

Tutorenprogramm - Strömungsmechanik II  
Sommersemester 2013  
Schleichende Strömungen

1. Aufgabe

1. Zweifache Integration:

$$\frac{\partial u}{\partial y} = \frac{1}{\eta} \frac{dp}{dx} y + C_1$$

$$u = \frac{1}{\eta} \frac{dp}{dx} \frac{y^2}{2} + C_1 y + C_2$$

Randbedingungen:

$$y = 0 : \quad u = u_\infty, v = 0$$

$$y = h(x) : \quad u = v = 0$$

$$\Rightarrow C_2 = u_\infty, \quad C_1 = -\frac{1}{h} \left( u_\infty + \frac{1}{\eta} \frac{dp}{dx} \frac{h^2}{2} \right)$$

$$\Rightarrow u = u_\infty \left( 1 - \frac{y}{h} \right) - \frac{dp}{dx} \frac{h^2}{2\eta} \frac{y}{h} \left( 1 - \frac{y}{h} \right)$$

Konstanter Volumenstrom:  $\dot{V} = \int_0^{h(x)} u dy = \text{konst.} = \frac{u_\infty h}{2} - \frac{h^3}{12\eta} \frac{dp}{dx}$

$$\Leftrightarrow \frac{dp}{dx} = 12\eta \left( \frac{u_\infty}{2h^2} - \frac{\dot{V}}{h^3} \right)$$

Integration mit  $p(0) = p(L) = p_\infty$ :

$$p(L) = p_\infty = p_\infty + 6\eta u_\infty \int_0^L \frac{dx}{h^2(x)} - 12\eta \dot{V} \int_0^L \frac{dx}{h^3(x)}$$

$$\Rightarrow \dot{V} = \frac{1}{2} u_\infty \frac{\int_0^L \frac{dx}{h^2(x)}}{\int_0^L \frac{dx}{h^3(x)}} = \frac{1}{2} u_\infty H$$

Spaltgeometrie:  $h(x) = h_1 - \frac{h_1 - h_2}{L} x$

$$\int_0^L \frac{dx}{h^2(x)} = \frac{L}{h_1 - h_2} \left( \frac{1}{h_2} - \frac{1}{h_1} \right)$$

$$\int_0^L \frac{dx}{h^3(x)} = \frac{L}{2(h_1 - h_2)} \left( \frac{1}{h_2^2} - \frac{1}{h_1^2} \right)$$

$$\Rightarrow H = \frac{\frac{L}{h_1 - h_2} \left( \frac{1}{h_2} - \frac{1}{h_1} \right)}{\frac{L}{2(h_1 - h_2)} \left( \frac{1}{h_2^2} - \frac{1}{h_1^2} \right)} = \frac{2h_1 h_2}{h_1 + h_2}$$

$$\Rightarrow \dot{V} = \frac{1}{2} u_\infty \frac{2h_1 h_2}{h_1 + h_2}$$

2.  $\dot{V}$  einsetzen:  $\frac{dp}{dx} = \frac{6\eta u_\infty}{h^2} \left(1 - \frac{H}{h}\right)$

Maximaler Druck  $p = p_{max}$  für  $h(x) = H$ :

$$h_1 - \frac{h_1 - h_2}{L}x = \frac{2h_1h_2}{h_1 + h_2}$$
$$\Rightarrow x = \frac{h_1(h_1 + h_2) - 2h_1h_2}{h_1 + h_2} \frac{L}{h_1 - h_2} = \frac{Lh_1}{h_1 + h_2}$$

Quelle: Herbst 2010

## 2. Aufgabe

$$\text{Druck: } p(x, y) = p_a + \rho g(h(x) - y) \quad \Rightarrow \quad \frac{\partial p}{\partial x} = \rho g \frac{\partial h(x)}{\partial x}$$

Geschwindigkeit:

Zweifache Integration:

$$\frac{\partial u(x, y)}{\partial y} = \frac{1}{\eta} \frac{\partial p}{\partial x} y + C_1$$

$$u(x, y) = \frac{1}{\eta} \frac{\partial p}{\partial x} \frac{y^2}{2} + C_1 y + C_2$$

Randbedingungen:

$$y = 0 : u = 0$$

$$y = h(x) : \frac{\partial u}{\partial y} = 0$$

$$\Rightarrow C_2 = 0, \quad C_1 = -\frac{1}{\eta} \frac{\partial p}{\partial x} h(x)$$

$$\Rightarrow u(x, y) = -\frac{1}{2\eta} \frac{\partial p}{\partial x} h^2(x) \left(2 - \frac{y}{h(x)}\right) \frac{y}{h(x)}$$

$$\Rightarrow u(x, y) = -\frac{1}{2\eta} \rho g \frac{\partial h(x)}{\partial x} h^2(x) \left(2 - \frac{y}{h(x)}\right) \frac{y}{h(x)}$$

konstanter Volumenstrom  $\dot{V}(x) = \text{konst.}$ :

$$\dot{V} = B \int_0^{h(x)} u(x, y) dy \neq f(x)$$

$$\Rightarrow \dot{V} = -B \frac{1}{2\eta} \rho g \frac{\partial h(x)}{\partial x} h^2(x) \left[ \frac{2}{h(x)} \frac{y^2}{2} - \frac{y^3}{3h^2(x)} \right]_0^{h(x)}$$

$$\Rightarrow \dot{V} = -B \frac{1}{3\eta} \rho g \frac{\partial h(x)}{\partial x} h^3(x)$$

Differentialgleichung für  $h(x)$ , Integration:

$$\int_0^x -\dot{V} \frac{3\eta}{\rho g B} dx = \int_{h_0}^{h(x)} h^3 dh$$

$$\Rightarrow -\dot{V} \frac{3\eta}{\rho g B} x = \frac{h^4(x)}{4} - \frac{h_0^4}{4}$$

$$\Rightarrow h(x) = \sqrt[4]{-\dot{V} \frac{12\eta}{\rho g B} x + h_0^4}$$

gültig für  $x \ll 0$  (ausreichende Entfernung zur Vorderkante)

Quelle: Frühjahr 2011