Tutorenprogramm - Strömungsmechanik II

Schleichende Strömungen

1. Aufgabe

1. Zweifache Integration:

$$\frac{\partial u}{\partial y} = \frac{1}{\eta}\frac{dp}{dx}y + C_1$$

$$u = \frac{1}{\eta}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2$$

Randbedingungen:

$$\begin{aligned} y = 0: &\quad u = u_\infty,\ v = 0 \\ y = h(x): &\quad u = v = 0 \end{aligned}$$

$$\Rightarrow \quad C_2 = u_\infty, \quad C_1 = -\frac{1}{h}\left(u_\infty + \frac{1}{\eta}\frac{dp}{dx}\frac{h^2}{2}\right)$$

$$\Rightarrow u = u_\infty\left(1 - \frac{y}{h}\right) - \frac{dp}{dx}\frac{h^2}{2\eta}\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

Konstanter Volumenstrom: $\dot{V} = \int_0^{h(x)} u dy = \text{konst.} = \frac{u_\infty h}{2} - \frac{h^3}{12\eta}\frac{dp}{dx}$

$$\Leftrightarrow \quad \frac{dp}{dx} = 12\eta\left(\frac{u_\infty}{2h^2} - \frac{\dot{V}}{h^3}\right)$$

Integration mit $p(0) = p(L) = p_\infty$:

$$p(L) = p_\infty = p_\infty + 6\eta u_\infty \int_0^L \frac{dx}{h^2(x)} - 12\eta\dot{V}\int_0^L \frac{dx}{h^3(x)}$$

$$\Rightarrow \dot{V} = \frac{1}{2}u_\infty \frac{\int_0^L \frac{dx}{h^2(x)}}{\int_0^L \frac{dx}{h^3(x)}} = \frac{1}{2}u_\infty H$$

Spaltgeometrie: $h(x) = h_1 - \frac{h_1 - h_2}{L}x$

$$\int_0^L \frac{dx}{h^2(x)} = \frac{L}{h_1 - h_2}\left(\frac{1}{h_2} - \frac{1}{h_1}\right)$$

$$\int_0^L \frac{dx}{h^3(x)} = \frac{L}{2(h_1 - h_2)}\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)$$

$$\Rightarrow H = \frac{\frac{L}{h_1 - h_2}\left(\frac{1}{h_2} - \frac{1}{h_1}\right)}{\frac{L}{2(h_1 - h_2)}\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)} = \frac{2h_1h_2}{h_1 + h_2}$$

$$\Rightarrow \dot{V} = \frac{1}{2}u_\infty \frac{2h_1h_2}{h_1 + h_2}$$

2. $\dot{V}$ einsetzen: $\frac{dp}{dx} = \frac{6\eta u_\infty}{h^2}\left(1 - \frac{H}{h}\right)$

   Maximaler Druck $p = p_{max}$ für $h(x) = H$:

   $h_1 - \dfrac{h_1 - h_2}{L}x = \dfrac{2h_1h_2}{h_1 + h_2}$

   $\Rightarrow x = \dfrac{h_1(h_1 + h_2) - 2h_1h_2}{h_1 + h_2}\dfrac{L}{h_1 - h_2} = \dfrac{Lh_1}{h_1 + h_2}$

Quelle: Herbst 2010

2. Aufgabe

Druck: $p(x,y) = p_a + \rho g(h(x) - y) \quad \Rightarrow \quad \frac{\partial p}{\partial x} = \rho g \frac{\partial h(x)}{\partial x}$

Geschwindigkeit:

Zweifache Integration:

$$\frac{\partial u(x,y)}{\partial y} = \frac{1}{\eta}\frac{\partial p}{\partial x} y + C_1$$

$$u(x,y) = \frac{1}{\eta}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

Randbedingungen:

$$y = 0 : u = 0$$

$$y = h(x) : \frac{\partial u}{\partial y} = 0$$

$$\Rightarrow \quad C_2 = 0, \quad C_1 = -\frac{1}{\eta}\frac{\partial p}{\partial x} h(x)$$

$$\Rightarrow \quad u(x,y) = -\frac{1}{2\eta}\frac{\partial p}{\partial x} h^2(x)\left(2 - \frac{y}{h(x)}\right)\frac{y}{h(x)}$$

$$\Rightarrow \quad u(x,y) = -\frac{1}{2\eta}\rho g \frac{\partial h(x)}{\partial x} h^2(x)\left(2 - \frac{y}{h(x)}\right)\frac{y}{h(x)}$$

konstanter Volumenstrom $\dot{V}(x) = \text{konst.}$:

$$\dot{V} = B\int_0^{h(x)} u(x,y)dy \neq f(x)$$

$$\Rightarrow \quad \dot{V} = -B\frac{1}{2\eta}\rho g \frac{\partial h(x)}{\partial x} h^2(x)\left[\frac{2}{h(x)}\frac{y^2}{2} - \frac{y^3}{3h^2(x)}\right]_0^{h(x)}$$

$$\Rightarrow \quad \dot{V} = -B\frac{1}{3\eta}\rho g \frac{\partial h(x)}{\partial x} h^3(x)$$

Differentialgleichung für $h(x)$, Integration:

$$\int_0^x -\dot{V}\frac{3\eta}{\rho g B}dx = \int_{h_0}^{h(x)} h^3 dh$$

$$\Rightarrow \quad -\dot{V}\frac{3\eta}{\rho g B}x = \frac{h^4(x)}{4} - \frac{h_0^4}{4}$$

$$\Rightarrow \quad h(x) = \sqrt[4]{-\dot{V}\frac{12\eta}{\rho g B}x + h_0^4}$$

gültig für $x \ll 0$ (ausreichende Entfernung zur Vorderkante)

Quelle: Frühjahr 2011