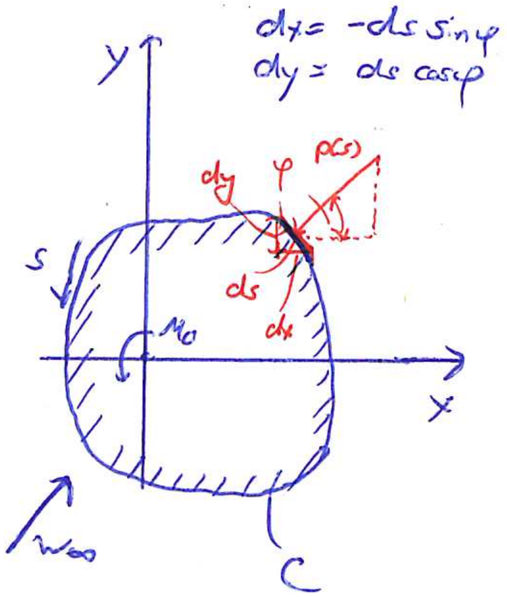
1. Blasiusche Formel: $P = i\, \rho/2 \oint w_z^2 \, dz$ (result. Kraft)
2. Blasiusche Formel: $M_0 = -\rho/2 \; Re \oint w^2 z \, dz$ (result. Moment)

Herleitung

Voraussetzungen: - zylindrischer Körper
- inkompressible, reibungslose Strömung

Kraft:

$$dP_x = -p\cos\varphi \, ds = -p\, dy$$
$$dP_y = -p\sin\varphi \, ds = p\, dx$$



$$\hookrightarrow \quad P_x = -\oint_C p\,dy \; ; \; P_y = \oint_C p\,dx \quad (*)$$

Moment auf Ursprung

$$dM_0 = x\, dP_y - y\, dP_x = p(x\,dx + y\,dy)$$
$$M_0 = \oint_C p(x\,dx + y\,dy) \quad (**)$$

Körperkontur ist Stromlinie

$$\hookrightarrow \quad \frac{dx}{dy} = \frac{u}{v} \Rightarrow u\,dy - v\,dx = 0$$

$$(*) \quad P_x = -\oint_C p\,dy - \rho \oint_C (u^2 dy - uv\,dx)$$
$$P_y = \oint_C p\,dx + \rho \oint_C (v^2 dx - uv\,dy)$$

mit Bernoulli: $p = \underbrace{p_{g\infty}}_{p_\infty + \rho/2\, w_\infty^2} - \rho/2 (u^2 + v^2)$ einsetzen

$$\oint p_{g\infty}\, dx = \oint p_{g\infty}\, dy = 0$$

$$\hookrightarrow \quad P_x = -\rho/2 \oint_C \left[(u^2 - v^2)\, dy - 2uv\, dx\right]$$
$$P_y = -\rho/2 \oint_C \left[(u^2 - v^2)\, dx + 2uv\, dy\right]$$

mit $w(z) = u - iv$; $dz = dx + i\,dy$

↳ komplexe Kraft $P = P_x - iP_y$

$$\Rightarrow \quad P = i\,\frac{\rho}{2} \oint_C w^2\,dz$$

Für die Kraft in der ζ-Ebene gilt:

$$P_\zeta = P_\xi - iP_\eta = i\,\frac{\rho}{2} \oint_P w_\zeta^2\,d\zeta$$

Integral über Profilkontur

$$↳ \quad w_\zeta = w_z \frac{dz}{d\zeta}$$

$$P_\xi - iP_\eta = i\,\frac{\rho}{2} \oint_{(P)} w_z^2 \left(\frac{dz}{d\zeta}\right)^2 d\zeta = i\,\frac{\rho}{2} \oint_{(K)} w_z^2 \frac{dz}{d\zeta}\,dz$$

Kreis K in z-Ebene

(**) $M_0 = \oint_{(C)} p(x\,dx + y\,dy) -$

$p = p_0 - \frac{\rho}{2}(u^2+v^2)$ | $- \rho \oint_{(C)} \left[x(v^2dx + uv\,dy) + y(uv\,dx - u^2dy)\right]$

$$↳ \quad M_0 = -\frac{\rho}{2} \oint_{(C)} (u^2 - v^2)(x\,dx - y\,dy) + 2uv(x\,dy + y\,dx)$$

↳ Komplexe Ebene

$$M_0 = -\frac{\rho}{2}\,\mathrm{Re} \oint_{(C)} w_z^2\, z\,dz$$
