Übung 8

$\boxed{A3}$ 1. Ges.: $w_z(z)$

$$F(z) = w_\infty e^{-i\alpha}(z-z_0) + w_\infty e^{i\alpha}\frac{R^2}{z-z_0} + \frac{i\Gamma}{2\pi}\ln(z-z_0)$$

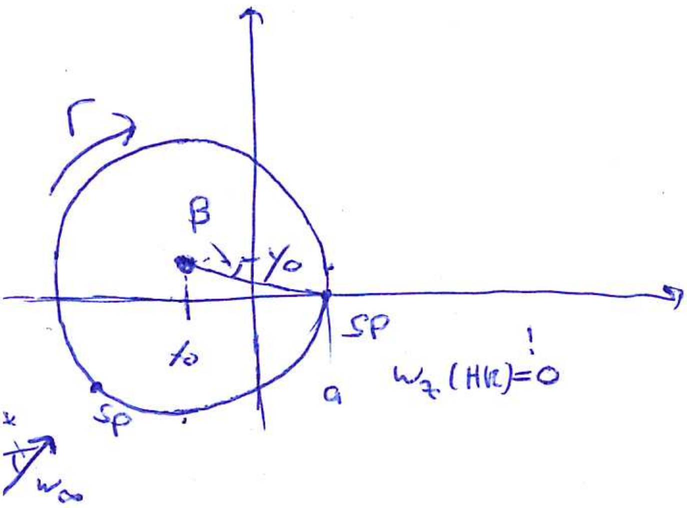
$$z_0 = x_0 + iy_0$$

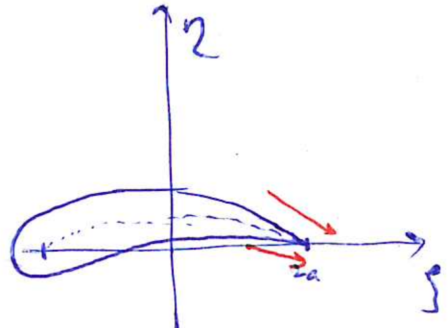
$$w_z(z) = \frac{dF}{dz} = w_\infty e^{-i\alpha} - \frac{w_\infty e^{i\alpha}R^2}{(z-z_0)^2} + \frac{i\Gamma}{2\pi(z-z_0)}$$

2) Kutta'sche Abflussbed.

z - Ebene

ζ - Ebene





HK bei: $z = a,\ \varphi = -\beta$

$$z_{HK} = z_0 + Re^{-i\beta} = a$$

$$w_z(z=a) \overset{!}{=} 0 = w_\infty e^{-i\alpha} - \frac{w_\infty e^{i\alpha}R^2}{(Re^{-i\beta})^2} + \frac{i\Gamma}{2\pi Re^{-i\beta}}$$

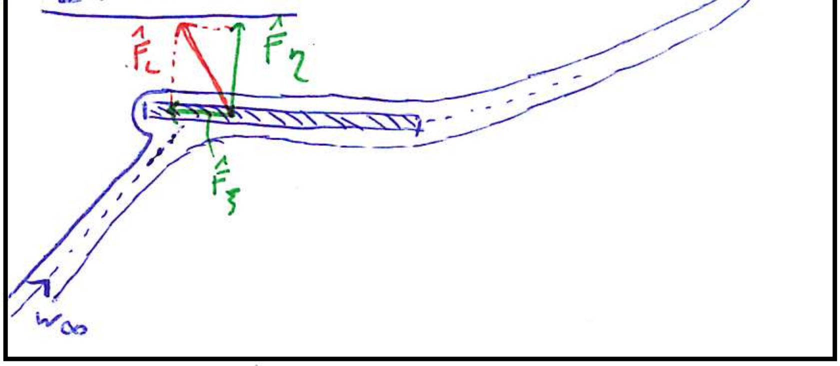
$$= e^{i\beta}\Big(w_\infty \underbrace{e^{-i\alpha}e^{-i\beta}}_{e^{-i(\alpha+\beta)}} - w_\infty \underbrace{e^{i\alpha}e^{i\beta}}_{e^{i(\alpha+\beta)}} + \frac{i\Gamma}{2\pi R}\Big) = 0$$

$$\Leftrightarrow \quad w_\infty\big(\cancel{\cos(\alpha+\beta)} - i\sin(\alpha+\beta) - \cancel{\cos(\alpha+\beta)} - i\sin(\alpha+\beta) + \frac{i\Gamma}{2\pi R} = 0$$

$$\Leftrightarrow \quad -i\,2w_\infty \sin(\alpha+\beta) + \frac{i\Gamma}{2\pi R} = 0$$

$$\boxed{\Gamma = 4\pi R\, w_\infty \sin(\alpha+\beta)}$$

③ Ebene Platte



$$F_\zeta = F_\xi - iF_\eta = \frac{i\rho}{2} \oint w_z^2 \frac{dz}{d\zeta}\, dz$$

$$w_z^2 = \Big(\underbrace{w_\infty e^{-i\alpha}}_{A_0} \underbrace{- \frac{w_\infty e^{i\alpha} R^2}{(z-z_0)^2}}_{A_2} + \frac{\overbrace{i\Gamma}^{}}{\underbrace{2\pi}_{A_1}(z-z_0)} \Big)^2$$

$$w_z^2 = \left(A_0 + \frac{A_1}{(z-z_0)} + \frac{A_2}{(z-z_0)^2} \right)^2$$

mit Hinweis: $\frac{1}{z-z_0} \approx \frac{1}{z} + \frac{z_0}{z^2}$

und $\frac{1}{(z-z_0)^2} \approx \frac{1}{z^2}\left(1 + \frac{2z_0}{z}\right)$

$$w_z^2 = \left(A_0 + \frac{A_1}{z} + \frac{A_1 z_0}{z^2} + \frac{A_2}{z^2} + \frac{2A_2 z_0}{z^3} \right)^2$$

$$= A_0^2 + \frac{2A_0A_1}{z} + \frac{A_1^2 + 2A_0A_1z_0 + 2A_2A_0}{z^2} + \mathcal{O}(z^{-3})$$

$$\frac{dz}{d\zeta} = \frac{1}{(d\zeta/dz)}$$

$$\frac{d\zeta}{dz} = 1 - \frac{a_1}{z^2} - \frac{2a_2}{z^3} - \frac{3a_3}{z^4} \dots$$

$$\frac{dz}{d\zeta} = \frac{1}{1 - \left(\frac{a_1}{z^2} + \frac{2a_2}{z^3} + \frac{3a_3}{z^4} \dots\right)}$$

mit Hinweis: $\frac{1}{1-\varepsilon} \approx 1 + \varepsilon$

$$\frac{dz}{d\zeta} = 1 + \frac{a_1}{z^2} + \mathcal{O}(z^{-3})$$

$$\frac{dz}{d\zeta} = 1 + \frac{a_1}{z^2} + \mathcal{O}(z^{-3})$$

$$F_\zeta = \frac{i\rho}{2} \oint \left(\underbrace{A_0^2}_{B_0} + \frac{\overbrace{2A_0A_1}^{B_1}}{z} + \frac{(\ldots)}{z^2} \right) \left(1 + \frac{a_1}{z} + \ldots\right) dz$$

$$F_\zeta = \frac{i\rho}{2} (i\,2\pi) \cdot \underbrace{2A_0A_1}_{B_1} = -2\pi\rho A_0A_1$$

Einsetzen von A_0 u. A_1

$$F_\zeta = -i\rho w_\infty \Gamma e^{-i\alpha} = \rho w_\infty \Gamma e^{-i(\alpha + \pi/2)}$$

4 $F_L = \| F_\zeta \| = \| \rho w_\infty \Gamma e^{-i(\alpha + \pi/2)} \|$

$$= \rho w_\infty \Gamma$$

Satz von Kutta Zhukhovski

$$F_L = 4\pi R \rho w_\infty^2 \sin(\alpha + \beta)$$