Investigations of Nasal Cavity Flows based on a Lattice-Boltzmann Method

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Outline

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• Lattice-Boltzmann Method (LBM)
• Results
• Extensions of the LBM
  • Local grid refinement
  • Heat diffusion and convection with a Thermal LBM
• Conclusion
Introduction

Anatomy of the nasal cavity

Images from http://www.ghorayeb.com/AnatomySinuses.html

- Surgically alleviate breathing problems caused by
  - deformation
  - allergies
  - malformations
Examples of pathological nasal cavities:

- deformation (rhino-anaplasty)
- malformation

What is the best surgical method for the individual patient?

**Analyze flow with simulations to support surgical decision process**
Lattice-Boltzmann method (LBGK)

- Gas-kinetic approach for the simulation of continuum flows
- Advantages:
  - efficient for the simulation of complex internal flows
  - easy automatic grid generation
- Solve LBGK-equation iteratively for PPDFs

\[ f_i (\vec{x} + \xi_i \delta t, t + \delta t) = f_i (\vec{x}, t) + \omega \delta t \cdot (f_{eq}^i (\vec{x}, t) - f_i (\vec{x}, t)) \]

equilibrium distribution function:

\[ f^{eq} (x_\alpha, t) = \rho t_p \left[ 1 + \frac{v_\alpha \xi_\alpha}{c_s^2} + \frac{v_\alpha v_\beta}{2c_s^2} \left( \frac{\xi_\alpha \xi_\beta}{c_s^2} - \delta_{\alpha\beta} \right) \right] \]

\[ \xi_i = \xi_0 \begin{cases} (\mp 1, 0, 0) ; (0, \mp 1, 0) ; (0, 0, \pm 1) & \alpha = 0..5 \\ (\mp 1, \mp 1, 0) ; (\mp 1, 0, \pm 1) ; (0, \pm 1, \mp 1) & \alpha = 6..17 \\ (0, 0, 0) & \alpha = 18 \end{cases} \]
Lattice-Boltzmann method (LBGK)

- Solution algorithm has two steps
  - collision step
    \[ f_{i}^{\text{coll}}(\vec{x}, t) = f_{i}(\vec{x}, t) + \omega \delta t \cdot (f_{i}^{\text{eq}}(\vec{x}, t) - f_{i}(\vec{x}, t)) \]
  - propagation step
    \[ f_{i}^{\text{prop}}(\vec{x} + \xi_{i} \delta t, t + \delta t) = f_{i}^{\text{coll}}(\vec{x}, t) \]

- Macroscopic variables from moments of particle prob. dist. funct.

  - density \( \rho = m \sum_{i=0}^{18} f_{i} = m \sum_{i=0}^{18} f_{i}^{\text{eq}} \)
  - momentum \( \rho v_{\alpha} = m \sum_{i=0}^{18} \xi_{i,\alpha} f_{i} = m \sum_{i=0}^{18} \xi_{i,\alpha} f_{i}^{\text{eq}} \)
  - energy \( \rho E = \frac{m}{2} \sum_{i=0}^{18} \xi_{i,\alpha}^{2} f_{i} = \frac{m}{2} \sum_{i=0}^{18} \xi_{i,\alpha}^{2} f_{i}^{\text{eq}} \)
Nasal cavity flows

- Application: Flow in a nasal cavity
- $Re = 710$ based on 125 ml/sec per averaged nostril

<table>
<thead>
<tr>
<th>$\frac{\delta p}{p_0}$</th>
<th>$ch_{left}$</th>
<th>$ch_{right}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00204</td>
<td>0.00192</td>
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</tbody>
</table>
Nasal cavity flows

- Backflow regimes and vortices
Nasal cavity flows
Wall shear stresses

- Calculation via derivative of 2nd order polynomial

\[ \tau_w = \eta \frac{du}{dy} \bigg|_{y=0} = \eta \cdot \Pi' \bigg|_{y=0}, \quad \Pi \text{ is the polynomial} \]
Wall shear stresses

- Calculation via derivative of 2nd order polynomial

\[ \tau_{w} = \eta \frac{du}{dy} \bigg|_{y=0} \]

These regions are potential candidates for surgical interventions
Local grid refinement (Hänel etc.)

- Overlay of cells from different levels are required
- Reconstruction required:
  - obtain macroscopic variables by bi- or tri-linear interpolation

Available PPDFs on both levels

Missing PPDFs from different level


Local grid refinement (Hänel etc.)

- Keep viscosity constant across levels

\[ \tau_f = m \left( \tau_c - \frac{1}{2} \right) + \frac{1}{2}, \quad m = \frac{\delta x_c}{\delta x_f} \]

- Reconstruction of incoming PPDFs
- Split of PPDFs in equilibrium and non-equilibrium parts

\[ f_{in}^{eq} = f_{i}^{eq} + f_{i}^{neq} \]

\[ f_{in,f}^{eq} = f_{i}^{eq} + \left( f_{in,c}^{eq} - f_{i}^{eq} \right) \frac{1}{m} \frac{\tau_f}{m} \frac{\tau_c}{\tau_f} \]

\[ f_{i}^{eq} = f_{i}^{eq} + \left( f_{in,f}^{eq} - f_{i}^{eq} \right) \frac{1}{m} \frac{\tau_f}{m} \frac{\tau_c}{\tau_f} \]

(transformation factors)
Nasal cavity flows with local grid refinement

- Same setup:
  - $Re = 710$ based on $125\text{ ml/sec per averaged nostril}$

- Simulation with three distinct grids:

<table>
<thead>
<tr>
<th></th>
<th>$G_s$</th>
<th>$G_{m2}$</th>
<th>$G_{m3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no levels</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>no cells</td>
<td>$4.744 \times 10^7$</td>
<td>$4.335 \times 10^7$</td>
<td>$2.277 \times 10^7$</td>
</tr>
<tr>
<td>reduction</td>
<td>0.0%</td>
<td>8.6%</td>
<td>52.0%</td>
</tr>
</tbody>
</table>
Nasal cavity flows with local grid refinement

right cavity

frontal cross section

partially removed right lower turbinate

left lower turbinate

right nostril

throat
Heat convection / diffusion with a TLBM

- To analyze heating capability solve convection-diffusion equation:

\[
\frac{\partial \rho_0 h}{\partial t} + \rho_0 v_\alpha h = \frac{\partial}{\partial x_\alpha} \left( \lambda \frac{\partial T}{\partial x_\alpha} \right)
\]

- Additionally solve Thermal LBGK-equation

\[
g_i (\vec{x} + \xi_i \delta t, t + \delta t) = g_i (\vec{x}, t) + \Omega_g \cdot (g^{eq}_i (\vec{x}, t) - g_i (\vec{x}, t))
\]

\[
g^{eq}_i (x_\alpha, t) = T t_p \left[ 1 + \frac{v_\alpha \xi_\alpha}{c_s^2} + \frac{v_\alpha v_\beta}{2 c_s^2} \left( \frac{\xi_\alpha \xi_\beta}{c_s^2} - \delta_{\alpha\beta} \right) \right]
\]

- obtain temperature by moment

\[
temperature \ T = \sum_{i=0}^{18} g_i = \sum_{i=0}^{18} g^{eq}_i
\]
Heat convection / diffusion with a TLBM

- First results \((Pr = 0.72)\):
Conclusion

• Analysis of the flow in the nasal cavity based on
  • velocity and wall shear stress distributions
  • streamline characteristics
  • backflow and vortex characteristics

• Local grid refinement:  
  • 8.6% reduction for two levels
  • 52.0% reduction for three levels
  • similar profiles compared to uniform grid solution

⇒ more efficient computation

• Heat convection / diffusion allows analysis of heating capability

⇒ will extent current simulations

• Fast and easy mesh generation with a Cartesian grid generator

⇒ efficient tool to simulate flows in highly intrinsic geometries
Thank you for your attention